Published by :
http://www.ijert.org

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 04, April-2016

Study of Static and Modal Analysis of Un-Crack and Crack Cantilever Beam using Fea

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Abstract **-Every faulty structure subjected to change in local flexibility which affects vibration response of the structure. Therefore it is necessary to detect the faults in the structure with its position. In this paper, aluminum un-crack and crack cantilever beam is used for analysis. Firstly, theoretical calculations had been done to obtain first three natural frequencies by solving the Euler equation for un-crack beam and cracked beam considering various crack positions for the beam. Secondly, static and modal analysis of un-crack and crack beam is performed to find deflection and natural frequencies of beam. Finally, the results are presented in tabular form to show the effect of crack in change of deflection and lower natural frequencies. ANSYS software was used for the analysis of un-crack and crack beam.**

Key words: ANSYS, Cantilever beam, Free Vibration, Static Analysis.

1. INTRODUCTION

Motion which repeats itself after a certain interval of time is called vibration. A vibration can caused due to external unbalanced force also. An unbalanced force created due to crack occurs into structure. Engineer's important task is to determine the effect of these damages on the stability characteristic of these structures. The presence of crack leads to reduction in stiffness also with an inherent reduction in natural frequency and increase in modal damping. The study of vibration analysis of cantilever beam with crack is a problem of practical interest and finds applications in aerospace, mechanical and civil engineering. Health monitoring and the analysis of damage in the form of crack in beam structures posses a vital mean.

In the literature, several studies deal with the effect of crack, crack detection by various methods.

Yang et al. performed the analytical study on the free and forced vibration of inhomogeneous Euler-Bernoulli beams containing open edge cracks. Analytical solutions were obtained for cantilever, with different end conditions to evaluate the dynamic response of the beam due to the edge crack.

Salawu and Williams presented an excellent review on the use of modal frequency changes for damage diagnostics. The observation that changes in structural properties cause changes in vibration frequencies was the impetus for using modal methods for damage identification and health monitoring.

The free and forced vibration analysis of a cracked beam was performed by S Orhan et al in order to identify the cracks in a cantilever beam. The study results suggest that free vibration analysis provides suitable information for the detection of single and two cracks. The Euler– Bernoulli beam model was assumed for the analysis.

R. Tiwari and M. Karthikey developed identification procedure for the detection, localization, and size of a crack in a beam based on forced response measurements. E Cam, S Orhan et. al used the impact echo method for the analysis of cracked beam structure. They analyzed vibrations as a result of impact shocks. ANSYS software was used for validation of results and simulations.

Kim and Zhao proposed a novel crack detection method using harmonic response. They conclude that slope response has a sharp change with the crack location and depth of the crack, and therefore it can used as a crack detection criterion.

S. Loutridis, E.Douk developed new method for crack detection in beams based on instantaneous frequency and empirical mode decomposition. It follows that the harmonic distortion increases with crack depth following definite trends and can be also used as an effective indicator for crack size.

Saavedra and Cuitio has performed the theoretical and experimental dynamic behaviors of different multi beams systems containing a transverse crack. In their analysis an analytical model of a cracked cantilever beam has been utilized, and natural frequencies are obtained through numerical methods. A genetic algorithm is utilized to monitor the possible changes in the natural frequencies of the structure.

Zheng and Kessissoglou have presented a method based on finite element method for detection of crack in faulty structural member. The results obtained from the proposed method are validated using experimental analysis.

In this paper Euler- Bernoulli beams is analyzed. First the natural frequency and deflection of Un-crack and crack cantilever beam is obtained using ANSYS with various conditions of beam with crack. The paper gives the effect of crack on natural frequency and deflection of beam with crack depth at different location. The study implies how natural frequency and deflection changes as the structure affected due to damage.

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2. THEORY

The stability and local flexibility of the beam depends on the material properties, physical dimensions, boundary conditions of the structure which play important role for the determination of its dynamic response. The characteristics of beam greatly depend on the position of crack, depth of crack, orientation of crack and number of cracks.

The beam with rectangular crack is clamped at left end and free at right end and uniform in rectangular cross section along its length. The crack is assumed to be open crack and no damping is considered in this study.

2.1 Governing Equation for Free Vibration of Beam:

The differential equation for transverse vibration of thin uniform beam is obtained with the help of strength of materials. The beam has cross section area A, flexural rigidity EI and density of material ρ.

Consider the small element dx of beam is subjected to shear force Q and bending moment M, as shown in figure

While deriving mathematical expression for transverse vibration, it is assumed that there are no axial forces acting on the beam and effect of shear deflection is neglected. The deformation of beam is assumed due to moment and shear force.

The net force acting on the element,

$$Q - \left(Q + \frac{\partial Q}{\partial x}dx\right) = dm * \text{acceleration}$$

$$-\frac{\partial Q}{\partial x}dx = (\rho A dx)\frac{\partial^2 y}{\partial t^2}$$

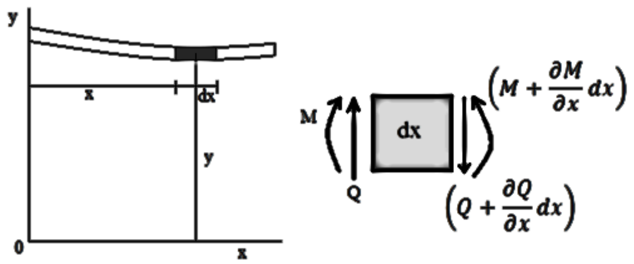

Fig.1. Shear Force and Bending Moment acting on beam element

$$\frac{\partial Q}{\partial x} + \rho A\frac{\partial^2 y}{\partial t^2} = 0$$

Considering the moments about A, we get

$$M - \left(M + \frac{\partial M}{\partial x}dx\right) + \left(Q + \frac{\partial Q}{\partial x}dx\right)dx = 0$$

$$-\frac{\partial M}{\partial x} + Q + \frac{\partial Q}{\partial x}dx = 0$$

So $Q = \frac{\partial M}{\partial x}$ higher order derivatives are neglected here $\left(\frac{\partial Q}{\partial x}dx = 0\right)$

Or

$$\frac{\partial Q}{\partial x} = \frac{\partial^2 M}{\partial x^2}$$

From the above two equations 1 and 2, we get

$$\frac{\partial^2 M}{\partial x^2} = \rho A\frac{\partial^2 y}{\partial t^2}$$

We know from strength of materials that

$$M = -EI\frac{\partial^2 y}{\partial x^2}$$

So

$$\frac{\partial^2 M}{\partial x^2} = -EI\frac{\partial^4 y}{\partial x^4}$$

Comparing equation 3and 4 we get,

$$\frac{\partial^4 y}{\partial x^4} + \left(\frac{\rho A}{EI}\right)\frac{\partial^2 y}{\partial t^2} = 0$$

This is the general equation for transverse vibration. Thus the natural frequency can be found out by this theory as,

$$\omega_n = C * \sqrt{\frac{EI}{\rho A l^4}}$$

Where,

E= Young's modulus of the material,

I= Moment of inertia,

A= Area of cross section,

l=length of the beam,

C= Constant depending mode of vibration,

C1=0.56 for first mode,

C2= 3.52 for second mode,

C3= 9.82 for third mode.

The moment of inertia can be found out by relation,

$$I = \frac{bd^3}{12}$$

Where,

b= width of the beam,

d= depth of the beam.

Due to presence of crack, moment of inertia of the beam changes and correspondingly the natural frequency also changes. For a constant beam material and cross section the reduced moment of inertia will be found by relation below.

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$$I_1 = I - I_c$$

Where,

I1=Moment of inertia of a cracked beam,

I=Moment of inertia of Uncracked beam,

Ic=Moment of inertia of cracked beam element.

Thus by the use of above equation, we can find out the different modes of natural frequencies for the cantilever beam.

2.2 Static Analysis:

If the free end of a cantilever beam is subjected to a point load, P, the beam will deflect into a curve. See Fig. 2. Larger the load, greater will be the deflection.

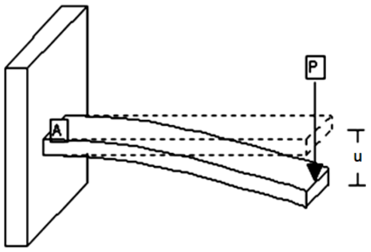

Fig. 2 Cantilever Beam Deflection under Load at Free End

Assuming the beam undergoes small deflections, is in the linearly elastic region, and has a uniform cross-section, the following equations can be used. The curvature of the beam, k, is equal to the second derivative of the deflection

$$K = \frac{\partial^2 w}{\partial x^2}$$

the curvature can also be related to the bending moment, M, and the flexural rigidity, EI,

$$K = \frac{M}{EI}$$

Where E is the Young modulus of the beam and I is the moment of inertia. The bending moment in a beam can be related to the shear force U, and the lateral load q, on the beam.

Thus,

$$M = EI\frac{\partial^2 w}{\partial x^2} \quad U = EI\frac{\partial^3 w}{\partial x^3} \quad q = -EI\frac{\partial^4 w}{\partial x^4}$$

For the load shown in Fig.4.3, the distributed load, shear force, and bending moment are:

$$q(x) = 0, w(x) = P, M(x) = -PL\left(1 - \frac{x}{L}\right)$$

Thus, the solution to above Equation is

$$\frac{\partial w}{\partial x} = \int_0^x M(x)dx = -\frac{PL}{EI}\left\{x - \frac{x^2}{2L}\right\}$$

$$\delta(x) = \int_0^x \frac{\partial u}{\partial x}dx = -\frac{PL}{EI}\left\{\frac{x^2}{2} - \frac{x^3}{6L}\right\}$$

At the free end of the beam, the displacement is:

$$\delta(L) = -\frac{PL^3}{3EI}$$

3. CRACK CONFIGURATIONS

In this study, static and modal analysis of an aluminium cantilever beam having a rectangular crack are studied. The dimension of the beam is 400*30*5 mm. The material properties , modulus of elasticity (E) is 70*109 N/m2, the density (ρ) 2700 kg/m3, poison's ration (μ) 0.3.

The various crack configurations of beam as are prepared to find how the crack affects the original characteristics of beam. Crack location at 100mm and 200mm with crack depth 1mm and 2mm are investigated to find deflection and natural frequency.

4. FINITE ELEMENT ANALYSIS

The ANSYS 14.5 was used for static and modal analysis of un cracked and cracked beams. In pre-processor, first the eight key points were crated and then straight line segments were formed. These straight lines were joined sequentially to create an area. Finally, the area was extruded along the normal plane and a three dimensional un crack and crack beam model was obtained in ANSYS as shown in Fig.3 and Fig.4. A three dimensional SOLID 185 element was selected to model the beam. Crack beam was mesh using tetrahedral element which is the best for crack configuration. The first three natural frequencies for each case were obtained. Similarly, static analysis was performed for each case to find the deflection. In static analysis, the magnitude of the applied load in the vertical direction was 10N. Cantilever boundary conditions were modelled by constraining all degrees of freedoms on the left end area.

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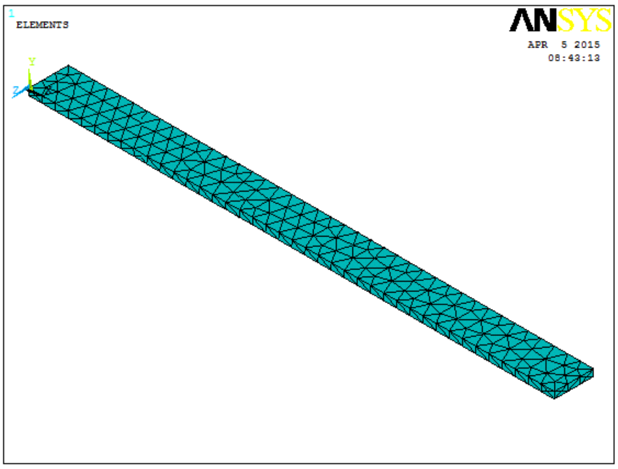

Fig.3 Cantilever Un crack Beam

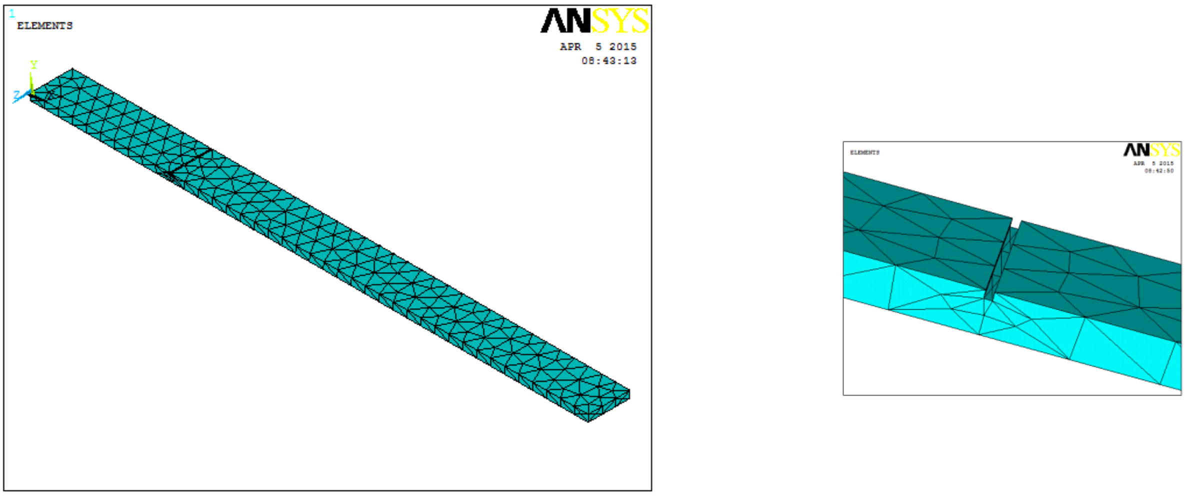

Fig.4 Cantilever Crack Beam

5. RESULTS:

Static and modal analysis of uncrack and crack cantilever beam was done for various condition of beam at various crack depth and locations to obtain deflection and natural frequencies.

The change in deflection with and without crack at various conditions of beam as shown table below. When the crack depth increases then stiffness decreases, displacement also increases. Deflection of cantilever beam for various conditions of beam as shown in Fig. 5 to Fig. 9.

Table -1 Deflection of Beam at Various Conditions

Beam Conditions	Beam without Crack	Crack Beam 1 mm depth & 100 mm length	Crack Beam 2 mm depth & 100 mm length	Crack Beam 1 mm depth & 200 mm length	Crack Beam 2 mm depth & 200 mm length
Deflection in mm	0.002904	0.003019	0.003053	0.002897	0.002949

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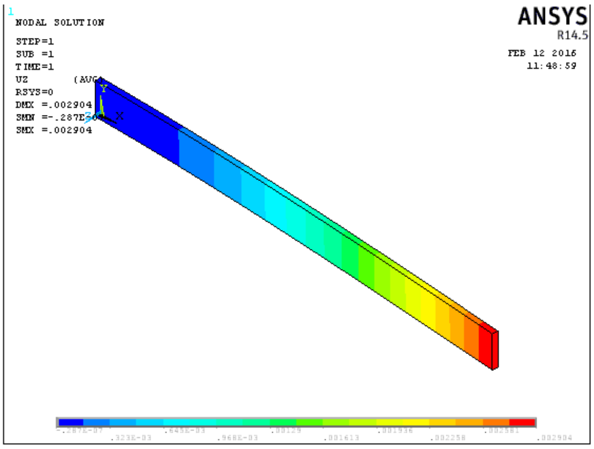


Fig.5 Deflection of Cantilever Beam

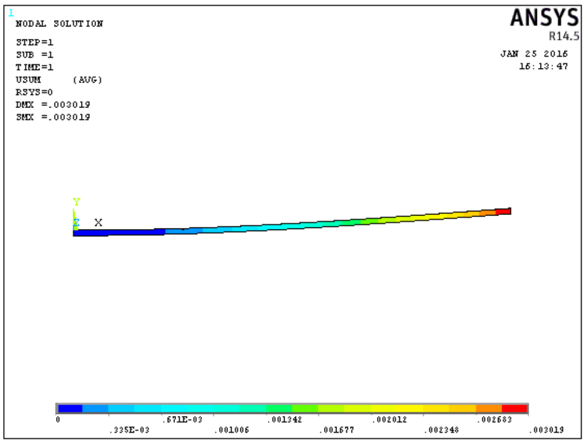


Fig. 6 Deflection Crack Cantilever Beam at Length 100mm and Crack Depth 1mm.

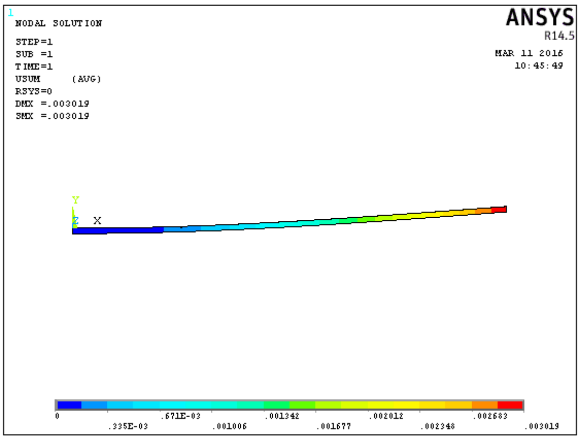
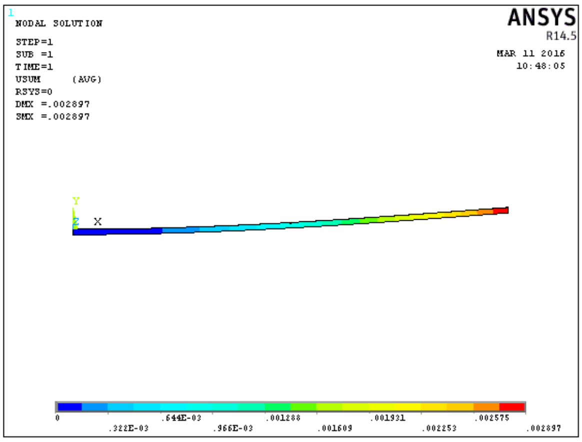


Fig.7 .Deflection Crack Cantilever Beam at
Length 100mm and Crack Depth 2mm



8. Deflection Crack Cantilever Beam
at Length 200mm and Crack Depth 1mm

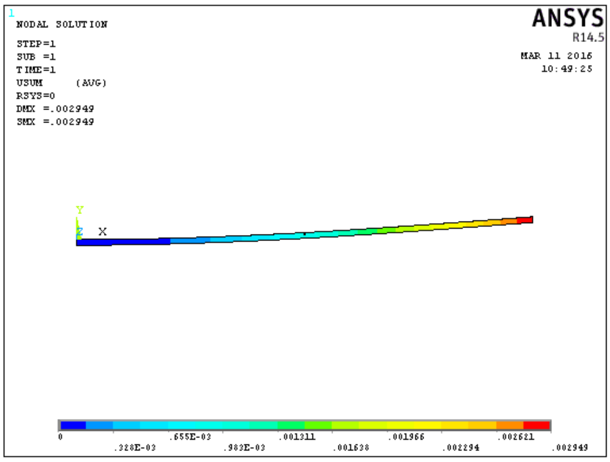


Fig.9. Deflection Crack Cantilever Beam
at length 200mm and Crack depth 2mm

The change in natural frequencies with and without crack at various conditions of beam as shown in following table. The natural frequency of the crack beam the larger than un crack beam. In un crack beam natural frequencies decreases, in increase in crack depth at the same location.

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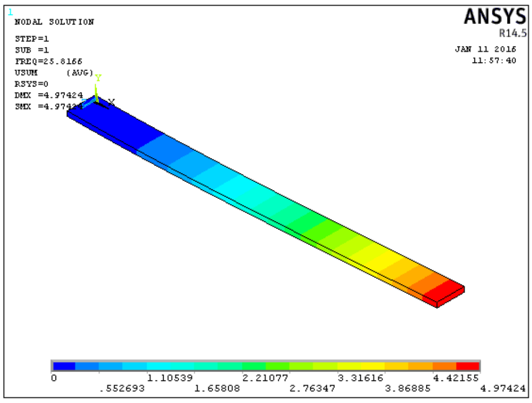
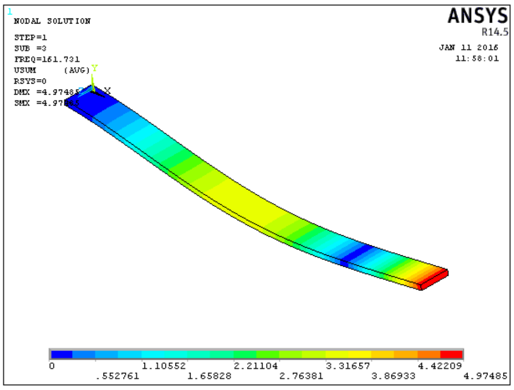
Table -2 Natural Frequency of Beam at Various Conditions

Beam Conditions		Beam without Crack	Crack Beam 1 mm depth & 100 mm length	Crack Beam 1 mm depth & 200 mm length	Crack Beam 2 mm depth & 100 mm length	Crack Beam 2 mm depth & 200 mm length
Natural Frequency in Hz	First	25.8166	46.3781	46.5864	45.709	47.0505
	Second	161.731	292.934	288.575	292.113	286.259
	Third	452.894	812.905	818.833	804.772	821.55

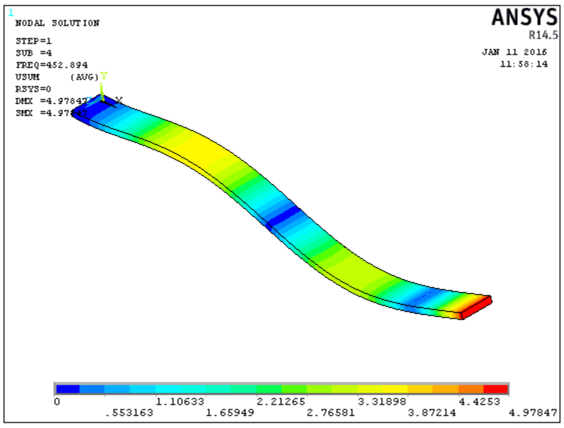
Natural Frequency of Cantilever Beam:



First Mode

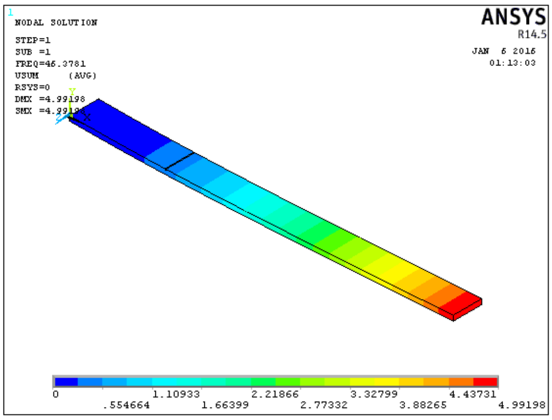


Second Mode



Third Mode

Crack Cantilever Beam with 1 mm Depth and 100 mm Lengths:



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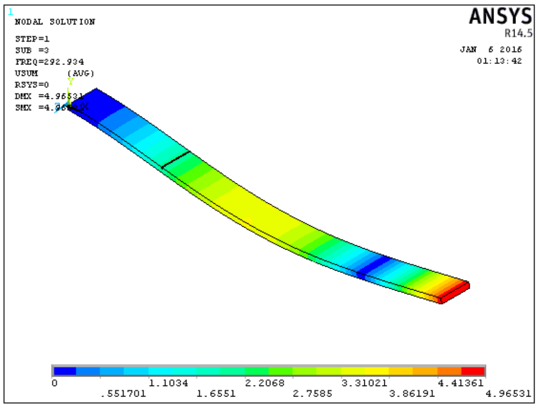

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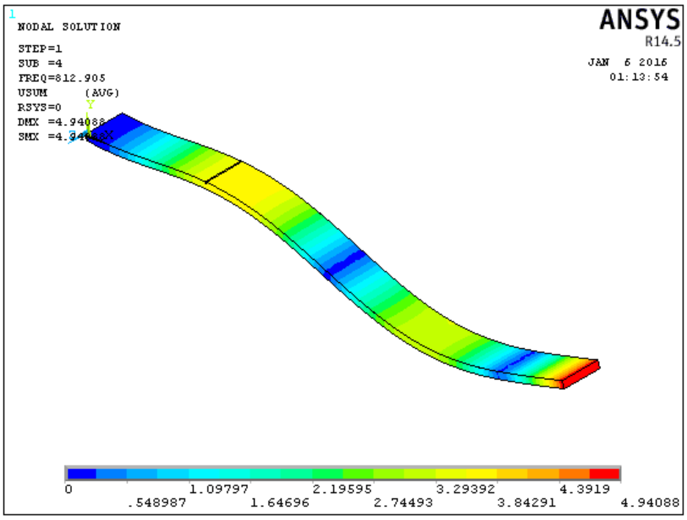
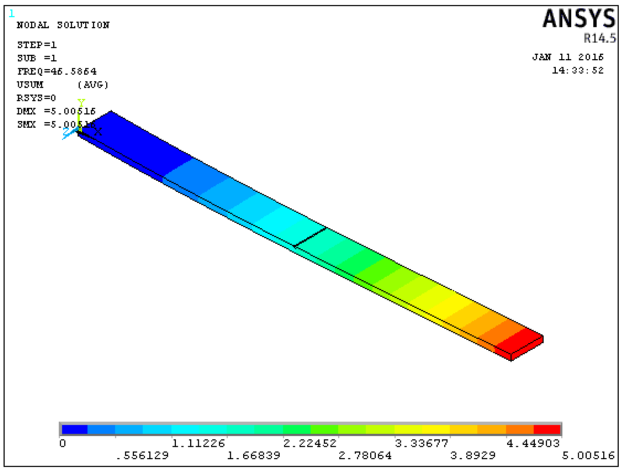


Third Mode

Crack Cantilever Beam with 1 mm Depth and 200 mm Lengths:



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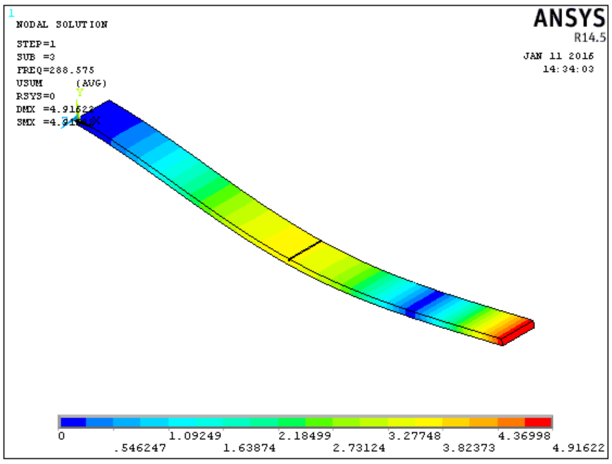

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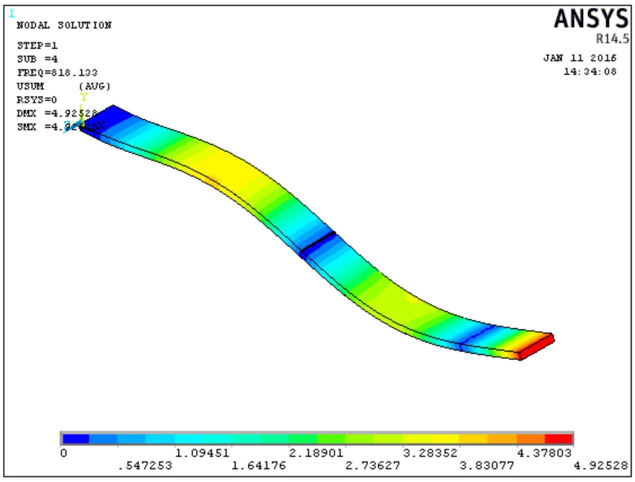

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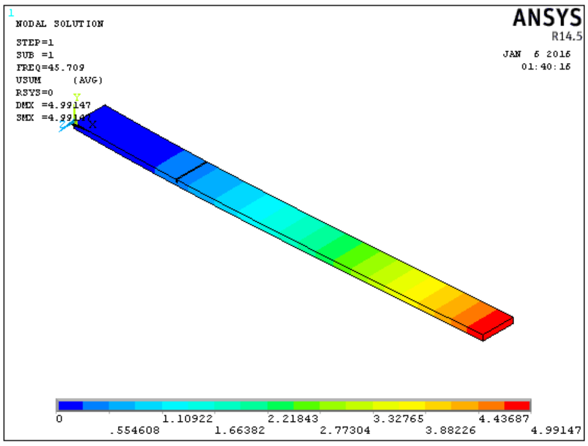
Crack Cantilever Beam with 2 mm Depth and 100 mm Lengths:



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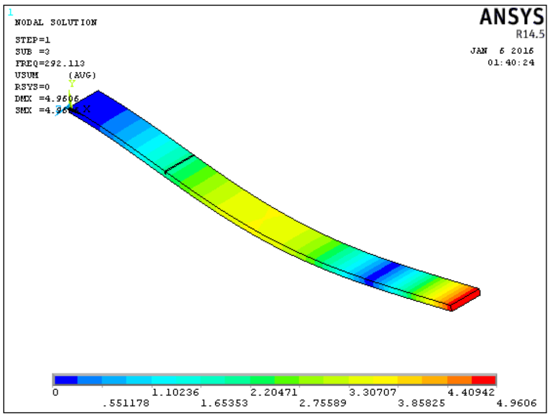

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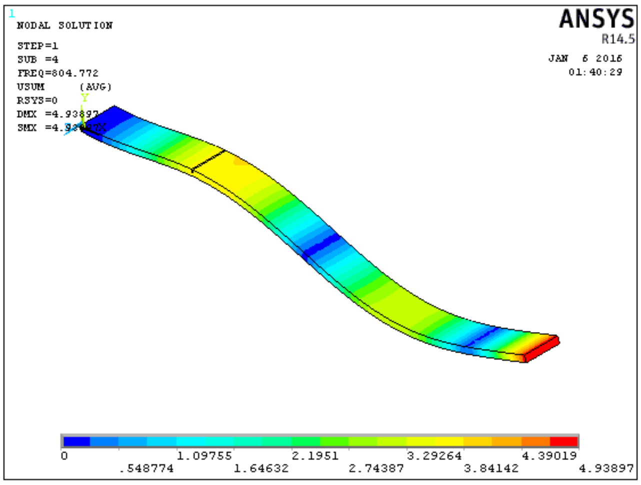

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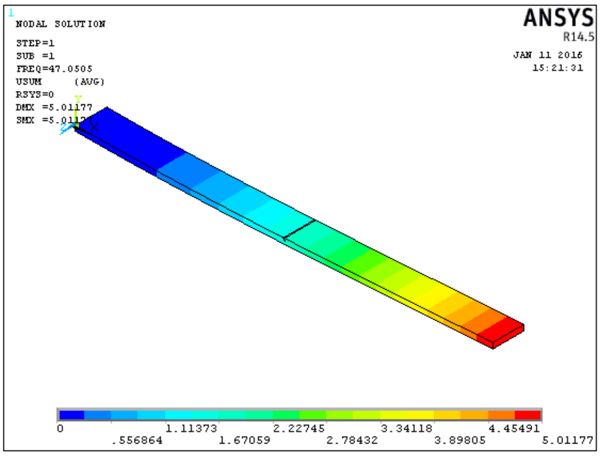
Crack Cantilever Beam with 2 mm Depth and 200mm Length:



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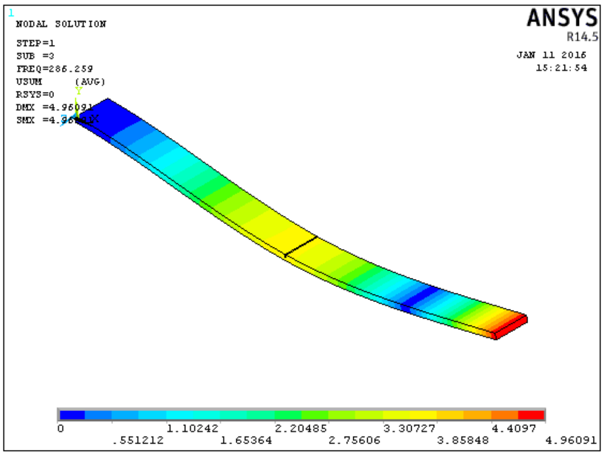

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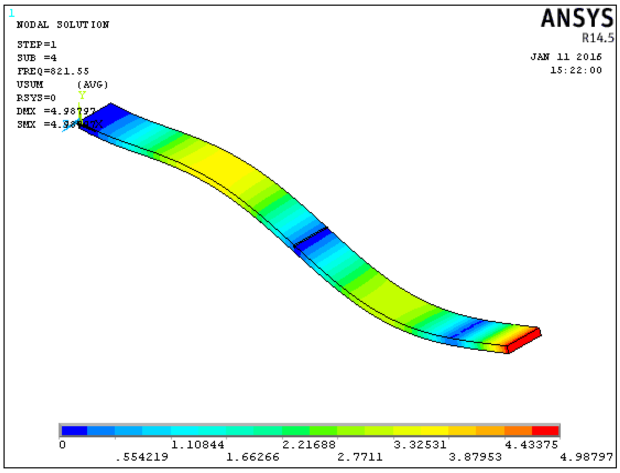

Third Mode

6. CONCLUSION:

In the present section, the natural frequency and deflection of beam is calculated using ANSYS. Results obtained from the various analyses can be concluded as follows:

1) The natural frequency of the crack beam is larger than uncrack beam for same c/s.

For crack beam: At same location - natural frequency decreases, in increase in the crack depth.

At increase in crack location- natural frequency increases, with same crack depth.

2) The amplitude of the crack beam is larger than uncrack beam for same c/s.

For crack beam: At same location – amplitude increases with increase in depth.

At increase in crack location- amplitude decreases, with same crack depth.

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