Study of MHD Flows Through Porous Media in Magnetic Graph Plane

Dr. Mukesh Chandra and Dr. B. K. Singh

Department of Mathematics
IFTM University, Moradabad-244102, (U.P.) INDIA.

ABSTRACT

In this paper, an attempt has been made to study of variably inclined MHD flows through Porous media in magnetic graph plane. The study of MHD flow of a steady homogeneous, incompressible, viscous fluid with finite electrical conductivity through porous media. In the last the expression for vorticity function has been obtained.

Keywords: Current density vector, Fluid pressure, Fluid density, Magnetic viscosity and porous media.

1. INTRODUCTION

Waterhouse and Kingston [5] studied steady, plane, inviscid and incompressible MHD flows, in which the velocity field and the magnetic field are constantly inclined to one another. Transformation techniques are employed for solving non-linear partial differential equations and hodograph transformation method is one of the strongest analytical method which has been widely used in continuum mechanics.

In this paper, we consider the steady plane variably inclined MHD flows of a viscous incompressible fluid with finite electrical conductivity through porous media. A Legendre transform function of magnetic flux-function is used to recast the equations in the magnetograph plane in terms of this transformed function.

2. FORMULATION OF THE PROBLEMS

Here, we shall consider the following notations

$p =$ fluid pressure

$\rho =$ fluid density

$\eta =$ coefficient of Viscosity

$\mu =$ magnetic permeability

$k =$ Permeability of the medium

$\upsilon_H =$ magnetic viscosity

$\vec{J} =$ curl $\vec{H}$ = Current density vector

$\vec{H} =$ magnetic field vector

$\vec{V} =$ velocity vector.

Magneto hydrodynamic flow of a steady homogeneous, incompressible, viscous fluid with finite electrical conductivity through porous media is given by [2]. Then the equations are given as follows:

(2.1) $$\nabla.\vec{V} = 0$$

(2.2) $$\nabla.\vec{H} = 0$$

(2.3) $$\nabla \times \left(\vec{V} \times \vec{H}\right) = \nabla\left(\nu_H \text{ curl } \vec{H}\right)$$

(2.4) $$\rho\left[\left(\vec{v}.\text{grad}\right)\vec{v}\right] = -\text{grad } p + \eta\nabla^2\vec{v} + \mu\vec{J} \times \vec{H} - \frac{\eta}{k}\vec{v}$$

Assuming that flow to be the two dimensional so that $\vec{v}$ and $\vec{H}$ lie in a plane defined by the rectangular coordinates (*x*, *y*) and all the flow variable's are functions of, *x* and *y*. In this regard, the above system of equations is replaced by the following equations:

(2.5) $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(2.6) $$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] + \frac{\partial p}{\partial x} = \eta\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \mu H_2\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right) - \frac{\eta}{K}u$$

(2.7) $$\rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] + \frac{\partial p}{\partial y} = \eta\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \mu H_1\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right) - \frac{\eta}{K}v$$

(2.8) $$uH_2 - vH_1 = \nu_H\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right) + C$$

(2.9) $$\frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} = 0$$

where *u*, *v* are the components of the velocity field $\vec{V}$ and H_1, H_2 the components of the magnetic field vector $\vec{H}$.

The vorticity and current density function is defined as

(2.10) $$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

(2.11) $$\Omega = \frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}$$

(2.12) $$h = p + \frac{1}{2}\rho q^2$$

where $q^2 = u^2 + v^2$

The above equations is replaced by following systems

(2.13) $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(2.14) $$\eta\frac{\partial \omega}{\partial y} - \rho\omega v + \mu\Omega H_2 + \frac{\eta}{k}u = -\frac{\partial h}{\partial x}$$

(2.15) $$\eta\frac{\partial \omega}{\partial x} - \rho\omega u + \mu\Omega H_1 - \frac{\eta}{k}v = \frac{\partial h}{\partial x}$$

(2.16) $$uH_2 - vH_1 = \nu_H\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right) + C$$

(2.17) $$\frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} = 0$$

(2.18) $$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \omega$$

(2.19) $$\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y} = \Omega$$

3. SOLUTION OF THE PROBLEMS

Consider variably inclined plane flow and let $\alpha = \alpha(x, y)$ be the variable angle in the (x, y) flow region, the equation (2.16) reduces in the form–

(3.1) $$uH_2 - vH_1 = qH\sin\alpha = C + \nu_H\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right)$$

(3.2) $$uH_1 + vH_2 = qH\cos\alpha = \left[C + \nu_H\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right)\right]\cot\alpha$$

where $H^2 = H_1^2 + H_2^2 \Rightarrow H = \sqrt{H_1^2 + H_2^2}$

Multiply equation (3.1) by H_2 and equation (3.2) by H_1, then

(3.3) $$uH_2^2 + vH_1H_2 = \left[C + \nu_H\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right)\right]H_2$$

(3.4) $$uH_1^2 + vH_1H_2 = \left[C + \nu_H\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right)\right]\cot\alpha H_1$$

Adding equation (3.3) and equation (3.4), then we get

(3.5) $$u = \left[C + \nu_H\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right)\right]\frac{(H_2 + H_1\cot\alpha)}{H_1^2 + H_2^2}$$

Multiply equation (3.1) by H_1, then we obtain

(3.6) $$uH_2H_1 - vH_1{}^2 = \left[C + \nu_H\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right)\right]H_1$$

Multiply equation (3.2) by H_2, then we obtain

(3.7) $$uH_1H_2 + vH_2{}^2 = \left[C + \nu_H\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right)\right]H_2\cot\alpha$$

Subtracting equation (3.7) from equation (3.6), we obtain

(3.8) $$v = \left[C + \nu_H\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right)\right]\frac{(H_2\cot\alpha - H_1)}{(H_1^2 + H_2^2)}$$

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Differentiating equation (3.8) with respect to 'x', we get

$$\frac{\partial v}{\partial x}=\frac{\partial}{\partial x}\left[(C+v_H\Omega)\frac{(H_2\cot\alpha-H_1)}{H_1^2+H_2^2}\right]\frac{\partial v}{\partial x}$$

Using equation (2.11) into equation (3.8), we get

$$\frac{\partial v}{\partial x}=(C+v_H\Omega)\frac{\partial}{\partial x}\left[\frac{(H_2\cot\alpha-H_1)}{H_1^2+H_2^2}\right]+\frac{(H_2\cot\alpha-H_1)}{H_1^2+H_2^2}\frac{\partial}{\partial x}(C+v_H\Omega)$$

$$=(C+v_H\Omega)\frac{\left[(H_1^2+H_2^2)\frac{\partial}{\partial x}(H_2\cot\alpha-H_1)-(H_2\cot\alpha-H_1)\frac{\partial}{\partial x}(H_1^2+H_2^2)\right]}{(H_1^2+H_2^2)^2}$$

$$+\frac{(H_2\cot\alpha-H_1)}{(H_1^2+H_2^2)}.v_H\frac{\partial\Omega}{\partial x}$$

i.e. $$\frac{\partial v}{\partial x}=(C+v_H\Omega)\left[\frac{1}{H^2}\left\{H_2\frac{\partial}{\partial x}(\cot\alpha)+\cot\alpha\frac{\partial H_2}{\partial x}-\frac{\partial H_1}{\partial x}\right\}-\right.$$

$$\left.\frac{(H_2\cot\alpha-H_1)}{H^4}\left(2H_1\frac{\partial}{\partial x}H_1+2H_2\frac{\partial}{\partial x}H_2\right)\right]+\frac{(H_2\cot\alpha-H_1)}{(H_1^2+H_2^2)^2}.v_H\frac{\partial\Omega}{\partial x}$$

(3.9) $$\frac{\partial v}{\partial x}=\frac{(C+v_H\Omega)}{H^2}\left[\left\{H_2\frac{\partial}{\partial x}(\cot\alpha)+\cot\alpha\frac{\partial H_2}{\partial x}-\frac{\partial H_1}{\partial x}\right\}-\right.$$

$$\left.\frac{(H_2\cot\alpha-H_1)}{H^4}\left(2H_1\frac{\partial H_1}{\partial x}+2H_2\frac{\partial H_2}{\partial x}\right)\right]+\frac{(H_2\cot\alpha-H_1)}{H^2}.v_H\frac{\partial\Omega}{\partial x}$$

Differentiating equation (3.5) partially with respect to 'y', we get

$$\frac{\partial u}{\partial x}=\frac{\partial}{\partial y}\left[(C+v_H\Omega)\frac{(H_2+H_1\cot\alpha)}{(H_1^2+H_2^2)}\right]$$

$$\frac{\partial u}{\partial y}=(C+v_H\Omega)\frac{\partial}{\partial y}\left[\frac{(H_2+H_1\cot\alpha)}{(H_1^2+H_2^2)}\right]+\frac{(H_2+H_1\cot\alpha)}{(H_1^2+H_2^2)}\frac{\partial}{\partial y}(C+v_H\Omega)$$

$$\frac{\partial u}{\partial y}=(C+v_H\Omega)\frac{\left[(H_1^2+H_2^2)\frac{\partial}{\partial y}(H_2+H_1\cot\alpha)-(H_2+H_1\cot\alpha)\frac{\partial}{\partial y}(H_1^2+H_2^2)\right]}{(H_1^2+H_2^2)^2}$$

$$+\frac{\left(H_2+H_1\cot\alpha\right)}{\left(H_1^2+H_2^2\right)}.\,v_H\,\frac{\partial\Omega}{\partial y}$$

i.e. $$\frac{\partial u}{\partial y}=\left(C+v_H\Omega\right)\left[\frac{1}{H^2}\left\{\frac{\partial H_2}{\partial y}+H_1\frac{\partial}{\partial x}(\cot\alpha)+(\cot\alpha)\frac{\partial\left(H_1\right)}{\partial y}\right\}-\right.$$
$$\left.\left\{\frac{\left(H_2\cot\alpha-H_1\right)}{H^4}\times\left(2H_1\frac{\partial H_1}{\partial x}+2H_2\frac{\partial H_2}{\partial x}\right)\right\}\right]+\frac{\left(H_2\cot\alpha-H_1\right)}{H^2}.\upsilon_H\,\frac{\partial\Omega}{\partial x}$$

(3.10) $$\frac{\partial u}{\partial y}=\frac{\left(C+v_H\Omega\right)}{H^2}\left[\left\{H_1\frac{\partial}{\partial y}(\cot\alpha)+(\cot\alpha)\frac{\partial H_1}{\partial y}+\frac{\partial H_2}{\partial y}\right\}-\right.$$
$$\left.\frac{\left(H_2+H_1\cot\alpha\right)}{H^2}\times\left(2H_1\frac{\partial H_1}{\partial y}+2H_2\frac{\partial H_2}{\partial y}\right)\right]+\frac{\left(H_2+H_1\cot\alpha\right)}{H^2}.v_H\,\frac{\partial\Omega}{\partial x}$$

The vorticity function is defined as:

(3.11) $$\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$$

Substituting the value of $\frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y}$ from equation (3.9) and (3.10) in equation (3.11), we get

$$\omega=\frac{\left(C+v_H\Omega\right)}{H^2}\left[\left\{H_2\frac{\partial}{\partial x}(\cot\alpha)+(\cot\alpha)\frac{\partial H_2}{\partial x}-\frac{\partial H_1}{\partial x}\right\}-\right.$$
$$\left.\frac{\left(H_2\cot\alpha-H_1\right)}{H^2}\times\left(2H_1\frac{\partial H_1}{\partial x}+2H_2\frac{\partial H_2}{\partial y}\right)+\frac{\left(H_2\cot\alpha-H_1\right)}{H^2}.v_H\,\frac{\partial\Omega}{\partial x}\right]-$$
$$\frac{\left(C+v_H\Omega\right)}{H^2}\left[\left\{H_1\frac{\partial}{\partial y}(\cot\alpha)+\cot\alpha\frac{\partial H_1}{\partial y}+\frac{\partial H_2}{\partial y}\right\}-\right.$$
$$\left.\left\{\frac{\left(H_2+H_1\cot\alpha\right)}{H^2}\times\left(2H_1\frac{\partial H_1}{\partial y}+2H_2\frac{\partial H_2}{\partial y}\right)\right\}\right]-\frac{\left(H_2+H_1\cot\alpha\right)}{H^2}.\upsilon_H\,\frac{\partial\Omega}{\partial y}.$$

or $$\left(C+v_H\Omega\right)\left[H_2\frac{\partial}{\partial x}(\cot\alpha)-H_1\frac{\partial}{\partial x}(\cot\alpha)+\frac{\partial H_1}{\partial x}+\left(\frac{\partial H_2}{\partial y}-\frac{\partial H_1}{\partial x}\right)\right.$$
$$\times\frac{\left(2H_2^2+2H_1H_2\cot\alpha\right)}{H^2}+\left(\frac{\partial H_2}{\partial x}+\frac{\partial H_1}{\partial y}\right)\times\frac{\left(2H_1H_2-2H_2^2\cot\alpha+\cot\alpha\right)}{H^2}+$$
$$v_H\left[\frac{\partial\Omega}{\partial x}\left(H_2\cot\alpha-H_1\right)-\frac{\partial\Omega}{\partial y}\left(H_2+H_1\cot\alpha\right)\right]=\omega H^2$$

On introducing Jacobians, we get

$$(3.12)\ \frac{J}{H^4}\left(\frac{\partial x}{\partial H_1}\left\{\left(C+v_H\Omega\right)\left(2H_2^2+2H_1H_2+\cot\bar{\alpha}+2\right)\right.\right.$$

$$\left.+v_H H^2\frac{\partial\bar{\Omega}}{\partial H_2}\left(H_2+H_1\cot\bar{\alpha}\right)\right\}-\frac{\partial x}{\partial H_2}\left\{\left(C+v_H\bar{\Omega}\right)\left(2H_2^2\cot\alpha-2H_1H_2-\cot\alpha\right)\right.$$

$$\left.+v_H H^2\frac{\partial\bar{\Omega}}{\partial H_1}\left(H_2+H_1\cot\bar{\alpha}\right)\right\}-\frac{\partial y}{\partial H_2}\left\{\left(C+v_H\bar{\Omega}\right)\left(2H_2^2+2H_1H_2\cot\bar{\alpha}+2\alpha\right)\right\}$$

$$-\frac{\partial y}{\partial H_1}\left\{\left(C+v_H\Omega\right)\left(2H_2^2\cot\bar{\alpha}-2H_1H_2\cot\alpha-\cot\alpha\right)\right\}$$

$$\left.-v_H H^2\frac{\partial\bar{\Omega}}{\partial H_2}\left(H_2\cot\bar{\alpha}-H_1\right)\right]=\bar{\omega}$$

Introducing partial differentiation equation in six unknown functions $x(H_1,H_2), y(H_1,H_2)$ and four transformed functions $\bar{\omega}(H_1,H_2), \vec{h}(H_1,H_2), \vec{\Omega}(H_1,H_2)$ and $\vec{\alpha}(H_1,H_2)$.

The solenodial equation implies the existence of magnetic flux function $\Phi(x,y)$, such that:

$$\frac{\partial\Phi}{\partial x}=-H_2,\frac{\partial\Phi}{\partial y}=-H_1,\frac{\partial L}{\partial H_1}=-y,\frac{\partial L}{\partial H_2}=x, L(H_1,H_2)=H_2x-H_1y+\Phi(x,y)$$

$$(3.13)\qquad \bar{\omega}=\frac{\bar{J}}{H^4}\left[\frac{\partial^2 L}{\partial H_1\partial H_2}\left\{\left(C+v_H\bar{\Omega}\right)\left(2H_2^2+2H_1H_2\cot\bar{\alpha}+2\right)+v_H H^2\frac{\partial\bar{\Omega}}{\partial H_2}\right.\right.$$

$$\left.\left\{\left(H_2+H_1\cot\bar{\alpha}\right)+\frac{\partial\bar{\Omega}}{\partial H_1}\left(H_2\cot\bar{\alpha}-H_1\right)\right\}-\right.$$

$$\frac{\partial^2 L}{\partial H_2^2}\left\{\left(C+v_H\bar{\Omega}\right)\left(2H_2^2\cot\bar{\alpha}-2H_1H_2-\cot\bar{\alpha}\right)\right.$$

$$\left.+v_H H^2\frac{\partial\bar{\Omega}}{\partial H_1}\times\left(H_2+H_1\cot\bar{\alpha}\right)\right\}+$$

$$\frac{\partial^2 L}{\partial H_1^2}\left\{\left(C+v_H\bar{\Omega}\right)\left(2H_2^2\cot\bar{\alpha}-2H_1H_2-\cot\bar{\alpha}\right)\right\}$$

$$\left.\left.-v_H H^2\frac{\partial\bar{\Omega}}{\partial H_2}\left(H_2\cot\bar{\alpha}-H_1\right)\right\}\right].$$

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