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Study of Effects of Temperature Modulation on Double Diffusive Convection in Oldroyd-B Liquids

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***Abstract* - The effect of modulation of temperature has been studied on double diffusive convection in viscoelastic Oldroyd-B liquid. A linear analysis has been done. The thermal Rayleigh number for the modulation problem is obtained using regular perturbation technique. It is observed that the effect of the stress relaxation parameter is to destabilize the system whereas strain retardation parameter and the Lewis number stabilize the system. Modulation is shown to give rise to super-critical motion.**

Key words: Rayleigh-Bénard convection, Double diffusion, Oldroyd-B liquids, Temperature modulation.

1. INTRODUCTION

Convection in Non-Newtonian fluids varies significantly from that in Newtonian fluids, one of which is viscoelastic fluids which are made up of two components – viscous component and elastic component. Viscoelastic fluids are the type of fluids in which the stress-strain relationship depends on time; polymers, human tissues and metals at high temperatures being classic examples. It has come to light that viscoelastic liquids are a working media in many problems in chemical and nuclear industries, geophysics, and engineering in biological systems etc. There have been quite a few works on convection and its onset in viscoelastic liquids (see Sekhar and Jayalatha [1] and references therein). Stationary and oscillatory instabilities for the Oldroyd-B viscoelastic model was studied by Li and Khayat [2,3]which gave a much needed information about the formation of pattern in viscoelastic fluid convection. Oscillatory convection in these fluids was also studied by Green. It was found that in a thin layer of the fluid when heated from below a large restoring force sets up an oscillating convective motion. Siddheshwar and Krishna [4] investigated the linear stability analysis of the Rayleigh–Benard convection problem in a Boussinesquian, viscoelastic fluid. They found that the strain retardation time should be less than the stress relaxation time for convection to set in as oscillatory motions in high-porosity media. Recently, nonlinear stability of thermal convection under g-jitter in a layer of viscoelastic liquid was studied by Siddheshwar [5]. Sharma [6] found that in Oldroyd-B liquids rotation has a destabilizing as well as a stabilizing effect in contrast to a Maxwell fluid. However, there are fewer studies on nonlinear convection as compared to that of linear studies in the case of these liquids.

Buoyancy driven convection occurs due to two components with different diffusivities-temperature and solute. This is popularly known as double diffusive convection (Mojtabi and Charrier-Mojtabi [7]). The study of double diffusive convection has emerged due to Stern [8]. Prior to this, Stommel*et al*. [9] noted that there was a significant potential energy available in the decrease of salinity with depth found in much of the tropical ocean. Modulation of thermal convective instability was initially studied by Rosenblat and Tanaka [10]. If a system has two diffusing components, instabilities occur depending on whether solute components is stabilizing or destabilizing. Further advancement took place when Siddheshwar and Pranesh[11] investigated the effect of temperature/gravity modulation on the onset of magneto-convection in weak electrically conducting fluids with internal angular momentum. Siddheshwar and Sri Krishna [12] investigated Rayleigh – Bénard convection in a viscoelastic fluid filled high porosity medium with non uniform basic temperature gradient. Bhadauria[13] studied temperature modulation of double diffusive convection in a horizontal fluid layer. A study on double diffusive magneto convection in viscoelastic fluids was done by Narayana *et. al.* [14]. Recently, Miloš *et. al.*[15] made an analysis of Rayleigh–Bénard convective instability in the presence of spatial temperature modulation. It was found that modulation delays the onset of convection. Pranesh and Sangeetha [16] investigated the effect of imposed time-periodic boundary temperature of small amplitude on electro convection under AC electric field in dielectric couple stress liquids.

In the present paper the effects of temperature modulation on double diffusive convection in a viscoelastic fluid satisfying the Oldroyd-B constitutive equation is studied.

Nomenclature	
d	thickness of the liquid
k	dimensionless wave number
Pr	Prandtl number
q	velocity
Ra	thermal Rayleigh number
Rs	solutal Rayleigh number
t	time
T	temperature
T_o	constant temperature of the upper boundary
T_R	reference temperature
Le	Lewis number
Greek symbols	
α	thermal expansion coefficient
ε	amplitude of modulation
κ	thermal diffusivity
κ_s	solutal diffusivity
λ_1	stress relaxation coefficient
λ_2	strain retardation coefficient
Λ	elastic ratio(λ_2 / λ_1)
μ	viscosity
ω	frequency of modulation
ρ	density
ρ_0	reference density

2. MATHEMATICAL FORMULATION

Consider a layer of a viscoelastic fluid, namely Oldroyd-B liquid, confined between two finite horizontal walls distance d apart. The parallel plates are maintained at different temperatures and concentrations so that there is density gradient. A Cartesian co-ordinate system is taken with origin in the lower boundary (fig 1).

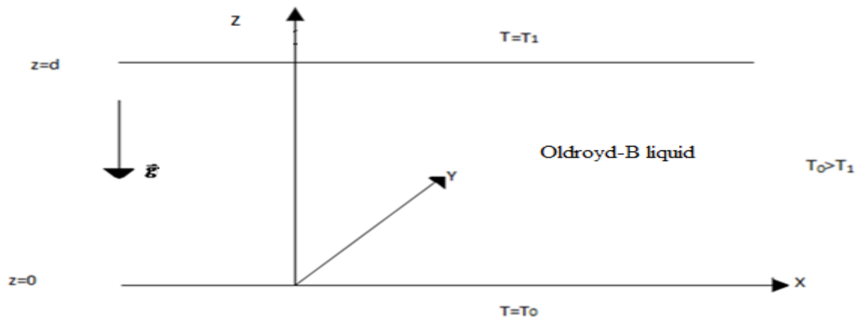

Fig 1: Physical configuration of the problem

The governing equations are:

Continuity Equation:

$$\nabla.\vec{q} = 0 \tag{1}$$

Conservation of momentum:

$$\rho_0 \frac{\partial \vec{q}}{\partial t} = -\nabla p + \rho \vec{g} + \nabla.\tau' \tag{2}$$

Rheological Equation:

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\tau' = \left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\left(\nabla \vec{q} + \nabla \vec{q}^{\,tr}\right) \tag{3}$$

Conservation of Energy:

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \kappa\nabla^2 T \tag{4}$$

Conservation of Species:

$$\frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = \kappa_s\nabla^2 S \tag{5}$$

Equation of State:

$$\rho = \rho_0\left[1 - \alpha(T - T_0) + \alpha'(S - S_0)\right] \tag{6}$$

The surface temperatures are:

$$\left.\begin{aligned} T &= T_R + (1/2)\Delta T(1 + \varepsilon\cos\omega t) \quad at \;\; z = 0, \\ T &= T_R - (1/2)\Delta T(1 - \varepsilon\cos(\omega t + \phi)) \quad at \;\; z = d \end{aligned}\right\} \tag{7}$$

3. BASIC STATE

In the basic state the fluid is at rest. Therefore, the parameters take the following form:

$$\vec{q} = \vec{q}_b = 0, p = p_b(z), \rho = \rho_b(z), S = S_b(z), T = T_b(z,t) \tag{8}$$

The temperature T_b, pressure p_b and density ρ_b satisfy

$$\frac{dp_b}{dz} + \rho_b\vec{g} = 0 \tag{9}$$

$$\frac{\partial T_b}{\partial t} = \kappa\frac{\partial^2 T_b}{\partial z^2} \tag{10}$$

$$\rho = \rho_0(1 - \alpha(T - T_0)), \tag{11}$$

The rheological equation takes the form

$$\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\left[\rho_0\frac{\partial\vec{q}}{\partial t} + \nabla p - \rho\vec{g}\right] = \mu\left(1 + \lambda_2\frac{\partial}{\partial t}\right)\nabla^2\vec{q} \tag{12}$$

The solution of eqn.(10) that satisfies the thermal boundary conditions is

$$T_b(z,t) = T_s(z) + \varepsilon\,\mathrm{Re}\left\{\left[a(\lambda)e^{\frac{\lambda z}{d}} + a(-\lambda)e^{\frac{-\lambda z}{d}}\right]e^{-i\omega t}\right\} \tag{13}$$

where

$$\lambda = (1 - i)\left(\frac{\omega d^2}{2\kappa}\right)^{\frac{1}{2}} \tag{14}$$

$$a(\lambda) = \frac{\Delta T}{2}\left(\frac{e^{-i\phi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right) \tag{15}$$

4. STABILITY ANALYSIS

We now superimpose the infinitesimal perturbations on the quiescent basic state to study the stability of the system. Let the basic state be perturbed by an infinitesimal perturbation as follows, where the primes denote perturbations.

$$\vec{q} = \vec{q}', p = p_b + p', \rho = \rho_b + \rho', T = T_b + T', S = S_b + S' \tag{16}$$

Substituting these equations in the governing equations and using the basic state solutions, we obtain the following equations for the perturbations

$$\frac{\partial T'}{\partial t} + w'\frac{\partial T'}{\partial z} + w'\frac{\partial T_b}{\partial z} = \kappa\nabla^2 T' \tag{17}$$

$$\frac{\partial S'}{\partial t} + w'\frac{\partial S'}{\partial z} + w'\frac{\partial S_b}{\partial z} = \kappa_s\nabla^2 S' \tag{18}$$

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$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\rho_0\frac{\partial(\nabla^2 w')}{\partial t}-\alpha\rho_0 g\nabla_1^2 T'+\alpha'\rho_0 g\nabla_1^2 S'\right]=\mu\left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^4 w' \quad (19)$$

We introduce the stream function ψ such that $u=\frac{\partial\psi}{\partial z}, w=-\frac{\partial\psi}{\partial x}$ and all the terms are independent of y. Using the following dimensionless new variables

$$w^*=\frac{w'}{\kappa/d}, t^*=\frac{t}{d^2/\kappa}, T^*=\frac{T'}{\Delta T}, S^*=\frac{S}{\Delta S}, \nabla^*=d\nabla, (x^*,y^*,z^*)=\left(\frac{x}{d},\frac{y}{d},\frac{z}{d}\right), \quad (20)$$

the resulting non-dimensional equations for the problem are:

$$\left[-\left(1+\Lambda_1\frac{\partial}{\partial t}\right)(1/\Pr)\nabla^2\frac{\partial}{\partial t}+\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^4\right]\frac{\partial\psi}{\partial x}=\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[Ra\nabla_1^2 T-Rs\nabla_1^2 S\right], \quad (21)$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)T+(\vec{q}.\nabla)T=\left(\varepsilon f(z,t)-1\right)\frac{\partial\psi}{\partial x}, \quad (22)$$

$$\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)S+(\vec{q}.\nabla)S=-\frac{\partial\psi}{\partial x}, \quad (23)$$

the dimensionless parameters that appear in these equations are the stress relation parameter, the strain retardation parameter, Prandtl number, Thermal Rayleigh number and Solutal Rayleigh number which are respectively given in eqn. (27).

$$\Lambda_1=\frac{\lambda_1\kappa}{d^2},\ \Lambda_2=\frac{\lambda_2\kappa}{d^2},\ Le=\frac{\kappa}{\kappa_s},\ \Pr=\frac{\mu}{\rho_0\kappa},\ Ra=\frac{\alpha\rho_0 g\Delta T d^3}{\mu\kappa},\ Rs=\frac{\alpha'\rho_0 g\Delta S d^3}{\mu\kappa} \quad (24)$$

5. LINEAR STABILITY ANALYSIS

In this section, we discuss the linear stability analysis considering marginal and over-stable states. To this end we neglect the Jacobians in eqns. (21) to (23). The linear version of these equations are:

$$\left[-\left(1+\Lambda_1\frac{\partial}{\partial t}\right)(1/\Pr)\nabla^2\frac{\partial}{\partial t}+\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^4\right]\frac{\partial\psi}{\partial x}=\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[Ra\nabla_1^2 T-Rs\nabla_1^2 S\right] \quad (25)$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)T=\left(\varepsilon f(z,t)-1\right)\frac{\partial\psi}{\partial x}, \quad (26)$$

$$\left(\frac{\partial}{\partial t}-(1/Le)\nabla^2\right)S=-\frac{\partial\psi}{\partial x}, \quad (27)$$

Eliminating T and S between eqns. (25) - (27), an equation for ψ is obtained in the form

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\left[\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{pr}\frac{\partial}{\partial t}-\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\nabla^2\psi$$
$$=\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\left(Ra\frac{\partial^2\psi}{\partial x^2}-\varepsilon f_1 Ra\frac{\partial^2\psi}{\partial x^2}\right)-\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\nabla^2\right)Rs\frac{\partial^2\psi}{\partial x^2} \quad (28)$$

6. PERTURBATION PROCEDURE

A regular perturbation technique is followed and the following expansions are used

$$\psi=\psi_0+\varepsilon\psi_1+\varepsilon^2\psi_2+....$$
$$R=R_0+\varepsilon R_1+\varepsilon^2 R_2+.... \tag{29}$$

Equation (29) is substituted into eqn. (28) and the coefficients of various powers of ε are equated to obtain the following system of equations

$$L\psi_0=0 \tag{30}$$

$$L\psi_1=\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)(Ra_1-Ra_0 f_1)\frac{\partial^2\psi_0}{\partial x^2}\right] \tag{31}$$

$$L\psi_2=\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\left[Ra_1\frac{\partial^2\psi_1}{\partial x^2}-Ra_0 f_1\frac{\partial^2\psi_1}{\partial x^2}+Ra_2\frac{\partial^2\psi_0}{\partial x^2}-Ra_1 f_1\frac{\partial^2\psi_0}{\partial x^2}\right] \tag{32}$$

where,

$$L=\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\left[\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{pr}\frac{\partial}{\partial t}-\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\nabla^2$$
$$-\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)Ra_0\frac{\partial^2}{\partial x^2}+\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\nabla^2\right)Rs\frac{\partial^2}{\partial x^2} \tag{33}$$

Each ψ_n is required to satisfy the boundary condition $\psi=\nabla^2\psi=\nabla^4\psi=0$ at z = 0, 1.

6.1. Solution to the zeroth order problem

The zerothorder problem is equivalent to the double diffusive problem of Oldroyd-B liquid in theabsence of temperature modulation.The marginally stable solution of the problem is the general solution of the equation. Eq. (30), obtained at o(ε^0) is the one used in the study of convection in a layer of Oldroyd-B liquid subjected to uniform temperature modulation. The marginal stable solutions are

$\psi_0=\sin(\pi\alpha x)\sin(\pi z)$ (34)

with the corresponding eigenvalue R_{a0} given by

$$R_{a0}=\frac{\delta^6}{a^2}+LeR_s+\varepsilon^2 R_2 \tag{35}$$

6.2. Solution to the first order problem

Substituting eqn. (34) in eqn. (31), we get

$$L\psi_1=\frac{-a^2k^2}{Le}R_{a1}\psi_0+\frac{a^2k^2 f}{Le}R_{a0}\psi_0 \tag{36}$$

where,

$$L(\omega,n)=Y_1+iY_2 \tag{37}$$

$$Y_1 = \frac{-k_n^8}{Le} + \frac{k_n^6\Lambda_1\omega^2}{pr} + k_n^4\omega^2 - \frac{\Lambda_1\omega^4k_n^2}{pr} + \frac{\omega^2k_n^4pr}{} + \omega^2\Lambda_2k_n^6 + \frac{\omega^2k_n^4}{Lepr}$$
$$\frac{\omega^2\Lambda_2k_n^6}{Le} + a^2Ra_0\left(\frac{k_n^2}{Le} - \omega^2\Lambda_1\right) - a^2Rs\left(k_n^2 - \omega^2\Lambda_1\right) \quad (38)$$

$$Y_2 = \frac{k_n^6\omega}{Lepr} + \frac{\omega\Lambda_2k_n^8}{Le} - \frac{\omega^3k_n^2}{pr} - \omega^3\Lambda_2k_n^2 + \omega k_n^6 - \frac{\Lambda_1\omega^3k_n^4}{pr} + \frac{\omega k_n^6}{Le} \quad (39)$$
$$-\frac{\Lambda_1\omega^3k_n^4}{Lepr} - a^2Ra_0\left(\omega + \frac{\omega\Lambda_1k_n^2}{Le}\right) + a^2Rs\left(\omega + \omega\Lambda_1k_n^2\right)$$

The solvability condition requires that the time-independent part of the right hand side of eqn. (36) should be orthogonal to the null operator L and this implies that $R_{a1} = R_{a3} = R_{a5}.... = 0$. Therefore, eqn. (36) becomes

$$L\psi_1 = \frac{a^2k^2f}{Le}R_{a0}\psi_0 \quad (40)$$

We use eqn.(32) to determine R_{a2}, the first non-zero correction to R_0. The steady part of eqn.(32) is orthogonal to sinπz. Taking time average of eqn.(40) and using eqn.(32) we get the following expression for the correction Rayleigh number.

$$R_2 = \left(\frac{R_0^2\pi^4\alpha^4}{2Le} - \frac{\lambda^2R_0^2\pi^2\alpha^2}{2Le}\right)\sum_{n=1}^{\infty}\frac{|B_n(\lambda)|^2}{|L(\omega,n)|^2}\left\{|L(\omega,n)| + |L^*(\omega,n)|\right\} \quad (41)$$

Where,

$$B_n(\lambda) = \frac{-2n\pi^2\lambda^2\left[e^{\lambda} - e^{-\lambda} + (-1)^n\left(e^{-\lambda-i\phi} - e^{\lambda-i\phi}\right)\right]}{e^{\lambda} - e^{-\lambda}\left[\lambda^2 + (n+1)^2\pi^2\right]\left[\lambda^2 + (n-1)^2\pi^2\right]} \quad (42)$$

Following are the thermal modulations considered:

Case A: In –Phase modulation

When the oscillating temperature field is symmetric so that the wall temperatures are modulated in phase (with φ = 0).

In this case n is even or odd.

Case B: Out-of –Phase modulation

When the wall temperature field is asymmetric, it corresponds to out-of-phase modulation (with φ = π).

In this case n is odd.

Case C: Only lower wall modulation

When the temperature of only the lower wall is modulated, the upper plate being held at a constant temperature, it corresponds to lower wall modulation (with φ = -i∞).

In this case n takes both even and odd values.

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9. RESULTS AND DISCUSSIONS

We now comprehend the effect of temperature modulation on the onset of double diffusive convection in a horizontal layer of an Oldroyd-B liquid for the relevant parameters. The linear stability problem is solved based on the method proposed by Venezian [17].The parameters of the system are Le, R_a, R_s, Pr, $\Lambda_1, \Lambda_2, \delta, \omega$ which influence the convection. The first six parameters are related to the fluid layer and the remaining are the external measures of controlling the convection.

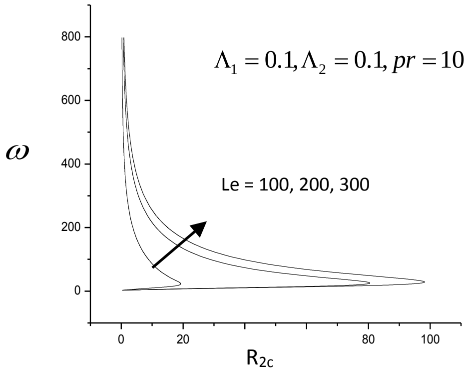

Fig 2: Case A: Graph of R_{2c} versus ω for different values of Le

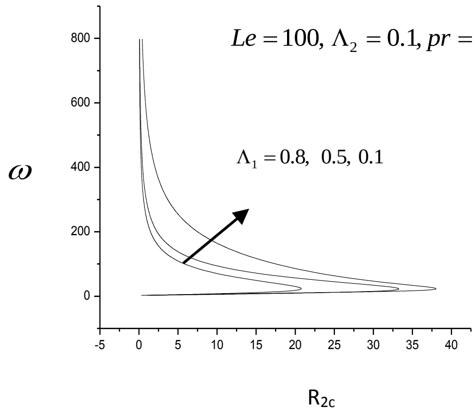

Fig 3: Case A: Graph of R_{2c} versus ω for different values of Λ_1

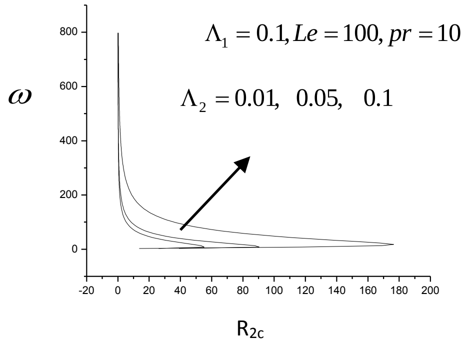

Fig 4: Case A: Graph of R_{2c} versus ω for different values of Λ_2

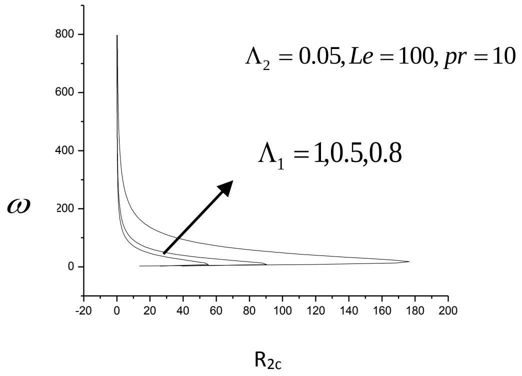

Fig 5: Case B: Graph of R_{2c} versus ω for different values of Λ_1

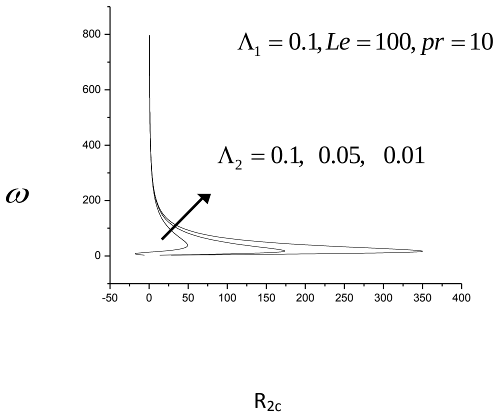

Fig 6: Case B: Graph of R_{2c} versus ω for different values of Λ_2

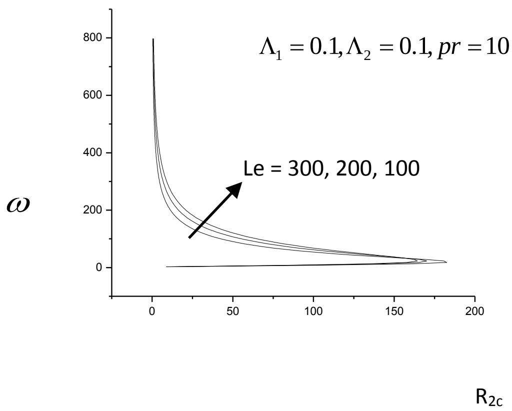

Fig 7: Case B: Graph of R_{2c} versus ω for different values of Le

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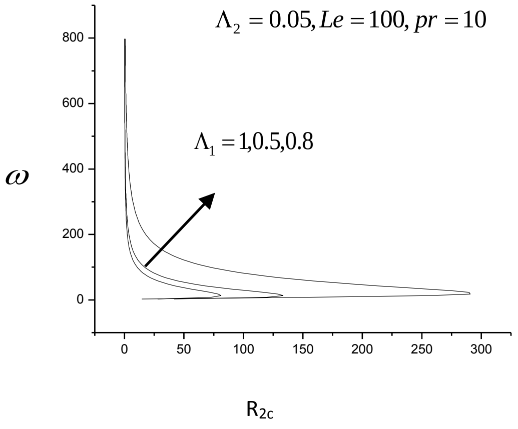

Fig 8: Case C: Graph of R_{2c} versus ω for different values of Λ_1

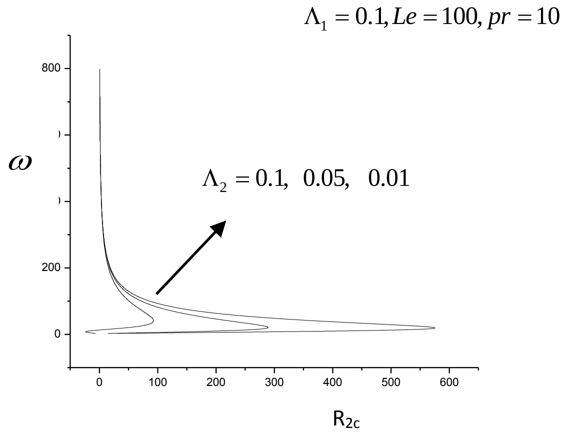

Fig 9: Case C: Graph of R_{2c} versus ω for different values of Λ_2

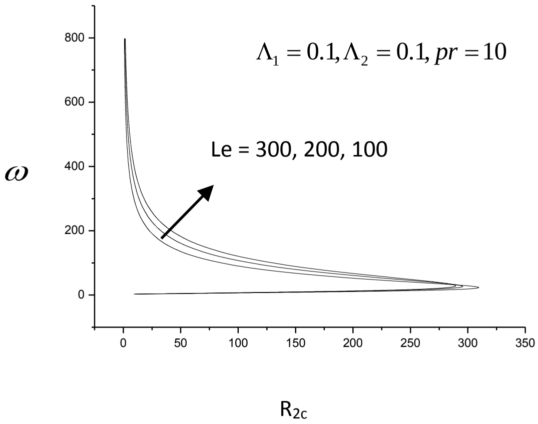

Fig 10: Case C: Graph of R_{2c} versus ω for different values of Le

Figs (2) – (4) show the correction Rayleigh number, R_{2c}, versus frequency of modulation, ω, under Case A (In-Phase modulation) for varying values of the Lewis number, Le, stress relaxation parameter, Λ_1, and strain retardation parameter, Λ_2. The effect of Le is to stabilize the system. The stress relaxation parameter destabilizes the system whereas strain retardation parameter stabilizes. Figs (5) – (7) are graphs showing case B (Out-of-Phase modulations) and figs (8) – (10) are graphs showing case C (only lower wall modulation). It can be seen from the graphs that the values of R_{2c} are larger in both of these cases. This is due to the fact that in the case of out of phase modulation the temperature has a linear gradient varying in time, o that the Rayleigh number is supercritical for half a cycle and subcritical during the other half cycle. These results on the various parameters do not change for the case of lower wall modulation.

10. CONCLUSIONS

The results of the study help in figuring out the effects of externally controlling the convection through modulations. They either advance or delay convection. The following conclusions are made:

1. In the case of in-phase modulation the various parameters cause delay in convection.
2. The parameters have opposing effects in the case of in-phase and out-of-phase modulations.
3. Lower wall modulations show the same results as that of out-of-phase modulations.
4. Modulation is an effective means of controlling convection to a large extent.
5. Lewis number and strain retardation decrease the heat/mass transfer for in-phase modulation.

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