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Study of diabetes model in fuzzy environment using UFM

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Abstract: According to WHO, public health refers to all organized measures to pre-vent disease, promote health and prolong life among the population as a whole. In very recent time several kinds of epidemic diseases have become most worrisome public health concerns. In this reason a large portion of people, all over the world, have great interest doing their research to generate new outcomes in not only the field of medical science,

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but also in biology, bio-medical engineering and epidemiology etc. In present scenario mathematics and epidemiology have a synergetic relationship. In this paper, we have formulated a diabetes model in fuzzy environment by envisaging all the parameters into the model as trapezoidal fuzzy numbers. The utility function method (UFM) has been used to confer the kinetics of the fuzzy diabetes model. The main qualitative properties of the proposed model like positivity and boundedness, uniform persistence and permanence have been explored. Existence of steady states of the system, local stability and global stability have been also elucidated. All the results and systematic consequences have been checked numerically and graphically using MATLAB software in elegant way.

Keywords: Diabetes disease, Types of Diabetes, WHO, Fuzzy Number, Trapezoidal Fuzzy Number, UFM, Local Stability, Numerical Simulation.

1 Introduction

Diabetes mellitus individualized by the incapability of the pancreas to regulate blood glucose levels in a hallmark. Insufficient insulin secretion of insulin by the diabetic pancreas leads to elevated blood glucose concentrations, poor maintenance of normoglycemia (defined as blood glucose 70–100 mg/dl), occasionally reaching 300 mg/dl. It is believed that in majority of the long-term diabetes problems are such as nephropathy and retinopathy, result from persistent hyperglycemia (arterial blood glucose 120 mg/dl)[1].

Diabetes mellitus, commonly referred to as a metabolic illness of complex aetiology is defined by persistent hyperglycaemia and abnormalities in the metabolism of carbohydrate, protein and fat brought on by deficiencies in insulin secretion, insulin action, or both. Diabetes mellitus can cause long-term harm, malfunction and organ failure. Thirst, polyuria, blurred eyesight, and weight loss are typical signs of diabetes mellitus or may present. The long-term exorcism of diabetes mellitus include progressive development of the specific complications of retinopathy with potential blindness, nephropathy which can effect renal failure, and/or neuropathy which increases risk of foot ulcers, amputation, Charcot joints, and features of autonomic malfunction, including sexual dysfunction. Diabetes can increases a person's risk of cardiovascular, cerebrovascular disease and peripheral vascular[2].

Various pathogenetic mechanisms are involved to develops the diabetes diseases. These include mechanisms that lead to resistance to the effects of insulin and those that kill the beta cells of the pancreas with consequent insulin insufficiency, and others that result in resistance to insulin action. The problems in the metabolism of fat, protein and carbohydrate are due to deficient action of insulin on target tissues which arises from either insensitive insulin or lack of insulin. Diabetes can also be founds in children, with different intense symptoms, very high blood glucose levels marked ketonuria, and glycosuria.

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Symptoms of diabetics mellitus are such as follows, increased thirst and urine volume, unexplained weight loss recurrent infections and in severe cases, drowsiness and coma etc. There is another type of diabetes named as gestational diabetes, which affects pregnant women. This type of diabetes is mainly found during pregnancy. The OGTT (Oral glucose tolerance test) is used to diagnosis gestational diabetes [3]. Mainly there are two different types of diabetes, type 1 diabetes (T1DM) and type 2 diabetes (T2DM). In both types of diabetes mellitus (DM) problem, the beta cell function and mass are decreased from the clinical onset and this is associated by a respondent disintegration of glycaemic possession. The phenomenon is more intense in type 1 diabetes (T1DM), and is mainly due to the autoimmune attack of auto reactive T cells against is let beta cells[4]. It is assumed that about 70-90 percentage of the beta cell mass is lost at the time of clinical presentation, which is usually abrupt, with acute metabolic decompensation. The pathogenesis is complex, in type 2 diabetes (T2DM), and in most cases, the reduction of beta cell function and mass is associated with different degrees of insulin resistance [5][6].
In the last few decades, mathematical models have been used to understand various aspects of diabetes mellitus (DM). Several researcher devoted there time to model DM such as DM progression [7, 8, 9] diagnostic test evaluations, long-term micro and macro-vascular complications [10, 11, 12], and blood glucose dynamics [13, 14]. Mathematical models to emulate blood glucose dynamics in DM have been classified, according to the complexity of their description, in two major groups [14]. The first group considers the whole-body models developed under a pharmacokinetic–pharmacodynamic (PKPD) approach, which is characterized by being structurally simple with a limited physiological interpretation. The second group considers the physiological based PKPD (PB-PKPD) models, which mathematically describes the physiological interactions between different subsystems of the human body [15].
In the field of epidemiology uncertainty[16, 17] plays a crucial role. Due to the lack of data, insufficient supply of data, technical fault in data collection and environmental fluctuations, uncertainty comes into the system. It is well established that changing climate affects the parameters of the ecological [18, 19, 20] or epidemiological systems. To handle the uncertainty into the system, nowadays the researchers are trying to frame there system with the help of fuzzy differential equations approach to get a more realistic scenario [21, 22].
Fuzzy differential equation [23, 24] is extensively used in various field like biology, physics and engineering. Nowadays fuzzy set theory plays a considerable role to model different real life situations. Lots of work are describe in fuzzy environment [25, 26, 27, 28]. The concept of fuzzy number and fuzzy set is a major concern today. Zadeh [29]and Dubois and Parade [30]are the pioneer of fuzzy set theory.
Nowadays, the researchers showing their interest on epidemiological models in fuzzy environment. The dynamical behavior of measles is discussed in [31], with the help of

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fuzzy. A prey–predator model with optimal harvesting policy is discussed in [32] with the help of imprecise nature of the parameters. The effect of harvesting is discussed in prey–predator system in fuzzy environment and by using the utility function method [33]. Fuzzy differential equation is also used to develop a cholera disease model by using the concept of UFM in [34]. The dynamics of fuzzy HTLV-I model with the help of the utility function method is studied in [35]. Our review of the literature discovered that so many researchers devoted their works on ecological and eco-epidemiological system on different aspect and so many researcher devoted their works on fuzzy environment but still there is a huge scope to explore the dynamics of epidemiological system in fuzzy environment by using the concept of UFM. In this research article we investigate the dynamics of diabetes mellitus model in fuzzy environment by using Utility Function Method.

1.1 Motivation

In recent time, millions of people are suffering from diabetes mellitus (DM) around the world; it is a metabolic disease that causes serious medical and financial difficulties. Due to the imprecision, uncertainty [36, 37, 38], and vagueness inherent in medical data, it is difficult to identify the disease. Though traditional mathematical and statistical models are used but unable to capture the patient symptoms, laboratory data and expert opinions. To overcome this situation in this era the use of fuzzy set theory increasing drastically. Fuzzy Set (IFS) theory integrate degrees of membership, and hesitation for handling uncertainty and imprecision. The Utility Function Method (UFM) provides a methodical approach to assign numerical utilities to different traits or outcomes, representing the relative relevance or desirability of each, in order to further improve the decision-making process. This technique aids in the conversion of multidimensional, complex fuzzy data into a single scalar value that can more clearly and confidently direct classification, ranking, and diagnosis in the context of diabetes modeling.

1.2 Novelty

The utility function method (UFM) and fuzzy Set theory are used in this study to produce a reliable and understandable model for the diagnosis and analysis of diabetes mellitus. The following are the study's primary innovative contributions:

1. Creation of a hybrid fuzzy-utility framework that uses fuzzy sets to represent imprecise and uncertain clinical indicators (such as blood pressure, insulin levels, age, BMI, and glucose levels) and aggregates them using utility functions to support efficient diagnostic conclusions.

2. By using the Utility Function Method to measure and balance the relative significance of various symptoms and diagnostic criteria, a more practical and clinically

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significant decision model may be created.

3. The implementation of a hesitation-handling mechanism that takes into consideration the ambiguity between membership and non-membership values, offering a more thorough comprehension of circumstances that are ambiguous or borderline.

4. The efficiency of the suggested model in comparison to conventional fuzzy models or crisp classification techniques, particularly when dealing with cases involving overlapping or ambiguous features, is demonstrated through validation using actual or simulated patient data.

This study advances the field by offering a new approach to decision-making that improves the accuracy of diabetes diagnosis while also providing a generalizable framework for other medical illnesses where subjective assessment is crucial.

2 Preliminaries

Definition 2.1 Fuzzy set(FS) [39]:
Let Y be a non-empty set. An FS $\tilde{T}$ in Y is considered by a set $\tilde{T} = \{y, M(y) : y \in Y\}$. The mapping $M(y) : Y \rightarrow [0, 1]$ is the membership function and M(y) is the membership value of y in Y of the fuzzy set $\tilde{T}$.
Definition 2.2 Trapezoidal fuzzy number (TrFN)[39]
A trapezoidal fuzzy number $\tilde{M} =< m_1, m_2, m_3, m_4 >$ which has the membership function $u_{\tilde{M}}(y)$ of TrFN on set of real number R defined as,

$$u_{\tilde{M}}(y) = \begin{cases} \frac{y-m_1}{m_2-m_1}, & if m_1 \leq y < m_2 \\ 1 & if m_2 \leq y < m_3 \\ \frac{m_4-y}{m_4-m_3}, & if m_3 < y \leq m_4 \\ 0, & otherwise \end{cases}$$

Definition 2.3 α-cut of a fuzzy number[40] A $\alpha-$cut of fuzzy number $\tilde{M}$ in Y is denoted by M_α and is defined as the following crisp set
$M_\alpha = \{y | u_{\tilde{M}}(y) \geq \alpha, y \in Y\}$ where $\alpha \in [0, 1]$,
M_α is a non-empty closed bounded set in Y and it can represented by, $M_\alpha = [M_{LT}(\alpha), M_{RT}(\alpha)]$, where $M_{LT}(\alpha)$ and $M_{RT}(\alpha)$ are the lower and upper bounds of the intervals respectively.

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The $\alpha-$cut of trapezoidal fuzzy number $\tilde{M} = (m_1, m_2, m_3, m_4)$ can be expressed as $[M_{LT}(\alpha), M_{RT}(\alpha)]$ where, $M_{LT}(\alpha) = m_1 + (m_2 - m_1)\alpha, M_{RT}(\alpha) = m_4 - (m_4 - m_3)\alpha.$

Definition 2.4 Utility Function Method(UFM): [41] In UFM method, the function is defined for each objectives f_i according to their relative importance and simply the utility function defined as $v_i f_i$, for $i - th$ objective, where v_i denote the scaler and represent the weight assigned to corresponding objection. Then total utility function is defined as follows,

$$U = \sum_{i=1}^{n} v_i f_i, v_i \leq 0$$

subject to the condition, $\sum_{i=1}^{n} v_i = 1$.

3 Development of the Mathematical Model

Our assumption in this model is that we consider a four dimensional diabetes mellitus model. We divide the human population in to two parts one is susceptible human population and another one is infected human population having diabetes. $P_U(t)$ the number of unaware pre-diabetes patients, $P_A(t)$ the number of aware diabetic patients, $D_C(t)$ the number of diabetics patients with complications and $D_w(t)$ the number of diabetics patients without complications. Our proposed model is similar to [42]

$$\begin{aligned}
\frac{dP_U(t)}{dt} &= \pi - pP_U(t)M - \beta_1 P_U(t) - \beta_2 P_U(t) - n_d P_U(t), \\
\frac{dP_A(t)}{dt} &= pP_U(t)M - \delta_1\beta_1 P_A(t) - \delta_2\beta_2 P_A(t) - n_d P_A(t), \\
\frac{dD_W(t)}{dt} &= \beta_1 P_U(t) + \delta_1\beta_1 P_A(t) + \gamma_1 D_C(t) - \gamma_2 D_w(t) - n_d D_w(t), \\
\frac{dD_C(t)}{dt} &= \beta_2 P_U(t) + \delta_2\beta_2 P_A(t) - \gamma_1 D_C(t) + \gamma_2 D_w(t) - h_1 D_C(t) - m_d D_C(t) - n_d D_C(t).
\end{aligned} \tag{1}$$

where $P_U(0) = P_{U0},\ P_A(0) = P_{A0},\ D_W(0) = D_{W0},\ D_C(0) = D_{C0}$.

The parameters of the above system are described in the following stanza n_d is the natural death rate and m_d death due to complication, π be the new cases diabetics patients, $\delta_1\beta_1$ in this cases β_1, rate of developing complications and δ_1 is the reduce rate of developing complication due to awareness and its lies between 0 and 1. $\delta_2\beta_2$ in this cases β_2, rate of developing complications and δ_2 is the reduce rate of developing complication due to awareness and its lies between 0 and 1. The term p is awareness conversion rate this rate at which unaware individuals become aware due to media effect. γ and γ_2 are the reduction rate from complication diabetic patients to without diabetic patients and another is vise versa. h_1 is the recovery rate from complication diabetic due to different kind of treatment.

To find the solution of (1), let

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$[\tilde{\frac{ds}{dt}}]_\alpha = [(\frac{ds}{dt})^\alpha_{LT}, (\frac{ds}{dt})^\alpha_{RT}]$

The deterministic system of the model (1), can be expressed as follows,

$[\frac{dP_U(t)}{dt}]^\alpha_{LT} = (\pi)^\alpha_{LT} - (p)^\alpha_{RT}(M)^\alpha_{LT}P_U(t) - (\beta_1)^\alpha_{RT}P_U(t) - (\beta_2)^\alpha_{RT}P_U(t) - (n_d)^\alpha_{RT}P_U(t)$

$[\frac{dP_U(t)}{dt}]^\alpha_{RT} = (\pi)^\alpha_{RT} - (p)^\alpha_{LT}(M)^\alpha_{RT}P_U(t) - (\beta_1)^\alpha_{LT}P_U(t) - (\beta_2)^\alpha_{LT}P_U(t) - (n_d)^\alpha_{LT}P_U(t)$

$[\frac{dP_A(t)}{dt}]^\alpha_{LT} = (p)^\alpha_{LT}(M)^\alpha_{LT}P_U(t) - (\delta_1)^\alpha_{RT}(\beta_1)^\alpha_{LT}P_A(t) - (\delta_2)^\alpha_{RT}(\beta_2)^\alpha_{LT}P_A(t) - (n_d)^\alpha_{RT}P_A(t)$

$[\frac{dP_A(t)}{dt}]^\alpha_{RT} = (p)^\alpha_{RT}(M)^\alpha_{RT}P_U(t) - (\delta_1)^\alpha_{LT}(\beta_1)^\alpha_{RT}P_A(t) - (\delta_2)^\alpha_{LT}(\beta_2)^\alpha_{RT}P_A(t) - (n_d)^\alpha_{LT}P_A(t)$

$[\frac{dD_W(t)}{dt}]^\alpha_{LT} = (\beta_1)^\alpha_{LT}P_U(t) + (\delta_1)^\alpha_{LT}(\beta_1)^\alpha_{LT}P_A(t) + (\gamma_1)^\alpha_{LT}D_C(t) - (\gamma_2)^\alpha_{RT}D_W(t) - (n_d)^\alpha_{RT}D_W(t)$

$[\frac{dD_W(t)}{dt}]^\alpha_{RT} = (\beta_1)^\alpha_{RT}P_U(t) + (\delta_1)^\alpha_{RT}(\beta_1)^\alpha_{RT}P_A(t) + (\gamma_1)^\alpha_{RT}D_C(t) - (\gamma_2)^\alpha_{LT}D_W(t) - (n_d)^\alpha_{LT}D_W(t)$

$[\frac{dD_C(t)}{dt}]^\alpha_{LT} = (\beta_2)^\alpha_{LT}P_U(t) + (\delta)^\alpha_{LT}(\beta_2)^\alpha_{LT}P_A(t) - (\gamma)^\alpha_{RT}D_C(t) + (\gamma_2)^\alpha_{LT}D_W(t) - (h_1)^\alpha_{RT}D_C(t) - (n_d)^\alpha_{RT}D_C(t)$

$[\frac{dD_C(t)}{dt}]^\alpha_{RT} = (\beta_2)^\alpha_{RT}P_U(t) + (\delta)^\alpha_{RT}(\beta_2)^\alpha_{RT}P_A(t) - (\gamma)^\alpha_{LT}D_C(t) + (\gamma_2)^\alpha_{RT}D_W(t) - (h_1)^\alpha_{LT}D_C(t) - (n_d)^\alpha_{LT}D_C(t)$

Using the concept of utility function method, we can write the above system of differential equation as follows,

$$\begin{aligned}
\frac{dP_U(t)}{dt} &= v_1(\frac{dP_U(t)}{dt})^\alpha_{LT} + v_2(\frac{dP_U(t)}{dt})^\alpha_{RT},\\
\frac{dP_A(t)}{dt} &= v_1(\frac{dP_A(t)}{dt})^\alpha_{LT} + v_2(\frac{dP_A(t)}{dt})^\alpha_{RT},\\
\frac{dD_W(t)}{dt} &= v_1(\frac{dD_W(t)}{dt})^\alpha_{LT} + v_2(\frac{dD_W(t)}{dt})^\alpha_{RT},\\
\frac{dD_C(t)}{dt} &= v_1(\frac{dD_C(t)}{dt})^\alpha_{LT} + v_2(\frac{dD_C(t)}{dt})^\alpha_{RT}
\end{aligned} \tag{2}$$

where v_1 and v_2 are two weight function such that $v_1 + v_2 = 1$ and $v_1, v_2 \geq 0$.
Then the system of equation (2) can be expressed as follows,

$$\begin{aligned}
\frac{dP_U(t)}{dt} &= h_{11} - h_{12}P_U(t),\\
\frac{dP_A(t)}{dt} &= h_{21}P_U(t) - h_{22}P_A(t),\\
\frac{dD_W(t)}{dt} &= h_{31}P_U(t) + h_{32}P_A(t) + h_{33}D_C(t) - h_{34}D_w(t),\\
\frac{dD_C(t)}{dt} &= h_{41}P_U(t) + h_{42}P_A(t) + h_{43}D_W(t) - h_{44}D_C(t).
\end{aligned} \tag{3}$$

with the initial condition, $P_U(0) = P_{U0}$, $P_A(0) = P_{A0}$, $D_W(0) = D_{W0}$, $D_C(0) = D_{C0}$
where, $h_{11} = v_1(\pi)^\alpha_{LT} + (1-v_1)(\pi)^\alpha_{RT}, h_{12} = v_1(p)^\alpha_{LT} + (1-v_1)(p)^\alpha_{LT}(M)^\alpha_{LT} + v_1(\beta_1)^\alpha_{RT} + (1-v_1)(\beta_1)^\alpha_{LT} + v_1(\beta_2)^\alpha_{RT} + (1-v_1)(\beta_2)^\alpha_{LT} + v_1(n_d)^\alpha_{RT} + (1-v_1)(n_d)^\alpha_{LT}, h_{21} = v_1(p)^\alpha_{LT}(M)^\alpha_{LT} + (1-v_1)(p)^\alpha_{RT}(M)^\alpha_{RT}, h_{22} = v_1(\delta_1)^\alpha_{RT}(\beta_1)^\alpha_{RT} + (1-v_1)(\delta_1)^\alpha_{LT}(\beta_1)^\alpha_{RT} + v_1(\delta_2)^\alpha_{RT}(\beta_2)^\alpha_{RT} + (1-v_1)(\delta_2)^\alpha_{LT}(\beta_2)^\alpha_{RT} + v_1(n_d)^\alpha_{RT} + (1-v_1)(n_d)^\alpha_{LT}, h_{31} = v_1(\beta_1)^\alpha_{LT} + (1-v_1)(\beta_1)^\alpha_{RT}, h_{32} = v_1(\delta_1)^\alpha_{LT}(\beta_1)^\alpha_{LT} + (1-v_1)(\delta_1)^\alpha_{RT}(\beta_1)^\alpha_{RT}, h_{33} = v_1(\gamma_1)^\alpha_{LT} + (1-v_1)(\gamma_1)^\alpha_{RT}, h_{34} = v_1(\gamma_2)^\alpha_{RT} + (1-v_1)(\gamma_2)^\alpha_{LT} + v_1(n_d)^\alpha_{RT} + (1-v_1)(n_d)^\alpha_{LT}, h_{41} = v_1(\gamma_2)^\alpha_{RT} + (1-v_1)(\gamma_2)^\alpha_{LT}, h_{42} = v_1(\delta_2)^\alpha_{LT}(\beta_2)^\alpha_{LT} + (1-v_1)(\delta_2)^\alpha_{RT}(\beta_2)^\alpha_{RT}, h_{43} = v_1(\gamma_2)^\alpha_{LT} + (1-v_1)(\gamma_2)^\alpha_{RT}, h_{44} = v_1(\gamma_1)^\alpha_{RT} + (1-v_1)(\gamma_1)^\alpha_{LT} + v_1(h_1)^\alpha_{RT} + (1-v_1)(h_1)^\alpha_{LT} + v_1(m_d)^\alpha_{RT} + (1-v_1)(m_d)^\alpha_{LT} + v_1(n_d)^\alpha_{RT} + (1-v_1)(n_d)^\alpha_{LT}$

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3.1 Positivity of the system

Theorem 1: Every solution of the system (3) are positive with initial values in the interval $[0, t]$ and $P_U(t) > 0, P_A(t) > 0, D_W(t) > 0, D_C(t) > 0$ for all $t \geq 0$.
Proof: From the 1st equation of the system (3), we get

$$\frac{dP_U(t)}{dt} + h_{12}P_U(t) = h_{11}$$

Integrating, in the above equation in the interval $[0, t]$, we get,

$$P_U(t) = (P_{U0} - \frac{h_{11}}{h_{12}})e^{-h_{12}t} > 0, \forall\ t$$

Now, the 2nd equation from the system (3) gives,

$$\frac{dP_A(t)}{P_A(t)} = \phi_1(P_A, P_U)dt, \text{ where } \phi_1(P_A, P_U) = \frac{h_{21}P_U(t)}{P_A(t)} - h_{22}$$

Integrating in the above equation in the interval $[0, t]$, we have,

$$P_A(t) = P_{A0}e^{\int_0^t \phi_1(P_A,P_U)dt} > 0, \forall\ t$$

Similarly, from the 3rd equation of the system (3), we have

$$\frac{dD_W(t)}{D_W(t)} = \phi_2(P_A, P_U, D_W)dt, \text{ where } \phi_2(P_A, P_U, D_W) = \frac{h_{31}P_U(t)}{D_W(t)} + \frac{h_{32}P_A(t)}{D_W(t)} + \frac{h_{33}D_C(t)}{D_W(t)} - h_{34}$$

After integrating the above equation we get,

$$D_W(t) = D_{W0}e^{\int_0^t \phi_1(P_A,P_U,D_W)dt} > 0, \forall\ t$$

From the 4th equation of the system (3), gives

$$\frac{dD_C(t)}{D_C(t)} = \phi_3(P_A, P_U, D_W, D_C)dt, \text{ where, } \phi_3(P_A, P_U, D_W, D_C) = \frac{h_{41}P_U(t)}{D_C(t)} + \frac{h_{42}P_A(t)}{D_C(t)} + \frac{h_{43}D_W(t)}{D_C(t)} - h_{44}$$

Integrating the above equation in the interval $[0, t]$,

$$D_C(t) = D_{C0}e^{\int_0^t \phi_3(P_A,P_U,D_W,D_C)dt} > 0, \forall\ t$$

Hence, the theorem is proved.

4 Equilibrium points

The converted system (3) have interior equilibrium point, $E_p^*(P_U^*, P_A^*, D_W^*, D_C^*)$ where $P_U^*(t) = \frac{h_{11}}{h_{12}}, P_A^*(t) = \frac{h_{21}h_{11}}{h_{12}h_{22}}, D_W^*(t) = \frac{1}{h_{44}h_{34}-h_{33}h_{43}}(\frac{h_{31}h_{11}h_{44}}{h_{12}} + \frac{h_{32}h_{21}h_{44}}{h_{22}} + \frac{h_{11}h_{33}h_{41}}{h_{12}} + \frac{h_{11}h_{21}h_{33}h_{42}}{h_{12}h_{22}}) D_C^*(t) = \frac{1}{h_{33}}[h_{34}D_W^*(t) - h_{31}P_U^*(t) - h_{32}P_A^*(t)]$.
The interior equilibrium point E_p^* is feasible when $h_{44}h_{34} > h_{33}h_{43}, h_{34}D_W^*(t) > h_{31}P_U^*(t) + h_{32}P_A^*(t)$

5 Local Stability Analysis

Here, we analyze the local stability analysis on the based of the following theorems:

Theorem 3: The system (3) is locally asymptotically stable of interior equilibrium point if $h_{34}h_{44} > h_{43}h_{33}$.
Proof: The variational matrix of the system (3) is given by,

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$$V_p^* = \begin{bmatrix} -h_{12} & h_{21} & h_{31} & h_{41} \\ 0 & -h_{22} & h_{32} & h_{42} \\ 0 & 0 & -h_{34} & h_{43} \\ 0 & 0 & h_{33} & -h_{44} \end{bmatrix}$$

Let ξ_p be the eigen value of V_p^*, then the characteristic equation at E_p^* is,

$$(h_{12} + \xi_p)(h_{22} + \xi_p)\{\xi_p^2 + (h_{33} + h_{44})\xi_p + h_{34}h_{44} - h_{43}h_{33}\} = 0 \quad (4)$$

The eigenvalues of V_p^* are $-h_{12}, -h_{22}$ and other eigen equation becomes,

$\xi_p^2 + (h_{33} + h_{44})\xi_p + h_{34}h_{44} - h_{43}h_{33} = 0$

Using the concept of Routh-Hurwitz criteria the above system is stable if $h_{33} + h_{44} > 0$ and $h_{34}h_{44} - h_{43}h_{33} > 0$ or $h_{34}h_{44} > h_{43}h_{33}$.

Therefore, the system (3) is LAS at interior point(E_p^*) when $h_{34}h_{44} > h_{43}h_{33}$.

6 Numerical Studies

In this section, we check and validity of all feasibility condition of fuzzy model about the diabetes. Here, all figures are plotted by MATLAB software. Discuss the dynamical behaviour of diabetes model in fuzzy environment, consider the initial condition are trapezoidal fuzzy number.

Table 1: The parameter values

Parameters	Values	Reference
$\tilde{\pi}$	$< 0.85, 0.95, 1.05, 1.15 >$	[Assumed]
$\tilde{p}$	$< 0.115, 0.145, 0.175, 0.205 >$	[45]
$\tilde{M}$	$< 1.185, 1.215, 1.245, 1.275 >$	[45]
$\tilde{\beta}_1$	$< 0.025, 0.045, 0.065, 0.085 >$	[Assumed]
$\tilde{\beta}_2$	$< 0.090, 0.110, 0.130, 0.150 >$	[45]
$\tilde{n_d}$	$< 0.011, 0.041, 0.071, 0.101 >$	[Assumed]
$\tilde{\delta}_1$	$< 0.001, 0.002, 0.003, 0.004 >$	[46]
$\tilde{\delta}_2$	$< 0.30, 0.50, 0.70, 0.90 >$	[Assumed]
$\tilde{\gamma}_1$	$< 0.02, 0.031, 0.041, 0.051 >$	[Assumed]
$\tilde{\gamma}_2$	$< 0.1, 0.2, 0.3, 0.4 >$	[Assumed]
$\tilde{h}_1$	$< 0.9, 0.11, 0.13, 0.15 >$	[Assumed]
$\tilde{m_d}$	$< 0.2, 0.025, 0.033, 0.05 >$	[Assumed]

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Consider the α-cut of the the parameters is given by,

$$(\pi)_{LT}^{\alpha} = 0.85 + (0.95 - 0.85)\alpha, (\pi)_{RT}^{\alpha} = 1.15 - (1.15 - 1.05)\alpha$$
$$(M)_{LT}^{\alpha} = 1.185 + (1.215 - 1.185)\alpha, (M)_{RT}^{\alpha} = 1.275 - (1.275 - 1.245)\alpha$$
$$(p)_{LT}^{\alpha} = 0.115 + (0.145 - 0.115)\alpha, (p)_{RT}^{\alpha} = 0.205 - (0.205 - 0.175)\alpha$$
$$(\beta_1)_{LT}^{\alpha} = 0.025 + (0.045 - 0.025)\alpha, (\beta_1)_{RT}^{\alpha} = 0.085 - (0.085 - 0.065)\alpha$$
$$(\beta_2)_{LT}^{\alpha} = 0.090 + (0.110 - 0.090)\alpha, (\beta_2)_{RT}^{\alpha} = 0.150 - (0.150 - 0.130)\alpha$$
$$(n_d)_{LT}^{\alpha} = 0.011 + (0.041 - 0.011)\alpha, (n_d)_{RT}^{\alpha} = 0.101 - (0.101 - 0.071)\alpha$$
$$(\delta_1)_{LT}^{\alpha} = 0.001 + (0.002 - 0.001)\alpha, (\delta_1)_{RT}^{\alpha} = 0.004 - (0.004 - 0.003)\alpha$$
$$(\delta_2)_{LT}^{\alpha} = 0.30 + (0.50 - 0.30)\alpha, (\delta_2)_{RT}^{\alpha} = 0.90 - (0.90 - 0.70)\alpha$$
$$(\gamma_1)_{LT}^{\alpha} = 0.020 + (0.031 - 0.020)\alpha, (\gamma_1)_{RT}^{\alpha} = 0.051 - (0.051 - 0.041)\alpha$$
$$(\gamma_2)_{LT}^{\alpha} = 0.2 + (0.2 - 0.1)\alpha, (\gamma_2)_{RT}^{\alpha} = 0.4 - (0.4 - 0.3)\alpha$$
$$(h_1)_{LT}^{\alpha} = 0.9 + (0.11 - 0.9)\alpha, (h_1)_{RT}^{\alpha} = 0.15 - (0.15 - 0.13)\alpha$$
$$(m_d)_{LT}^{\alpha} = 0.2 + (0.025 - 0.2)\alpha, (m_d)_{RT}^{\alpha} = 0.05 - (0.05 - 0.033)\alpha$$

Using the values of $\tilde{\pi}, \tilde{p}, \tilde{M}, \tilde{m_d}, \tilde{n_d}, \tilde{h_1}, \tilde{\beta_1}, \tilde{\beta_2}, \tilde{d_2}, \tilde{\delta_1}, \tilde{\delta_1}, \tilde{\gamma_1}, \tilde{\gamma_2}$, are taken from Table 1 and the α-cut mentioned as $\alpha \in (0, 0.4, 0.8, 1)$ and from this the system is clearly locally asymptotically stable. We draw he following time series taking the parametric values from table 1. Also to observe the behavior of the system's solutions $P_U(t)$, $P_A(t)$, $D_W(t)$ and $D_C(t)$ from Figure 1(a) for $\alpha = 0$ initially grow all the four variables then parallel to time axis; Figure 1(b) for $\alpha = 0.4$; Figure 1(c) for $\alpha = 0.8$; Figure 1(d) for $\alpha = 1$ for time(t) $\in [0, 200]$ with weighted function $V_1 = 1, V_2 = 0$ $P_U(t)$, $P_A(t)$, and $D_C(t)$ graph illustrate initially increasing and all figure signify locally asymptotically stable in nature which validated the analytical result. Figure 2(a) for $\alpha = 0$; Figure 2(b) for $\alpha = 0.4$; Figure 2(c) for $\alpha = 0.8$ and Figure 2(d) for $\alpha = 1$ for time(t) $\in [0, 200]$ with weighted function $V_1 = 0.4, V_2 = 0.6$ $P_U(t)$, $P_A(t)$, $D_W(t)$ and $D_C(t)$ and all figure clearly show the locally stable in nature at the given equilibrium points since all the initially increasing but after some time its goes steady way. Figure 3(a) for $\alpha = 0$; Figure 3(b) for $\alpha = 0.4$; Figure 3(c) for $\alpha = 0.8$; Figure 3(d) for $\alpha = 1$ for time(t) $\in [0, 200]$ with weighted function $V_1 = 0.8, V_2 = 0.2$ and all figure show its local asymptotical stable character of all variables of the system. Figure 4(a) for $\alpha = 0$; Figure 4(b) for $\alpha = 0.4$; Figure 4(c) for $\alpha = 0.8$; Figure 4(d) for $\alpha = 1$ for time(t) $\in [0, 200]$ with weighted function $V_1 = 0, V_2 = 1$ and $P_A(t)$ value increasing with increasing value of α and the all figure show its stable character of all variables of the system which validated the analytical result.

7 Conclusion

In this paper, the modeling of diabetes populations through compartmental analysis employs mathematical frameworks, primarily systems of differential equations, to categorize

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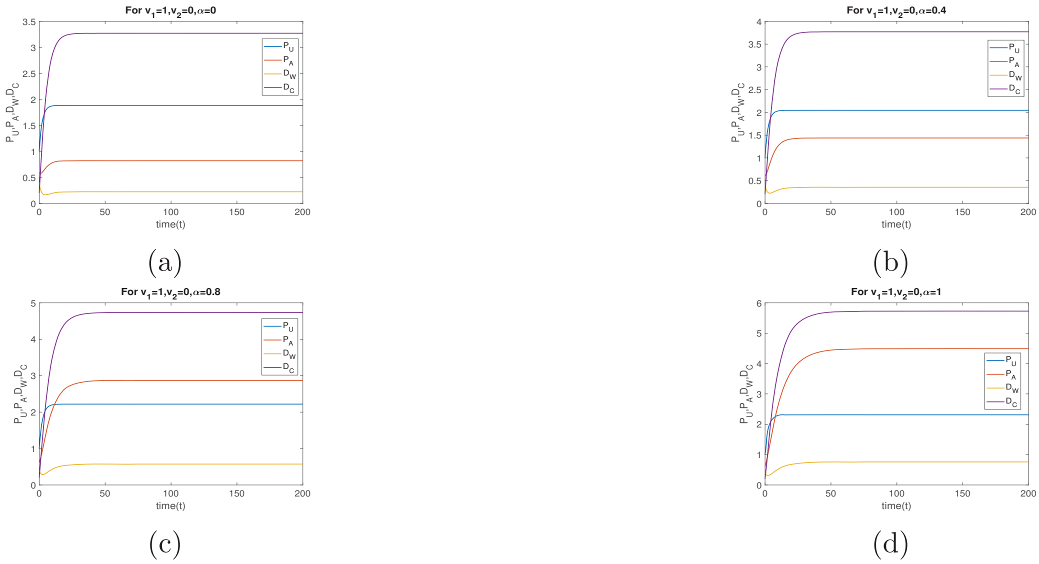

Figure 1: The figures represents time series solutions of the system (3) using the set of parameter values given in Table 1 with weighted function $V_1 = 1, V_2 = 0$ and $\alpha(= 0, 0.4, 0.8, 1)$ respectively.

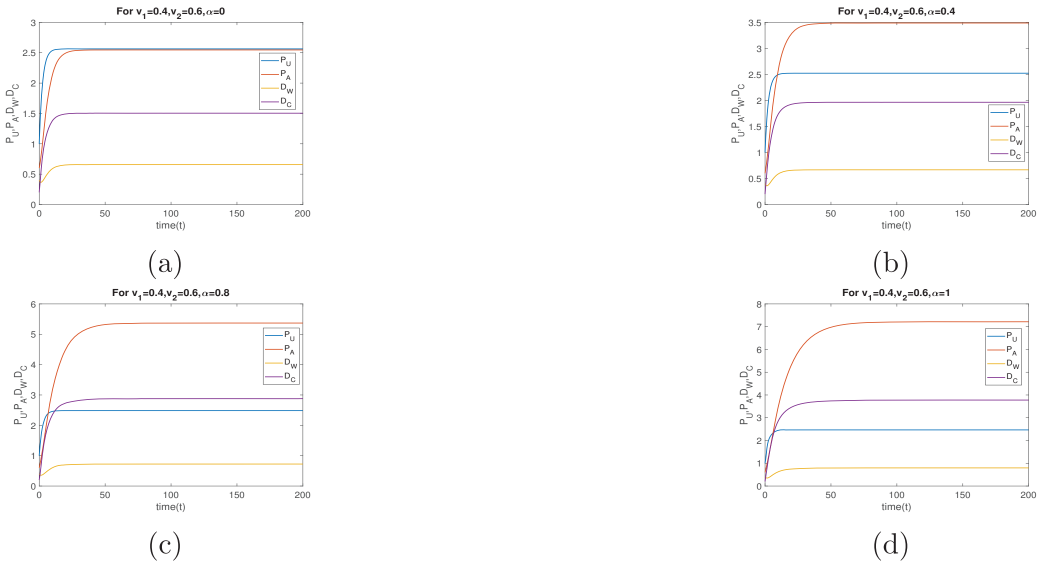

Figure 2: The system's(3) time series solutions using the set of parameter values given in Table 1 and taking the weight functions as $V_1 = 0.8, V_2 = 0.2$; $\alpha(0, 0.4, 0.8, 1)$ respectively.

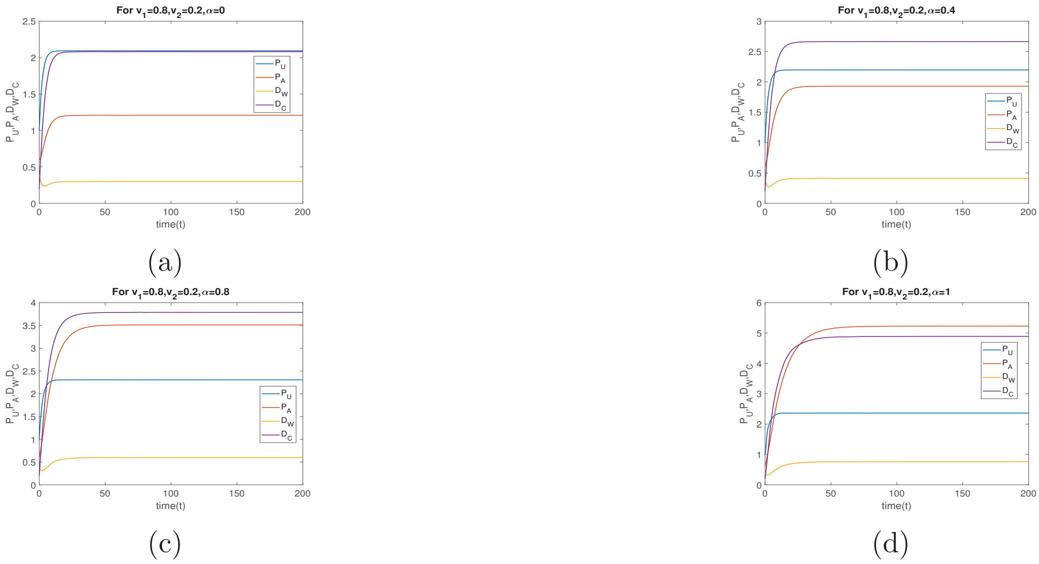

Figure 3: The figures depicts the time series solutions of the system (3) with weighted function $V_1 = 0.4, V_2 = 0.6$ and using the set of parameter values given in the above Table 1 also for different values of $\alpha(= 0, 0.4, 0.8, 1)$.

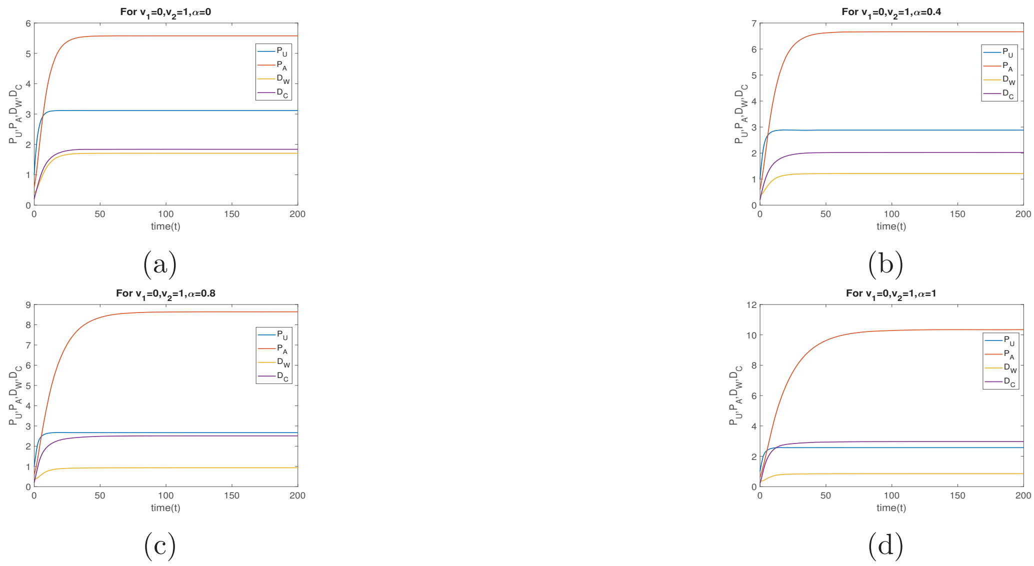

Figure 4: The time series illustrates solutions of the system (3) using the set of parameter values given in Table 1 and with weighted function $V_1 = 0, V_2 = 1$ to draw respective graph we also consider $\alpha(0, 0.4, 0.8, 1)$.

a population into specific health state to analyze and forecast the progression of diabetes, the emergence of complications and the effect of interventions. This study investigates a diabetes model where all parameters are evaluated within an uncertain context. To tackle various uncertainties and enhance the realism of the model, all parameters are represented as TIFn. The diabetes model has been considered incorporating fuzziness as a result of the inherent variability in all biological parameters. Here, the diabetes model divided into three separate compartment as P_U, P_A, D_W, D_C The stability analysis of the fuzzy model has been done and also shown stability condition. This research includes local stability analyses of the four dimensional diabetes model. In numerical simulation, all results are check and verify, we draw the different figure for different values $\alpha(= 0, 0.4, 0.8, 1)$ with different combination of weight function V_1, V_2. It is concluded that the diabetes influence of human body have been greatly affected by imprecise value of the parameters. So, we can say that the fuzzy model is more realistic than the corresponding crisp model since crisp once are the particular case of fuzzy model.In future, the proposed model can be extended using fractional calculus in intuitionistic fuzzy environment.

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Conflict of Interest: The authors declare no conflicts of interest.

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