Study: Investigation Of Phase Noise Channel Information Rate By Using Repeat Accumulate Code Design

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Abstract

Investigation of information rate is a major problem in communication. This paper considers the problem of investigation of information rate in phase noise channel which limits the performance. Lower bound for the information rate is calculated for M-ary phase shift keying modulations (M-PSK) and differential binary phase shift keying (DBPSK) modulation. Moreover, a closed form upper bound for the information rate is calculated for M-ary phase shift keying modulations (M-PSK). Finally, consider non systematic irregular repeat accumulate (RA) codes design for this phase noise channel trying to give new insights on the codes to be employed for such an application.

Keywords **─***Phase noise channel, Information rate, repeat accumulate code, iterative detection and decoding.*

1.Introduction

Investigation of information rate in a phase noise channel is one of the major impairment in communications. In some communication links, low-cost transmit and/or receive oscillators produce the phase noise. In the optical coherent systems, laser's phase noise strongly degrades the performance [2].

Previously, this phase noise is generally modeled as a Wiener process, although more accurate phase noise have been recently introduced in consumer-grade equipments to be employed in second generation digital video broadcast systems(DVB-S2) [3]. In this paper, mainly consider the random-walk (Wiener) phase noise model because it is characterized by single parameter which allows effective tuning of its strength, and also a receiver designed for the Wiener phase noise model also performs well than more accurate phase noise model. This paper suggest Wiener phase noise model because there is no significance practical difference between simplified and more accurate model [3].

The detailed problem of detection for phase noise affected channel is addressed in many papers [3]-[6]. But none of the paper gives better performance if we except the case of a constant phase offset error [7]-[9] and the case of ideal interleaving [10].The ultimate aim of the paper is to analyze and compute the information rate (IR), i.e., the average mutual information between the input and output when the channel inputs are independent and uniformly distributed (i.u.d) random variables, of the phase noise channel. Addition to design specific repeat accumulate codes through Extrinsic Information Transfer (EXIT) [11] charts for such an application, we discuss the common practice of employing, in satellite applications.

In [3], two new iterative decoding algorithms are introduced for strong face noise affected channel, achieve near-coherent performance with very low complexity. Noncoherent sequence detection algorithms for combined demodulation and decoding of coded linear modulation symbols transmitted over additive white Gaussian noise is discussed in [4]. In that paper, random rotation of the signal phase is assumed to be constant during the entire transmission which increases the complexity of the receiver with the duration of the transmission.

Evaluation of information rate (IR) for finite-state channel is introduced in [12].Although considered phase noise channel is not finite-state, this approach will be pursued by restoring to a proper auxiliary channel and deriving lower bounds on the information rate achievable by a maximum likelihood decoder for the auxiliary channel.

This paper introduce specific repeat accumulate code design for this phase noise channel trying to give new insights on the codes to be employed such an application, one alternative code design is described in [13]. In that paper, low-density parity-check (LPDC) codes over rings are designed by dividing the codeword into sub-blocks of adjacent symbols under the assumption that the phase variations over each of them are small. Two classes of check nodes are then created: the "global check nodes", spread across many sub-blocks that converge irrespective of possible rotations of a multiple of the rotational invariance angle for the employed constellation. The phase ambiguity problem can be solved by inserting "local check nodes" on each sub-block. The joint detection and decoding process is performed accordingly at the receiver end. First, the global check nodes are used and different phase estimates are produced, one for each sub-block. The phase ambiguities of the sub-blocks are then solved by using local check nodes. But this approach has the following drawbacks. Fire one is, it is not possible to tackle large amounts of phase noise, such as amount of phase noise considered in this paper. In addition, in the case of stronger phase nose and lower optimal sub-block size, increase the decoding complexity. Finally, the detection and decoding procedure has an intrinsic loss of few tenths of dBs since it does not include the local check nodes during the first stage and the degraded performance of the turbo phase estimator with reduced sub-block size. On other hand, the codes described in this paper, phase ambiguity is solved through the intrinsic differential encoding, do not require an ad-hoc coding procedure.

The remainder of this paper is organized as follows. The problem setup is discussed in section II. The lower bond of the information rate is evaluated in section III. The upper bonds of information rate for M-ary phase shift keying modulations (M-PSK) are discussed in section IV. In section V, the specific repeat accumulates code design through Extrinsic Information Transfer (EXIT) chart is discussed. The numerical results are presented in section VI. Finally , section VII provides some concluding remarks.

2.Problem Setup

CONSIDER the sequence of transmission of independent, identically and uniformly distributed complex M-ary modulation symbols $X = \left\{x_k^{(i)}\right\}_{i=0}^{M-1}$ over an additive white Gaussian channel which is affected by an unknown time-varying channel phase. Linear modulation performed at the transmitter side and assuming that one sample per symbol is adequate, if transmit and receive filters are such that there is absence of intersymbol interference, the observation model is given by,

$$y_k = x_k e^{j\theta_k} + w_k \qquad (1)$$

where $W = \{w_k\}$ noise samples are independent and identically distributed, complex, circularly symmetric, Gaussian random variables, each with variance $2\sigma^2$ and mean zero. The received samples are $Y = y_k$.

The phase noise process $\{\theta_k\}$ is commonly modeled as random-walk phase noise model or Wiener phase noise model which is described by

$$\theta_{k+1} = \theta_k + \Delta_k \qquad (2)$$

where, discrete-time real Gaussian process is $\{\Delta_k\}$ with variance σ_Δ^2 and mean zero and θ_0 is uniformly distributed in the interval $[0,2\pi)$. It follows that,

$$p(\theta_k \backslash \theta_{k-1}, \theta_{k-2},, \theta_0) = p(\theta_k \backslash \theta_{k-1}) = p\Delta(\theta_k - \theta_{k-1}) \qquad (3)$$

where, $p\Delta(\varphi)$ as the probability density function of the increment Δ_k mod 2π, which is given by,

$$p\Delta(\varphi) = \sum_{l=-\infty}^{\infty} g(0, \sigma_\Delta^2; \varphi - l2\pi) \ \ , \ \varphi \in [0,2\pi) \ (4)$$

Having denoted by $g(\eta, \rho^2; x)$ a real Gaussian pdf with mean η, variance ρ^2, and argument x. The sequence of phase increments $\{\Delta_k\}$ is supposed unknown to both transmitter and receiver and statistically independent of x and w.

Also consider the following finite-state auxiliary channel model in which the channel phases $\boldsymbol{\theta} = \{\theta_k\}$ belong to a finite set such that

$$\theta_k \in \left\{\frac{2\pi}{L} l\right\}_{l=0}^{L-1} \qquad (5)$$

The larger the number of discretization levels L, the better the approximation. The transition probabilities of the discretized model is given by

$$P_{i,j} = P\left(\theta_{k+1} = \frac{2\pi}{L} j \,\middle|\, \theta_k = \frac{2\pi}{L} i\right)$$

$$= \int_{\left(j-i-\frac{1}{2}\right)\frac{2\pi}{L}}^{\left(j-i+\frac{1}{2}\right)\frac{2\pi}{L}} \sum_{l=-\infty}^{\infty} g(0, \sigma_\Delta^2; \varphi - l2\pi)\, d\varphi$$

$$= \sum_{l=-\infty}^{\infty} \left[Q\left(\left(j-i-\frac{1}{2}\right)\frac{2\pi}{L\sigma_\Delta} - \frac{2\pi l}{\sigma_\Delta}\right) - Q\left(\left(j-\frac{1}{2}\right)\frac{2\pi}{L\sigma_\Delta} - \frac{2\pi l}{\sigma_\Delta}\right)\right] \qquad (6)$$

where $Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$ is the Gaussian Q function.

3.Evaluation of the Information Rate

A lower bound for the information rate of the channel at hand is the information rate achievable by a receiver designed for the auxiliary channel with discretized phase, defined by (1) and (5), when the actual channel is the original one with Wiener phase noise. We also expect that the larger the value of L, the tighter this lower bound. This issue, which is an instance of mismatched decoding [15], cannot be addressed in closed form, but can be solved by means of the simulation based method described in [12], which only requires the existence of an algorithm for exact maximum a posteriori (MAP) symbol detection over the auxiliary channel. For the considered auxiliary channel, MAP symbol detection is an instance of the well known Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm working on a trellis with L states.

The achievable IR for the mismatched receiver can be evaluated as

$$I(x;y) = \lim_{n\to\infty} \frac{1}{n} I(x^n;y^n)$$

$$= \lim_{n\to\infty} \frac{1}{n} E\left\{\log\frac{p(y^n|\, x^n)}{p(y^n)}\right\}\left[\frac{bits}{symb}\right] \qquad (7)$$

Where $p(y^n|x^n)$ and $p(y^n)$ are probability density functions according to the auxiliary channel model, while the outer statistical average is with respect to the input and output sequences evaluated according to the actual channel model [11]. Both $p(y^n|x^n)$ and $p(y^n)$ can be evaluated recursively through the forward recursion of the MAP detection algorithm matched to the auxiliary channel model [12]. Let us recall that the mismatched receiver can assure error-free transmissions when the transmission rate at the modulator input does not exceed I(x; y) bit/symbol.

4. Upper Bound for M-PSK Modulations

It would be also desirable to find some closed-form bounds for the IR. To obtain a closed-form result, a simple hypothesis, which is however largely verified in all practical channel conditions, is to consider $\sigma_\Delta \ll 2\pi$, such that the pdf of the phase increment (4) is practically Gaussian. We need the following preliminary result:

Theorem 1: For an M-PSK modulation, in the absence of thermal noise (i.e., $\sigma = 0$), the average mutual information is a non-decreasing function of *M*.

Theorem 2: For any M-PSK modulation over the Wiener phase noise channel, with a parameter σ_Δ, the average mutual information is upper bounded by

$$I(x;y) \le \log_2\left(\frac{\sqrt{2\pi/e}}{\sigma_\Delta}\right) \qquad (8)$$

5. RA Code Design

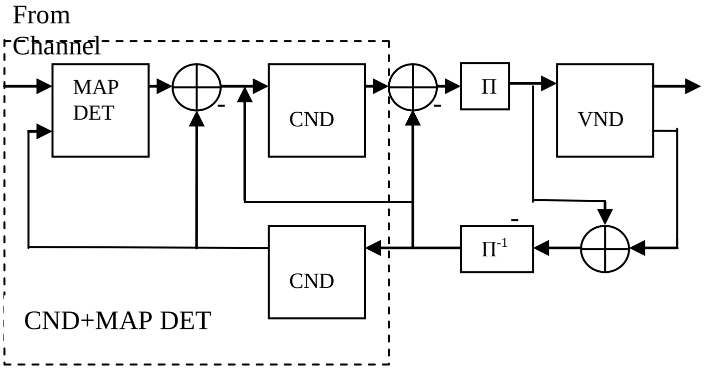

Fig. 1. Iterative receiver for a nonsystematic RA code

This chapter provides a possible solution to the problem of finding good codes for this scenario. We use EXIT charts [11] and linear programming to find the parity check degree distribution of a nonsystematic irregular RA code exhibits the largest rate. We apply a design strategy similar to that applied in [14] for a multiple-input multiple-output channel.

The iterative receiver for this case, whose scheme is shown in Fig. 1, is the serial concatenation of three soft-input soft-output (SISO) blocks: the variable node decoder (VND), the check node decoder (CND) and the MAP SISO detector for a differentially encoded modulation (MAP DET). This MAP SISO detector is based on phase quantization and is described in [6]. An interleaver (II) is placed between the CND the VND. For the optimization procedure, it is necessary to join the CND and the MAP SISO detector in a macroblock (CND+MAP DET in the figure) [14]. It is worth noticing those pilot symbols are not necessary (as it would be in the absence of differential encoding), since the detector exploits the differential nature of the transmitted data in order to avoid phase ambiguities. We use here a different notation than that of [14]. Let D_v be the number of different variable node degrees and denote these degrees by $d_{v,i}$, i=1, D_v…. The average variable node degree is

$$\overline{d_v} = \sum_{i=1}^{D_v} a_i d_{v,i} \quad (9)$$

Where a_i is the fraction of nodes having degree $d_{v,i}$. Moreover, we denote by λ_i the fraction of edges incident to variable nodes of degree $d_{v,i}$. Clearly both the a_i and λ_i sum up to one. Similarly, we denote by $D_c, d_{c,i}, b_i$ *and* ρ_i the number of different check node degrees, the i-th degree, and the fractions of check nodes and edges having degree $d_{c,i}$ respectively. Since the number of edges at the VND and CND are the same, we have $\overline{d_c}$=R$\overline{d_v}$, R being the code rate.

In order to obtain a linear optimization problem, the check node distribution has to be chosen a priori. The optimization algorithm consists of finding a variable node degree distribution, namely the $d_{v,i}$ along with their λ_i (for a given value of D_v) such that the code rate R is maximized and the EXIT chart tunnel is open.

Let us consider the average mutual information from the variable node decoder, available in [14], and that going into the VND. This latter average mutual information is a cumbersome function, computable only through numerical simulations, parameterized by the thermal noise and phase noise variances (σ^2 *and* σ_Δ^2, *respectively*) and by the check node degree distribution [14]. The optimization problem at hand can be expressed as a linear programming problem, similar to those reported in [15], where the objective function to be maximized is the code rate R, which is equivalent to

$$\max_{\{\lambda_i\}_i} \sum_i \lambda_i / d_{v,i} \quad (10)$$

and the constrains are the open tunnel condition [14], that can be formulated as to have an increasing average mutual information exchanged by the SISO components, and

$$\sum_i \lambda_i = 1, \sum_i \lambda_i / d_{v,i} \leq \overline{d_c} \quad (11)$$

the second ensuring a code rate not greater than one.

The optimization algorithm proceeds as follows. First of all, the check node distribution and the minimum and maximum tolerance values for the variable node degree are chosen. Since we are considering a non-systematic RA code, the iterative procedure can start only in the presence of check nodes with degree one. Indeed, a small amount of degree-one check nodes must be inserted in order to allow the bootstrap of the decoding algorithm (the so-called code doping [14]). We impose a biregular structure for the check node degrees, i.e., =1 with b_1 and $d_{c,2}$= d_c with b_2=0.8, where d_c is also optimized by the optimization program.

The signal-to-noise ratio is incremented step by step starting from a minimum value. For each value, several candidate values of d_c are tried, from d_c = 2 to d_c =10, the linear programming problem is solved for each value and the distribution $\{\lambda_i\}_i$ that maximizes the code rate, as well as the rate itself, are evaluated. The value of d_c which guarantees the largest rate is chosen. It is worth noticing, however, that the linear problem could not have a valid solution (i.e., a solution for which all constrain are satisfied). This happens when the signal-to-noise ratio value is too small with respect to the modulation and the channel conditions to have a reliable communication.

6. Numerical Results and Discussions

This section present, for different modulation schemes, information rate of channel with Wiener phase noise as a function of E_b/N_0, E_b being the received signal energy per information bit and N_0 is the noise power spectral density. The unconstrained capacity of the AWGN channel is given by [1],

$$C = \log_2\left(1 + R\frac{E_b}{N_0}\right) \quad (12)$$

and the information rate which instead depends on the modulation format.

In order to obtain very lower bounds for the information rate in the presence of phase noise, in the simulations used L=1024 discretization levels for the channel phase and sequence of length $n = 10^5$ symbols. Also verified that a further increase of these values do not lead to significantly different results. This number of discretization level is much higher than that necessary for a practical receiver, based on phase discretization, to obtain near optimal performance.

In fig.2, the information rate of quaternary PSK (QPSK) modulation is compared with information rate and capacity of AWGN channel. Two different values of σ_Δ are considered. In the phase noise channel, the energy loss is proportional to the increment of variance of the phase noise, i.e., for greater σ_Δ value greater energy loss. At an information rate of 1bit/symbol, the phase noise channel with variance $\sigma_\Delta = 2$ degrees exhibits 0.2 dB loss with respect to constrained AWGN capacity, the loss increases about 0.7 dB at $\sigma_\Delta = 6$ degrees. The information rate of QPSK modulation for any channel cannot go above 2bits/symbol.

In Fig.3, The upper bound (8) information rates of M-PSK modulations for different M values are plotted together with. The phase noise variance $\sigma_{\Delta} = 6$ degrees, the upper bound of information rate $I(x;y) \leq 3.85$ bits/symbol. ALL modulations such that $\log_2 M > 4$ are not able to reach their maximum value $\log_2 M = 4$ not even for very large signal to noise ratio values. The bound is tight, since the curves of all constellations with $M \geq 16$ converge to the horizontal lines for a sufficiently large.

In Fig.4, the amplitude/phase shift keying (APSK) with 32 symbols are compared with constrained capacity of AWGN channel. The APSK constellations are suited for satellite communication because of their robustness to channel non-linearities. 32-PSK is in particular built by three concentric PSKs (an inner 4-PSK, a medium 12-PSK and outer 16-PSK). Due to higher density of the considered constellation with respect to QPSK, the energy loss for a given IR from the AWGN curve is larger than QPSK. At an information rate of 3.5 bits/symbol, losses are 0.6 dB and 1.3 dB for the phase noise channel with variance $\sigma_{\Delta} = 2$ degrees and $\sigma_{\Delta} = 6$ degrees respectively.

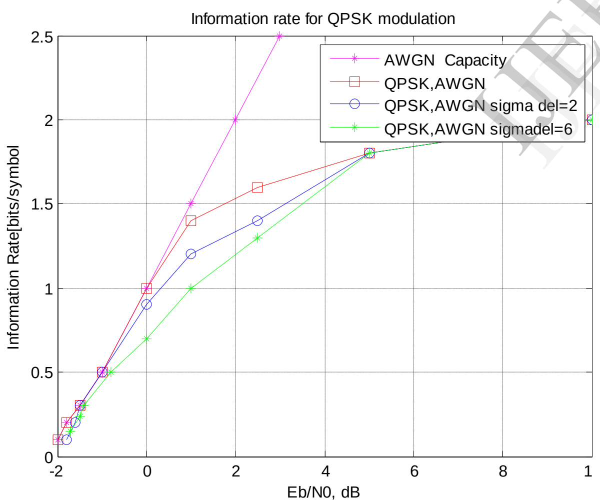

Fig.2. Information rate for QPSK modulation

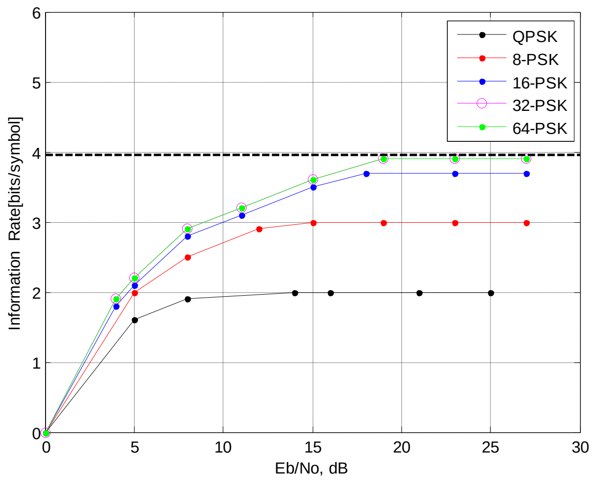

Fig.3. Upper bound of Information rate for an M-PSK modulation

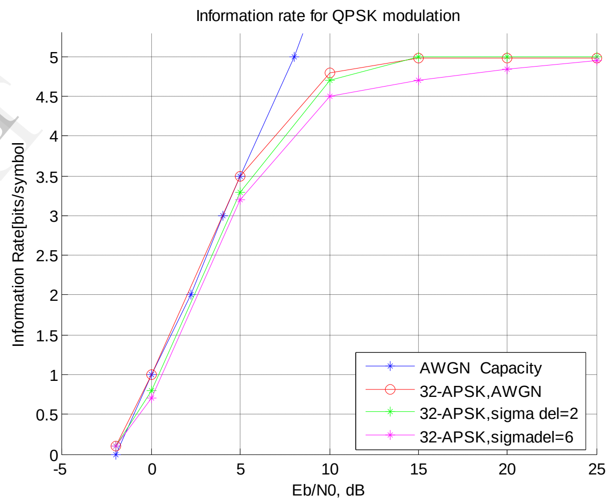

Fig.4. Information rate for a 32-APSK modulation

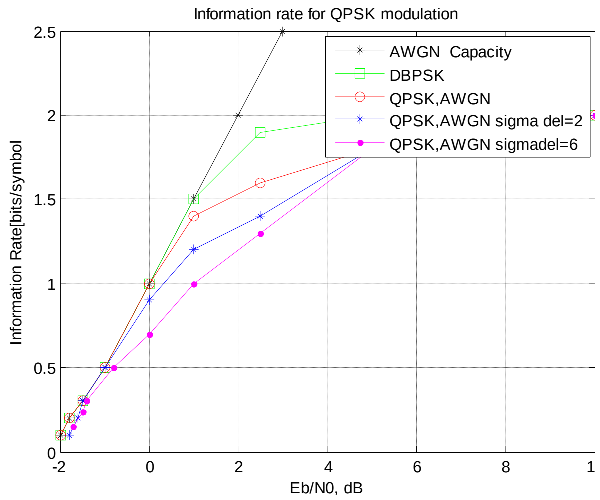

Fig.5. Information rate of QPSK modulation compared with DBPSK modulation.

In fig.5 the information rate of QPSK and DPSK modulations are compared with AWGN channel capacity. The DBPSK modulation with AWGN capacity gives better performance than QPSK modulation with AWGN capacity.

The AWGN channel, larger constellations of modulation scheme yield better information rate. This paper takes into account following modulations: M-PSK with $\log_2 M = 2,3,4,5,6$ and 32-APSK, on a channel with $\sigma_\Delta = 6$ degrees. The corresponding figure is not reported because it should not be clear do to more number of curves. But manually checked various E_s/N_0, the modulation which reaches the largest rate. The results are shown in TABLE 1.

TABLE 1

BEST MODULATION FOR DIFFERENT SIGNAL-TO-NOISE RATIO VALUES

E_b/N_0	Modulation
.< 0.2dB	16-QAM
0.2dB <.. <10 dB	16-QAM
10 dB <.	256 QAM

From the results, some interesting observations can be drawn. First, amplitude modulations are outperformed compare to PSK modulations. This can be explained by the fact that the information conveyed by the amplitude variations are not affected by the time-varying phase but affected by thermal noise only. At low signal-to-noise ratio, less dense constellations perform better.

DESIGNED CODES FOR CHANNEL WITH PHASE NOISE AND COMPARISON WITH AWGN.TWO VALUES OF HAVE BEEN CONSIDERED

TABLE II

AWGN				
E_s/N_0	d_c	$d_{v,i}$	λ_i	R
-4 dB	3	2 8 22 23	0.28 0.34 0.36 0.02	0.395
1 dB	4	3 14 15 23	0.57 0.21 0.08 0.14	0.74

TABLE III

PN with $\sigma_\Delta = 6$ degrees				
E_s/N_0	d_c	$d_{v,i}$	λ_i	R
-4 dB	3	2 7 8 25	0.28 0.34 0.36 0.02	0.37
1 dB	4	3 14 15 24	0.57 0.21 0.18 0.13	0.73

Finally, in Fig.6 show the best code rate which is the function of E_b/N_0 ($E_b/N_0 = RE_s/N_0$, E_s is the received signal energy per information symbol), compared with the IR , for a BPSK modulation with a phase noise characterized by $\sigma_\Delta = 20$ degrees. At the receiver end, the MAP SISO detector is employed with L=12 phase discretization levels. This value is sufficient to give a practically optimal performance. Set the maximum variable node degree to 25 and the minimum to 3. RA codes for both the AWGN channel and the channel affected by phase noise were designed, for different signal to noise ratio vales. As it can be seen, for both channel models the optimization procedure used was able to obtain codes with a theoretical threshold with only a negligible loss with respect to the constrained capacity, at least for rates not larger than 0.8.Better results for such large rates could be obtained by considering also variable nodes of degree 2.

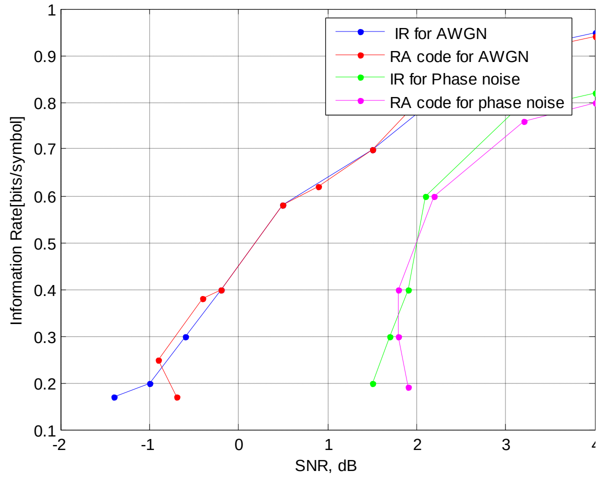

Fig .6. Code rate compared with information rate for a channel with a phase noise with σ_Δ = 20 degrees

The optimized degree distributions, for the AWGN and phase noise channel, are substantially different and codes designed for the AWGN channel exhibit a significant loss on the phase noise channel. An example is shown in table II and III where, for a phase noise channel with $\sigma_\Delta = 6$ degrees, the parameters of the optimized codes are shown for two values of E_s/N_0 and compared with those of the codes optimized for the AWGN channel.

7. Conclusion

The evaluation of information rate for a Wiener phase noise affected channel has been analyzed. The information rate is evaluated based upon the approach in [12] for a channel with memory. The lower bound of information rate is evaluated for different M-ary PSK, DBPSK and 32-APSK modulation. A simple closed-form of upper bound for the information rate of M-ary PSK modulation has been presented. Finally, a procedure for the optimization of irregular repeat accumulate code was carried out, based on EXIT charts, showing that codes with negligible losses with respect to the information rate of both AWGN and phase noise channel.

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References

[1] T.M. Cover and J.A. Thomas, *Element of Information Theory*. John Wiley & Sons , Inc., 1991.

[2] G.Colavope, T.Foggi, E.Forestireri, and G. Prati, *"Robust* multilevel coherent optical systems with linear processing at the receiver, *J.Light-wave Technol.*, vol. 27, pp. 2357-2369, July 2099.

[3] G.Colavolpe, A. Barbieri, and G.Caire, "Algorithm for iterative decoding in the presence of strong phase noise" , *IEEE J.Sel. Areas commun*, vol.23, pp. 1748-1757, Sep. 2005.

[4] G.Colavolpe and R.Raheli, "Noncoherent sequence detection," *IEEE Trans. Commun*., vol. 47 , pp. 1376-1385, Sep. 1999.

[5] G. Colavolpe, "On LPDC codes over channels with memory", IEEE Trans . Wireless Commun., vol.47, pp.1757 – 1766, July 2006.

[6] A.Barbieri and G.Colvalope , "Soft-output decoding of rotationally invariant codes over channels with phase noise," *IEEE Trans. Commun*., vol. 55 , pp. 2125-2133, Nov. 2007.

[7] M.Pleg and S.Shamai (Shitz), "On the capacity of the blockwise incoherent MPSK channel," *IEEE Trans. Commun*., vol. 46, pp.603-609, May 1998.

[8] G.Colavolpe and R.Raheli, "The capacity of the noncoherent channel ", *European Trans. Telecommun*., vol. 12, pp. 289-296, July/Aug. 2001.

[9] R. Nuriyev and A. Anastasopoulos, "Capacity and coding for the block-independent noncoherent AWGN channel", *IEEE Trans. Inf. Theory*, vol.51, pp. 866-883, Mar. 2005.

[10] M. Katz and S.S. (Shitz),"On the capacity-achieving distribution of the discrete-time noncoherent and partially coherent AWGN channel", *IEEE., Trans . Inf. Theory* , vol.50 , pp.2257-2270, Oct. 2004.

[11] S.ten Brink, "Convergence behavior of iteratively decoded parallel concatenated codes", *IEEE Trans. Commun.*, vol. 49, pp. 1727-1737, Oct. 2001.

[12] D.M Arnold, H.A. Loeliger, P.O.Vontobel, A.Kavcic, and W.Zeng, "Simulation - based computation of information rates for channels with memory"' *IEEE Trans . Inf. Theory*, vol. 52, pp.3498-3508, Aug. 2006.

[13] S.Karuppasami and W.G. Cowley," Construction and iterative decoding of LPDC codes over rings for phase-noisy channels ", EURASIP J. wireless commun. Netw., vol. 2008, 2008 . Article ID 3854212357.

[14] S. ten Brink and G. Kramer, "Design of repeat-accumulate codes for iterative detection and decoding," IEEE Trans. Signal Process., vol. 51, pp. 2764-2772, Nov. 2003.

[15] Roumy, S. Guemghar, G. Caire, and S. Verdu, "Design methods for irregular repeat-accumulate codes," IEEE Trans. Inf. Theory, vol. 50, pp.1711-1727, Aug. 2004.