Study and Analysis of Tool-Chip Interface Friction in Orthogonal Metal Cutting by Finite Element Method

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***Abstract*—Using the capabilities of finite element models, it has recently become possible to deal with complicated conditions in metal cutting. Finite element modeling makes it possible to model several factors that are present during the chip formation including friction at the chip tool interface, temperature, stress, strain, and strain rate. The aim of understanding of metal cutting is to find ways to have high quality machined surfaces, while minimizing machining time and tooling cost. In the present work, an Arbitrary Lagrangian Eulerian (ALE) finite element formulation is used to simulate the continuous chip formation process in orthogonal cutting. The ALE is an effective way to simulate the chip formation. Since ALE utilizes the combination of Lagrangian and Eulerian formulations which reduces the element distortion that causes several numerical problems.**

The study involves the turning of AISI 4140 steel using a cemented carbide cutting tool. The material properties are extracted from previously published work, including the Johnson and Cook material model. The effect of feed rate and friction coefficient on cutting forces, thrust force, stresses, strain, and temperature need to be studied. Model solutions will be obtained by using commercially available finite element package ABAQUS/Explicit, V-6.9. Model verification is accomplished by comparing the predicted results with published experimental results.

Friction behaviour at the tool-chip interface is one of the complicated subjects in metal cutting that still needs a lot of work. Several models have been presented in the past with different assumptions. In the current model, the Coulomb friction model, which assumes a constant friction coefficient, is used to model the friction. The effect of the constant friction model is considered by analyzing the FE simulation results for several friction coefficient values and comparing with the previous published experimental work. The different results obtained in FE analysis were analysis by using design of experiments in order find the dominant parameter in FE analysis of orthogonal metal cutting.

Keywords*—*ALE FE Formulation; Adoptive meshing; Explicit multi-coupled dynamic analysis; Coulomb friction model; 2D simulation in ABAQUS/Explicit V-6.9.; Optimization of simulation results.

I. INTRODUCTION

Importance of metal cutting operations can be by the observation that nearly every device in use in our society has one or more machined surface. As we know Metal cutting is a highly nonlinear and coupled thermo-mechanical process, where the coupling is introduced through localized heating and temperature rise in the workpiece, which is caused by the rapid plastic flow in the workpiece and by the friction along the tool–chip interface. For example, temperature rise of up to 1000°C has been reported in the literature [30] [5]. As such, metal cutting modeling has not been easy. Early analysis of metal cutting was based on simple models, such as the shear-angle approach proposed [6] [25] [31], Oxley also involved in proposing the thin zone and thick zone shear plane theory and the slip-line field theory by Lee and Shaffer [24]and Kudo [23] based on rigid-perfectly plastic material behaviour.

The plane-strain orthogonal metal cutting process is the one where the direction of relative movement of wedge-shaped cutting tool is perpendicular to its straight cutting edge and it has been extensively studied. Since, it provides reasonably good modeling of the chip formation on cutting edge of many metal removal processes, such as turning, milling, drilling, etc.

A computational approach using the finite element method soon became a mainstream for the analysis of machining after it has been developed. Because, it provides a nearly exact displacement and/or velocity field depending on the assumptions made while building the model for orthogonal metal cutting operation. Of course, it is continuing to find even more usage in response to quick and revolutionary developments in computer hardware.

There by many researchers related to metal cutting process have started focusing on determining the best cutting conditions, tool geometries for the process efficiency. As we were using simplified analytical method for determining the optimum parameters and related parameters, but this simplified analytical method had some limitations i.e., it cannot be applicable for complex cutting processes, where large number of interrelating parameters are present. Over the last two decades, Finite Element Method (FEM) has been most frequently used in metal cutting analysis. Various outputs and characteristics of the metal cutting process such as cutting force, shear stress, temperature, chip shape, etc., can be predicted by using FEM without doing any experiment. Therefore, FEM is applied to study and simulate metal cutting processes by using various finite element simulation techniques which are developed during the years.

An early finite element model developed by Usui and Shirakashi [59] treated steady-state metal cutting based on empirical data and assuming a rate-independent deformation behavior. Reference [54] considered the effect of friction between the chip and the tool rake face but was limited to very low cutting speeds and low strain rates and assumed as a rigid-plastic deformation. Like this there are many more development have been observed during years which are listed in the literature.

During the Finite Element (FE) analysis of metal removal process, the FEM should consider some parameters related to simulation geometry, material properties of workpiece and the tool. The material model also called flow stress which has the major role to play in the FE simulation of metal removal process because it covers the behavior of workpiece at high strain, strain rate and temperature during various process parameters, cutting conditions etc. the material model mainly depends on the temperature, strain and strain rate.

Most of the researchers normally use Jhonson and Cook (JC) material model as it take into account of effect of temperature, strain hardening and strain rates on material properties, however different techniques to model tool-chip interface were proposed by many researchers based on their experimental data.

The aim of the thesis is to evaluate the role of friction at the chip-tool interface and to develop a finite element model for varying feed rates. This model is performed as a two-dimensional (orthogonal) Arbitrary Lagrangian Eulerian (ALE) finite element model using ABAQUS/Explicit version-6.9. Then we have to study the effect of the constant friction coefficient on cutting and thrust forces, chip thickness, and tool-chip contact length were studied and analyzed using FE simulation.

The specific objectives of the thesis are as follows:

- To develop a finite element model by using the Arbitrary Lagrangian Eulerian (ALE) technique.
- To study the contact mechanism at the tool-chip interface in order to obtain better understanding of the friction behavior by considering a range of friction coefficient.
- To compare the obtained results of the proposed finite element model with the previously published measured data of workpiece AISI 4140 steel [1] [2]. The Comparison includes the feed force, cutting force, chip thickness, and contact length, as well as the shear stress, normal stress, and temperature distribution along the rake face.

A. *Introduction to metal cutting*

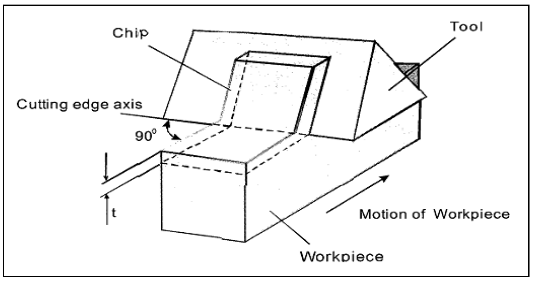

Fig. 1. Orthogonal cutting geometry

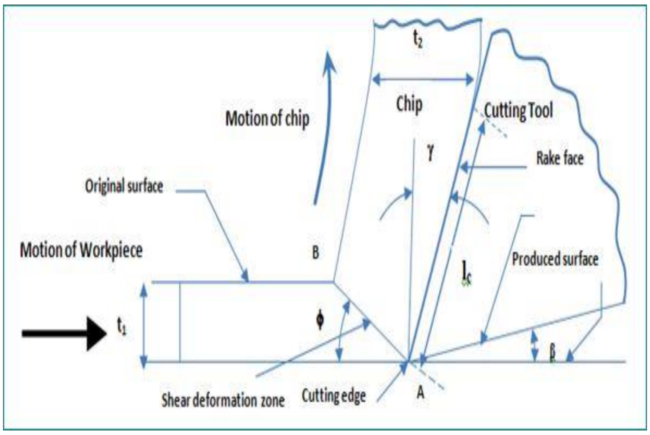

Fig. 2. Schematic illustration of two dimensional orthogonal cutting

II. LITERATURE REVIEW

Models based on the advanced numerical techniques such as Finite Element Method (FEM) provide a viable alternative for solving such problems. By using such techniques detailed information on the state of the material undergoing deformation is determined and even more realistic conditions that prevail during machining can be incorporated.

A. *Historical Review of FEM in metal cutting*

In metal cutting, the use of FEM has increased considerably well since 1970s, 1980s and 1990s. This trend suggests that the application of FEM will continue to grow. This is primarily because of the classical shear angle approach [6] [31], slip line field solution [23] and well-known thick zone model [21]. Oxley and Palmer studied metal cutting at low cutting speeds and derived slip line fields by using modified Hencky relationships. These are based on the simplistic assumptions and cannot account for some important features, such as flow stress characteristics of workpiece material, tool-chip interface friction, strain hardening built up edges (BUE). In addition rapid improvement of computer hardware and software makes the previously complicated Finite Element calculations feasible.

Huang and Black presented a detailed evaluation of the chip separation criteria [32]. Klamecki developed one of the first FE models of cutting [7]. This model of the incipient chip formation was a three-dimensional model and strain-rate effect and friction were not included. Later, this model was improved by applying thermo-elastic-plastic deformation. However, temperature softening of work material assumed overcoming the strain-rate effects and the latter was neglected. Reference [8] used FE simulations to predict temperatures in steady state orthogonal machining. Reference [25] performed rigid-plastic FE simulations of steady state orthogonal cutting process. Their model was able to solve the simultaneous equations of stress equilibrium, stress-strain increment relationship, heat transfer, material flow stress and frictional stress characteristics in the chip-tool interface. They also developed a friction model including both sticking and sliding actions.

Reference [26] used an updated Lagrangian FEM model and utilized a chip separation approach based on effective strain criteria. Reference [19] also developed a thermo-elastic-plastic FEM model based on updated Lagrangian method and used strain energy density as a chip separation criterion.

Reference [17] developed an Arbitrary Lagrangian Eulerian (ALE) formulation to simulate orthogonal cutting to avoid frequent re-meshing for chip separation. Reference [16] also used ALE formulation to simulate 3-D cutting process. Reference [28] used an updated Lagrangian implicit formulation with automatic re-meshing for chip separation. Ueda and Reference [27] used a numerical approach and simulated the chip formation in the cutting process. Reference [18] introduced an explicit dynamic FEM method to simulate orthogonal cutting with sharp edge but a restricted contact cutting tool. Reference [15] used ALE formulation to simulate cutting but using a sharp edge tool. Reference [9] used ALE formulation exclusively to simulate cutting with blunt edge tools.

Reference [11] developed flow stress and friction models and simulated chip flow in orthogonal cutting and milling with a round edge tool. Reference [12] simulated adiabatic shearing and serrated chip formation when cutting with a sharp edge cutting tool using Lagrangian analysis with re-meshing and by employing a damage criteria. Reference [14] used dynamic explicit Lagrangian analysis and reference [10] constitutive model and data for metals subjected to large strains, high strain rates and high temperatures shear failure criteria in conjunction with removal of damaged elements to simulate serrated chip formation using a sharp cutting edge tool. Reference [13] developed an explicit Lagrangian model with re-meshing capability, utilized a similar damage criteria for adiabatic shearing and simulated serrated chip formation and deformations at microstructure-level when orthogonal cutting with a round edge tool.

The specific mesh formulations used for models of orthogonal machining are Lagrangian, Eulerian, and Arbitrary Lagrangian Eulerian. Eulerian and Arbitrary Lagrangian Eulerian. The advantages and disadvantages of these formulations are discussed below.

B. *Outcome of Literature Review*

Simulation of orthogonal metal cutting operation is very popular in recent days and there are many of researchers working in this area. Since from the second half this century, a lot work has been carried out for developing successful models of orthogonal metal cutting operations. During this period, many of different aspects of the models have been studied. For example, there are varieties of separation criteria to separate chip from the workpiece. The same is also true for friction condition and material modeling used in the metal cutting operation works. In addition, most of the researchers have verified their results with only one or two process variable, such as cutting or thrust forces.

However, the systematic way to consider more number of output parameters variation are not being considered in most of review papers. Therefore, this work is intended to validate more number of output parameters of FE model of orthogonal metal cutting operation with the experimental results (i.e., with experimental results of previously conducted experiments by various researchers). This work also includes the design of FE analysis results where we will be able to obtain the most influencing parameters on the FE analyzed results. Since good agreement can be obtained in individual results by tuning the process parameters, but in this is expected to give interaction results which are more helpful in design point of view. At the end, the effects of various process parameters, such as friction or chip separation criteria, feed will be clear.

III. FINITE ELEMENT MODELLING

A. *Model Definition and Assumptions*

Finite Element Model was developed by using ABAQUS/Explicit V6.9 for the simulation of orthogonal metal cutting of AISI 4140 steel with continuous chip under plane strain condition (plane strain condition is an assumption for two dimensional analysis in any case either in FEM or in case of analytical Model).The ABAQUS/Explicit V6.9 is Finite Element Analysis software which can simulate large deformation accompanied by elastic, plastic, thermal, and friction effects. The element type used is bilinear (four-noded) with reduced integration and hourglass control to deal with the large deformation. The dynamic displacement-temperature explicit method is followed to analyze this FE model with a combination of input parameter such as temperature. In order to simplify the analysis, a perfectly rigid cutting tool is assumed because of the significantly high elastic modulus of most of the tool materials.

To simplify the analysis, it is better to consider some parameters from the beginning so that the goal of the analysis will be achieved efficiently.

The basic assumptions which are required to be made in the present model are considered as follows:

- The initial temperature of both the workpiece and tool is 25°C (room temperature).
- The cutting tool is sharp with a 5µm cutting edge radius.
- Constant cutting velocity of 200 m/min. is taken.
- Tool wear is neglected in this study so that the running time will be reduced.
- Dry cutting i.e. without coolant is assumed.
- Homogenous material is used to model the workpiece material.

B. *Material Properties*

In FEM model, workpiece and tool material properties and its flow stress model are important inputs. The workpiece material is AISI 4140steeland is modelled by using Jhonson and Cook (JC) material model (JC material model is available in the FEA software's material model library of ABAQUS/Explicit V6.9). Cutting tool material is of Cemented Carbide of grade P10.The details of tool and workpiece material properties are given in following sections.

TABLE I. TOOL MATERIAL PROPERTIES

Density	10600kg.m^{-3}
Young's modulus	520GPa
Poisson's ratio	0.22
Specific heat	200J.kg^{-1}.°C^{-1}
Conductivity	25W m^{-1}°C^{-1}
Expansion	7.2µm m^{-1} °C^{-1}

C. *Work piece material properties*

The workpiece material used for the plane strain orthogonal metal cutting simulation is AISI 4140steel. The physical properties of workpiece material are given in TABLE II. [1] [2] [3]

TABLE II. PHYSICAL PROPERTIES OF WORKPIECE MATERIAL

Density	7800 kg-m-3
Young's modulus	210 GPa
Poisson's ratio	0.3
Specific heat	561 J.kg-1°C-1
Melting temperature in °C	1520°C
Inelastic heat fraction	0.9
Conductivity	42.6 Wm-1°C-1
Expansion	14.6 µm m-1°C-1

D. *Johnson Cook equation*

$$\bar{\sigma} = (A + B(\bar{\varepsilon})^n)\left(1 + C\ln\left(\frac{\dot{\bar{\varepsilon}}}{\dot{\bar{\varepsilon}}_0}\right)\right)\left(1-\left(\frac{T-T_{room}}{T_{melt}-T_{room}}\right)^m\right)$$

TABLE III. JOHNSON COOK EQUATION COEFFICIENTS

Material	**A(MPa)**	**B(MPa)**	**n**	**C**	**M**	$\dot{\varepsilon}_0$
AISI 4140	598	768	0.2092	0.0137	0.807	0.001

TABLE IV. CUTTING CONDITIONS AND TOOL GEOMETRY

Cutting conditions	Feed (mm/rev)	0.1, 0.2, 0.3
	Coefficient of friction	0.24,0.32,0.4
	Cutting speed m/min	200
Tool geometry	Rake angle (degree)	6
	Clearance angle(degree)	6
	Cutting edge radius(µm)	5

The sequence of steps required for FE modeling using ABAQUS/Explicit v6.9 is described below.

- Part Modeling
- Assigning Material properties and Cross Sectional Properties
- Assembling the Parts
- Mesh Generation
- Applying Boundary Conditions
- Applying Interaction properties
- ALE Adaptive Meshing
- Defining the type analysis
- Field output and History output requests
- Submit the job for simulation
- Visualization of Results

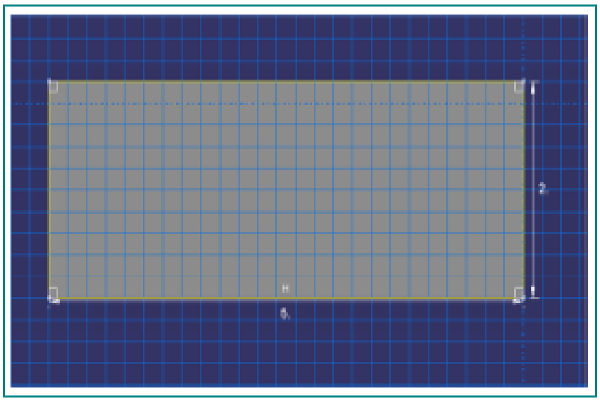

Fig. 3. Modeling of workpiece

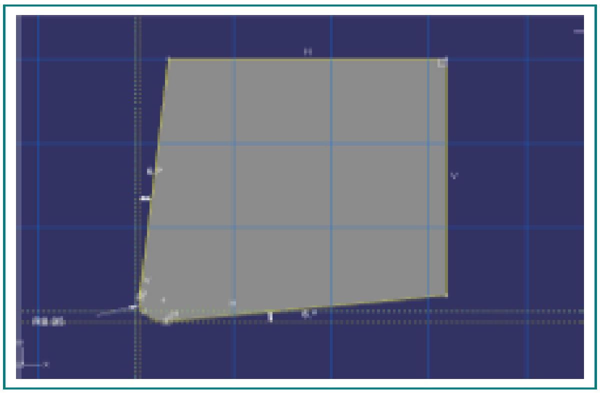

Fig. 4. Modeling of cutting tool

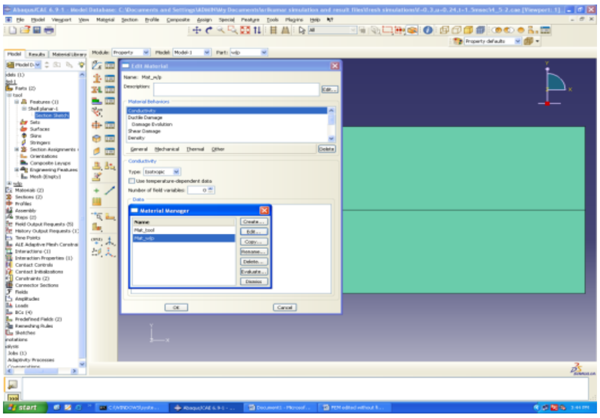

Fig. 5. Assigning material properties

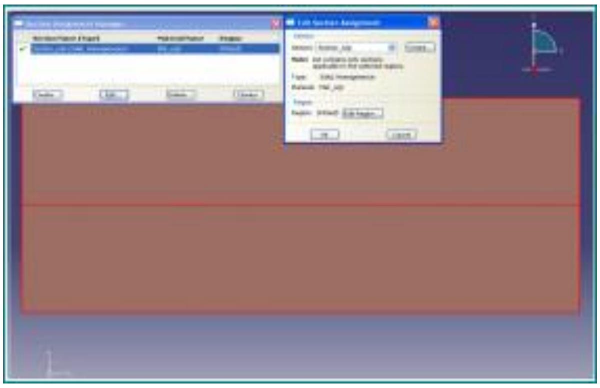

Fig. 6. Assigning cross-sectional properties

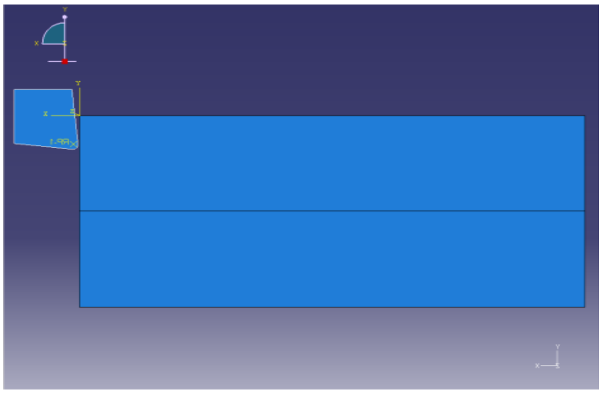

Fig. 7. Showing assembly of workpiece and tool model

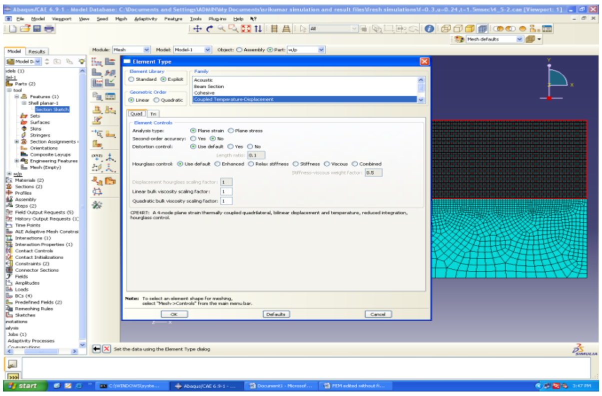

Fig. 8. Shows the selection of element type for workpiece

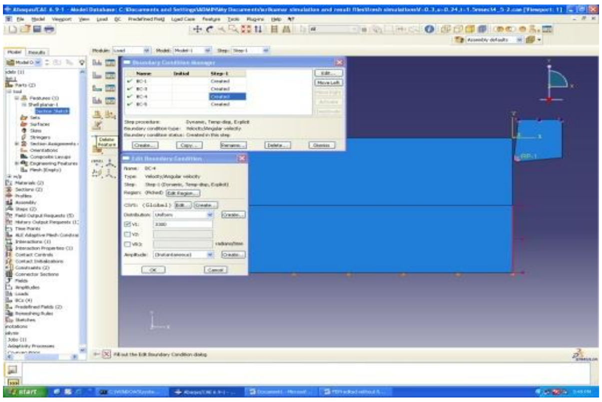

Fig. 9. Shows the boundary condition and velocity of the workpiece

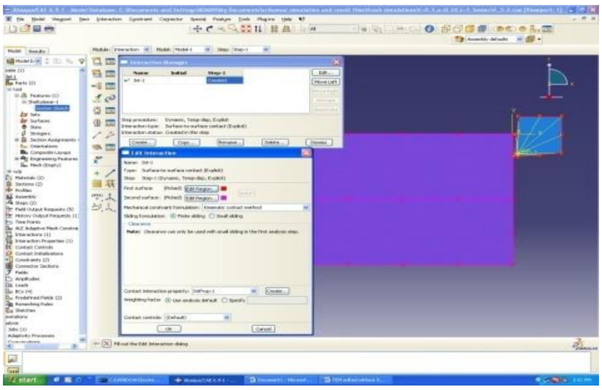

Fig. 10. Defining the interaction between tool and workpiece

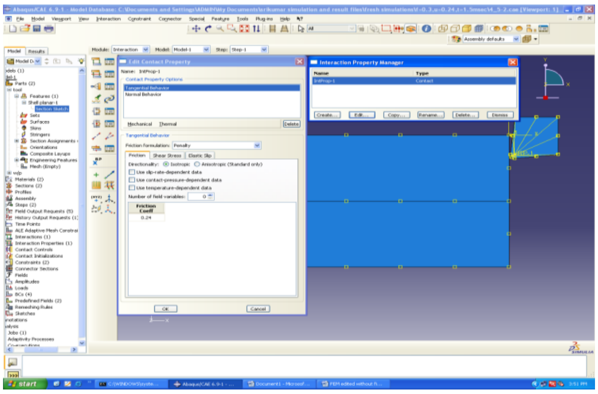

Fig. 11. Assigning friction value at tool-chip interface

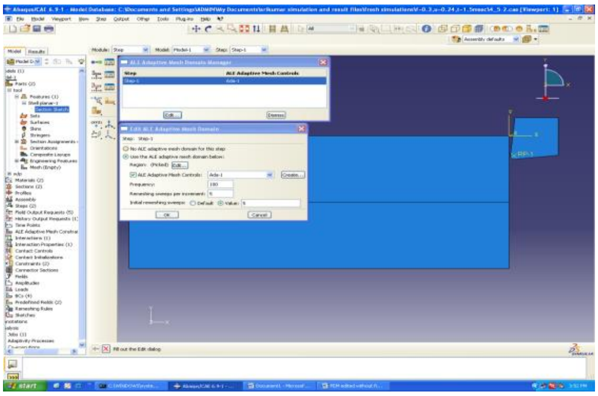

Fig. 12. Applying ALE adoptive meshing

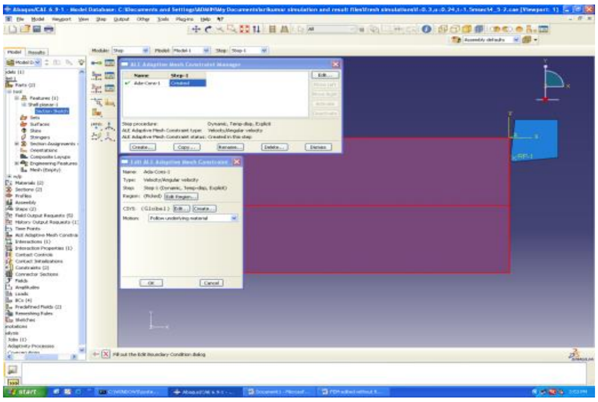

Fig. 13. Applying ALE adoptive mesh constraint

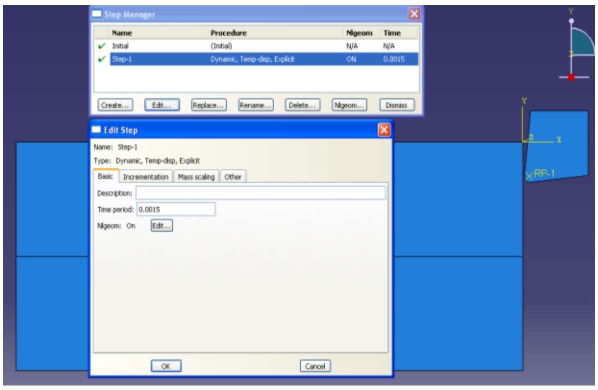

Fig. 14. Defining the tool simulation time period

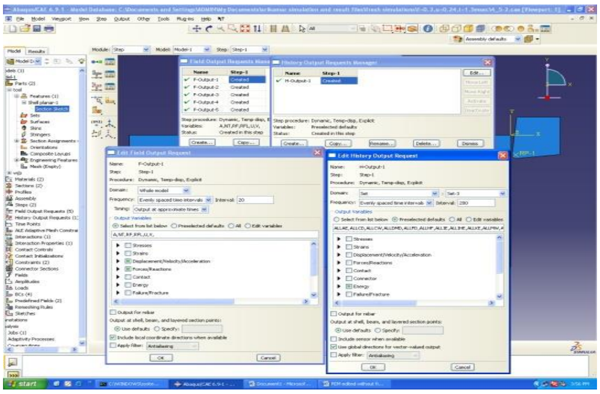
Fig. 15. The two types of output request windows are shown

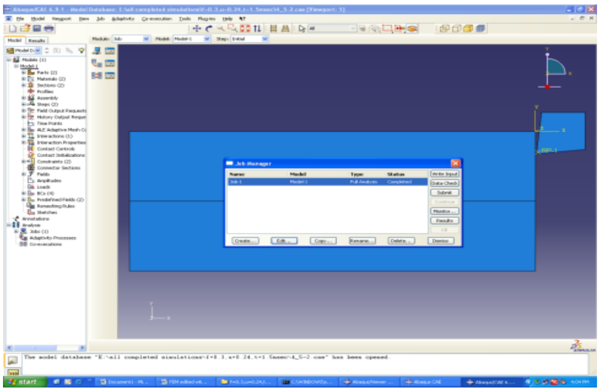
Fig. 16. Simulation completed for full analysis

Visualization of Results:

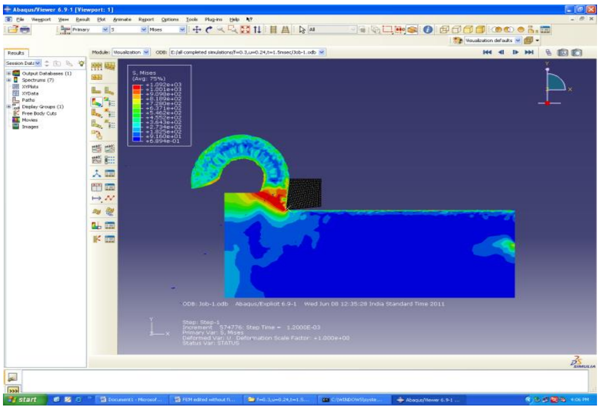
Fig. 17. Von-mises stress variation

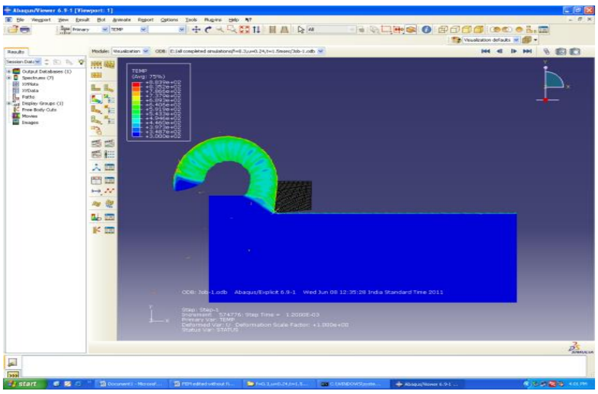
Fig. 18. Temperature effect on chip and rake force of cutting tool

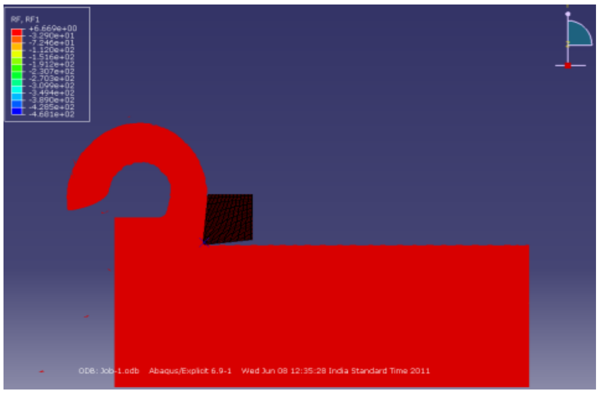
Fig. 19. The reaction force (cutting force) RF1

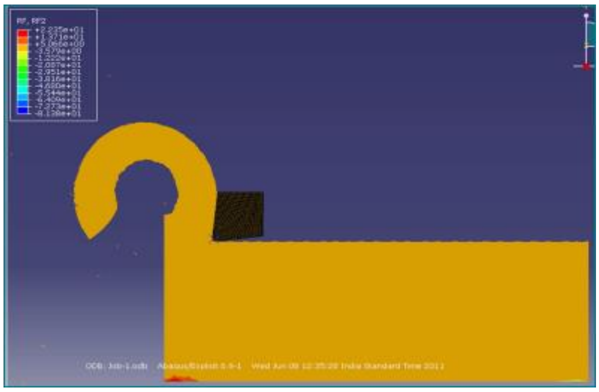
Fig. 20. The reaction force (thrust force) RF2

IV. RESULTS AND DISCUSSION

TABLE V. RESULTS OBTAINED OUT OF EXPERIMENTS

The friction value obtained out of 3 trials of experiment	Coefficient of friction
trial 1	0.23
trial 2	0.30
trial 3	0.40

Three trials of experiments were done to estimate the error in different values of coefficient of friction.

TABLE VI. SOME OF THE OTHER OUTCOMES OF EXPERIMENT

Serial No.	Feed (mm/rev)	Cutting force (N)	Thrust force (N)	Chip thickness (mm)
1	0.05	105	57	0.09
2	0.1	205	95	0.19
3	0.15	300	117	0.29
4	0.2	395	135	0.34
5	0.25	460	140	0.42
6	0.3	540	145	0.5
7	0.35	615	155	0.58

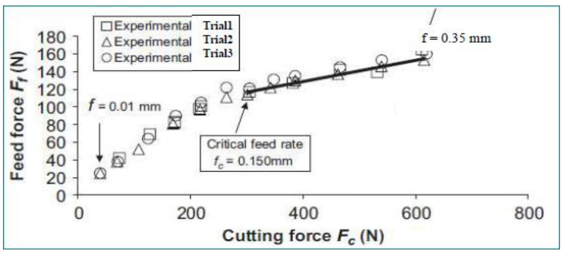

Fig. 21. Forces decomposition in the Albrecht's method obtained from Experimental tests for all 12 feed values [4].

TABLE VII. THE SIMULATION RESULTS FOR CUTTING FORCE, THRUST FORCE AND CHIP THICKNESS.

Serial No.	Feed (mm/rev)	Coefficient of friction	Cutting force (N)	Thrust force (N)	Chip thickness (mm)
1	0.1	0.24	203	63.45	0.186
2	0.2	0.24	310.9	65	0.288
3	0.3	0.24	450.3	69	0.4

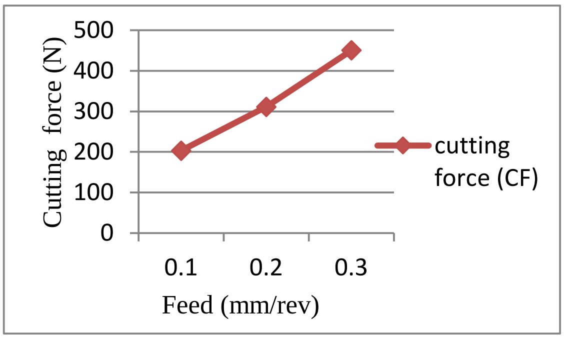

Fig. 22. Graph of feed rate versus thrust force obtained in FE modeling

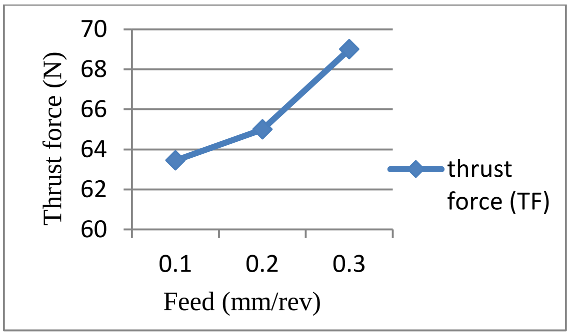

Fig. 23. Graph of feed rate versus cutting force obtained in FE modeling

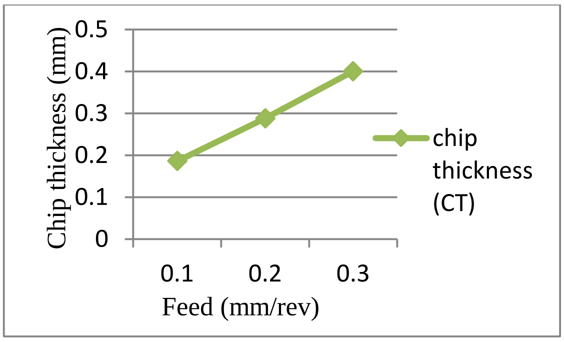

Fig. 24. Graph of feed rate versus chip thickness obtained in FE modeling

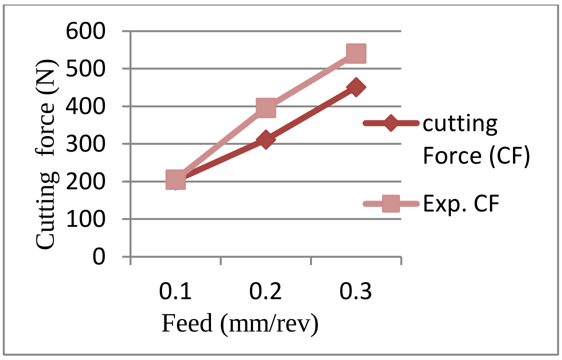

Fig. 25. Graph of comparison of cutting force

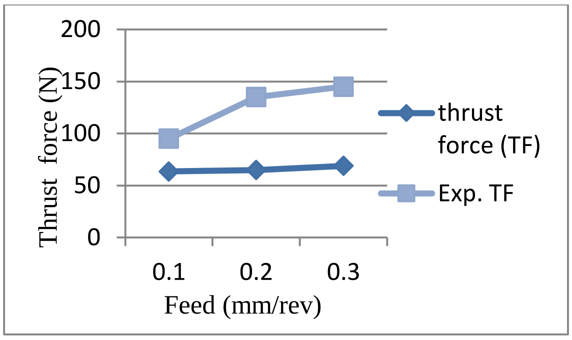

Fig. 26. Graph of comparison of thrust force

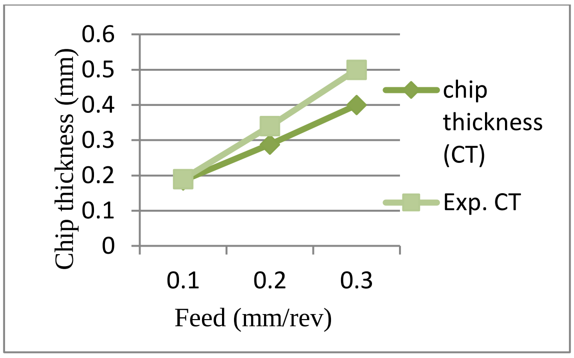

Fig. 27. Graph of comparison of chip thickness

V. CONCLUSION AND FUTURE WORK

Following conclusions are drawn from the present work

- Finite Element simulation model for orthogonal metal cutting was successfully made and validated.
- Linear model is fitted for cutting force, chip thickness, and chip-tool contact length. Whereas 2 factorial interaction (2FI) model was fitted for thrust force and temperature.
- The response surface equation for cutting force, thrust force, temperature, chip thickness, and chip-tool contact length are obtained.
- In the model of cutting force, the feed rate is more significant than coefficient of friction with 56.73%, 39.36% contribution in total variability of model, respectively.
- In thrust force model: the coefficient of friction, feed rate and combined effect of feed and coefficient of friction are significant factors with 52.83%, 32.8% and 9.73% contribution in the total variability of model, respectively.
- For temperature model: The main effects and interaction between main effect are observed significant. The feed rate is most influencing factor with 60.85% contribution. Next to feed rate is coefficient of friction with 35.92% of contribution. The least influencing factor is interaction effect of feed and coefficient of friction with 2.86% contribution.
- Linear model was fitted for chip thickness. Both feed rate and coefficient of friction are significant; feed rate provides primary contribution of 55.33% and coefficient of friction with 44.66% contribution.

- Tool-chip contact length model: in this feed rate is dominating parameter with 52.94% contribution and second most dominating parameter is coefficient with only 41.76% contribution in the total variability of model.
- The effect of coefficient of friction at tool-chip interface is quite significant on thrust force and temperature. But still further effect of coefficient friction on other parameters can be studied by varying friction value to higher range.
- The contour plots of thrust force and temperature model indicates that the different thrust force on each streamline can give different feed rate, coefficient of friction and the different temperature on each streamline can give different feed and coefficient of friction. These contour plots can be used for selecting cutting parameters for providing desired output.

Future work

- Perform more complicated models of metal cutting processes like 3-dimensional oblique metal cutting process.
- Study the effect of the workpiece hardness in the metal cutting model because it influences the results of the cutting, thrust force and the shape of the chip. Also the effect of hardness on residual stresses can be studied and analysed.
- Different workpiece and tool material combinations for FE analysis can be tried and verified with experimental results.
- Study and analysis of orthogonal metal cutting by considering the effect of friction, feed rate, cutting velocity, tool geometry variation.

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