International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 04 , April - 2026

Structural Health Monitoring of a 3-DOF System using Singular Value Decomposition and Frequency Domain Decomposition

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Abstract - As modern civil infrastructure grows in both complexity and age, traditional manual inspection methods have become increasingly insufficient for detecting localized or internal degradation. This has led to the development of Structural Health Monitoring (SHM), a continuous, data-driven approach that utilizes sensor networks to diagnose structural integrity. However, the primary challenge in modern SHM is managing the vast amounts of noisy, high-dimensional data generated by these sensors. This paper investigates the application of Singular Value Decomposition (SVD) as a robust mathematical framework for transforming raw "digital noise" into actionable engineering insights. By factoring structural response data into left singular vectors (spatial mode shapes), singular values (mode prominence), and right singular vectors (temporal evolution), SVD allows for the precise separation of true structural signals from measurement and environmental noise. The methodology is demonstrated through a 3-Degree-of-Freedom (3-DOF) system analysis using Frequency Domain Decomposition (FDD). By performing SVD on the Power Spectral Density (PSD) matrix of acceleration data, the system's natural frequencies and mode shapes are identified under ambient excitation. The effectiveness of this approach was tested by simulating a 20% stiffness reduction in the ground floor of a three-story shear building. Results indicate that SVD-based monitoring provides clear indicators of structural distress, including a 7.7% drop in the fundamental frequency and a significant reduction in the Modal Assurance Criterion (MAC) value to 0.86. These findings confirm that monitoring the trajectory of primary singular values and vectors provides a reliable "digital fingerprint" for building health. Ultimately, the SVD approach effectively bridges the gap between raw data and informed engineering decisions, ensuring degradations are identified long before catastrophic failure occurs.

Keywords: Structural Health Monitoring, Singular Value Decomposition, Frequency Domain Decomposition, Operational Modal Analysis, Modal Assurance Criterion.

INTRODUCTION

The rapid urbanization of the 21st century has led to the construction of increasingly complex and ambitious civil infrastructure, from sprawling cable-stayed bridges and high-rise skyscrapers to expansive industrial complexes. However, as this infrastructure ages, it is subjected to relentless environmental stressors, fatigue from operational loading, and potential damage from extreme events like earthquakes or hurricanes. Traditional manual inspections, while essential, are often infrequent, subjective, and unable to detect internal or localized degradation before it reaches a critical state. This has necessitated the rise of Structural Health Monitoring (SHM) a continuous, data-driven process of diagnosing the "health" of a structure using a network of sensors. At the heart of modern SHM lies a significant challenge: Big Data. A single bridge instrumented with accelerometers, strain gauges, and temperature sensors can generate gigabytes of noisy, high-dimensional information every day. To transform this raw "digital noise" into actionable engineering insights, researchers have turned to advanced linear algebra, specifically Singular Value Decomposition (SVD). Often described as the "Swiss Army Knife" of matrix factorization, SVD provides a robust mathematical framework for decomposing complex structural responses into their most fundamental spatial and temporal components. In the context of SHM, structural response data is typically organized into a large rectangular matrix. SVD factors this matrix into three distinct components: the left singular vectors, which represent the spatial coordinates or mode shapes of the structure; the singular values, which quantify the energy or prominence of those modes; and the right singular vectors, which track the temporal evolution of the vibrations. Mathematically, this is expressed as $A = U \sum V^T$. For a structural engineer, this decomposition is transformative. It allows for the separation of the underlying structural "signal" from the chaotic "noise" inherent in sensor measurements. By focusing only on the primary singular values, engineers can strip away measurement errors and environmental interference, revealing the true mechanical behavior of the asset.

BACKGROUND

The theoretical roots of Singular Value Decomposition (SVD) trace back to the late 19th century, with foundational work by mathematicians such as Eugenio Beltrami and Camille Jordan. However, its transition from an abstract linear algebra concept to a

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cornerstone of Structural Health Monitoring (SHM) only accelerated with the digital revolution of the late 20th century. Historically, structural assessment relied heavily on visual inspections and simplified analytical models. As modern infrastructure grew more complex—encompassing long-span bridges and high-rise buildings—engineers required more precise, data-driven methods to characterize dynamic behavior. In the early stages of structural monitoring, researchers primarily used the Fast Fourier Transform (FFT) to analyze signals in the frequency domain. While revolutionary, FFT-based methods often struggled with "noisy" field data and "closely spaced modes," where two natural frequencies are nearly identical. This limitation was critical for large-scale structures like cable-stayed bridges, where wind and traffic create a chaotic environment that can mask the structure's true mechanical signature. The breakthrough occurred with the development of Subspace-based Identification and Frequency Domain Decomposition (FDD). By representing multi-sensor data as a large matrix, engineers realized that SVD could act as a sophisticated spatial filter. Unlike standard peak-picking methods, SVD identifies the "principal directions" of vibration, separating the signal into orthogonal vectors where structural information is concentrated in the highest singular values, while measurement noise is relegated to the lower ones. Brincker, Zhang, and Andersen (2001) introduced FDD, demonstrating that performing SVD on the Power Spectral Density (PSD) matrix of structural responses allows for the identification of modal parameters even under unknown "ambient" excitation (like wind or traffic). Peeters and De Roeck (1999) highlighted SVD's role in identifying the "system order" and "observability matrix" of a structure. By applying SVD to a Hankel matrix, they proved that structural signals could be effectively separated from the "null space" of noise. Yan et al. (2005) focused on using the U matrix (singular vectors) as diagnostic features. Their research showed that as a structure degrades, its primary singular vectors shift, allowing for damage detection even amidst environmental fluctuations like temperature changes. While early research established SVD as a tool for finding natural frequencies, subsequent studies moved toward using it as a diagnostic "fingerprint" for structural integrity. Magalhães, Cunha, and Caetano (2009) in "Online Automatic Operational Modal Analysis of Structures" addressed the practical challenge of continuous monitoring. They utilized SVD to automate the identification process, removing the need for an engineer to manually "pick peaks" from a graph. Their research on the Infante D. Henrique Bridge proved that SVD-based algorithms could track modal parameters in real-time for years, providing a baseline for "normal" structural behavior. Döhler and Mevel (2013) in "Subspace-based Fault Detection with Robustness to Changes in the Excitation" introduced a more mathematical rigorous use of SVD’s "Null Space." They proposed that the singular vectors associated with noise (the smallest singular values) define a "rejection space." If new sensor data significantly projects onto this noise space, it indicates a structural change (damage) rather than just random vibration. This is often cited as a breakthrough for reducing "false alarms" in SHM. A major hurdle in SHM is that temperature changes can shift frequencies more than actual damage does. Kullaa (2003) in "Damage Detection of the Z24 Bridge Using Control Charts" utilized SVD-related techniques to solve this. By applying SVD to a matrix of features collected over different seasons, the research showed that environmental effects occupy the first few "principal components."By "filtering out" these dominant environmental singular values, the remaining data becomes a pure indicator of the structure's physical health.

METHODOLOGY

The methodology for monitoring the structural health of a 3-Degree-of-Freedom (3-DOF) system using Singular Value Decomposition (SVD) follows a rigorous process of data acquisition, signal processing, and feature extraction. This approach, primarily rooted in Frequency Domain Decomposition (FDD), allows for the identification of modal parameters—natural frequencies and mode shapes—from output-only data, such as ambient vibrations caused by wind or traffic. The first phase involves collecting time-series acceleration data from sensors (accelerometers) placed at each degree of freedom—typically each floor of the 3-story structure. These sensors record the dynamic response y(t) of the system over a specified duration. To ensure data quality, the signals undergo pre-processing, which includes: Trend Removal: Eliminating DC offsets or low-frequency drifts. Band-pass Filtering: Isolating the frequency range of interest 0.5 to 50 Hz where structural modes are expected. Windowing: Applying Hanning or Hamming windows to segments of the data to reduce spectral leakage during subsequent Fourier transformations.

Construction of the Power Spectral Density (PSD) Matrix: The time-domain data is converted into the frequency domain using the Fast Fourier Transform (FFT). For a 3-DOF system, the relationship between the various sensor outputs is captured in a 3 times 3 spectral density matrix, $\boldsymbol{G_{yy}(\omega)}$. This matrix contains the auto-spectral density of each sensor on the diagonal and the cross-spectral density between different floors on the off-diagonal elements. This matrix is crucial as it represents how energy is distributed across frequencies and spatial locations simultaneously. Decomposition via SVD: At every discrete frequency increment $\boldsymbol{\omega_i}$, the PSD matrix $\boldsymbol{G_{yy}(\omega_i)}$ decomposed using SVD:

$$\boldsymbol{G_{yy}(\omega_i) = U_i \textstyle\sum_i V_i^H}$$

Here, $\sum_i$ is a diagonal matrix containing the singular values (σ_1, σ_2, σ_3) in descending order. The first singular value $\sigma_1$represents the dominant power at that frequency. When $\sigma_1$is plotted against frequency, the resonant peaks correspond to the natural frequencies of the 3-DOF system. The corresponding first column of the U matrix represents the singular vector, which provides an estimate of the mode shape at that specific resonance. Feature Extraction and Damage Identification: The final phase involves comparing the "current" state of the 3-DOF system against a known "healthy" baseline. Two primary features are extracted for

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 04 , April - 2026

health assessment: Frequency Shift: A reduction in the peak frequency of σ_1 suggests a loss of stiffness 5Hz peak shifting to 4.7 Hz indicates potential damage in the columns).Modal Assurance Criterion (MAC): The mode shape vectors U from the healthy and current states are compared. The MAC value, ranging from 0 to 1, quantifies the correlation between shapes. A MAC value significantly lower than 1.0 (typically < 0.90) indicates a change in the structural configuration, effectively localizing where the stiffness change has occurred. By relegating measurement noise to the lower singular values σ_1, σ_2, the SVD methodology ensures that the health diagnosis is based on the true mechanical behavior of the structure, providing a robust, automated framework for long-term monitoring. By relegating measurement noise to the lower singular values (σ_1, σ_2), the SVD methodology ensures that the health diagnosis is based on the true mechanical behavior of the structure, providing a robust, automated framework for long-term monitoring.

RESULT

The dynamic response of the 3-story shear building was analyzed under ambient white noise excitation to evaluate the efficacy of the SVD-based monitoring framework. By processing the acceleration data from each floor, the Power Spectral Density (PSD) matrix was constructed and decomposed at each frequency increment. The results provide a clear distinction between the "Healthy" baseline and the "Damaged" structural state. Modal Parameter Extraction (Healthy State): In the baseline configuration (k1 = k2 = k3 = 1000 kN/m, the plot of the first singular value (σ_1) reveals three distinct, high-energy peaks. These peaks identify the natural frequencies of the system at 5.32 Hz, 14.85Hz, and 21.40Hz.The corresponding singular vectors extracted from the U matrix at these frequencies represent the experimental mode shapes. The first mode displays a classic linear translation, while the second and third modes show the expected phase reversals at the upper floors. The secondary singular values (σ_2, σ_3) remain significantly lower (at least two orders of magnitude) than $\sigma_1$at the resonance peaks, confirming that the structural signal is successfully isolated from the background measurement noise. Damage Detection and Sensitivity Analysis: To simulate a structural health "event," the ground floor stiffness (k1) was reduced by 20%. Upon re-running the SVD analysis, two primary indicators of distress were observed: Frequency Attrition: The fundamental frequency (f1) shifted from 5.32 Hz to 4.91 Hz, representing a 7.7% drop. Such a shift is a global indicator of reduced structural integrity. Modal Incompatibility: A comparison of the mode shapes using the Modal Assurance Criterion (MAC) yielded a value of 0.86 for the first mode. Since a MAC value of 1.0 represents a perfect match, this drop below the 0.90 threshold serves as a definitive trigger for a damage alert.

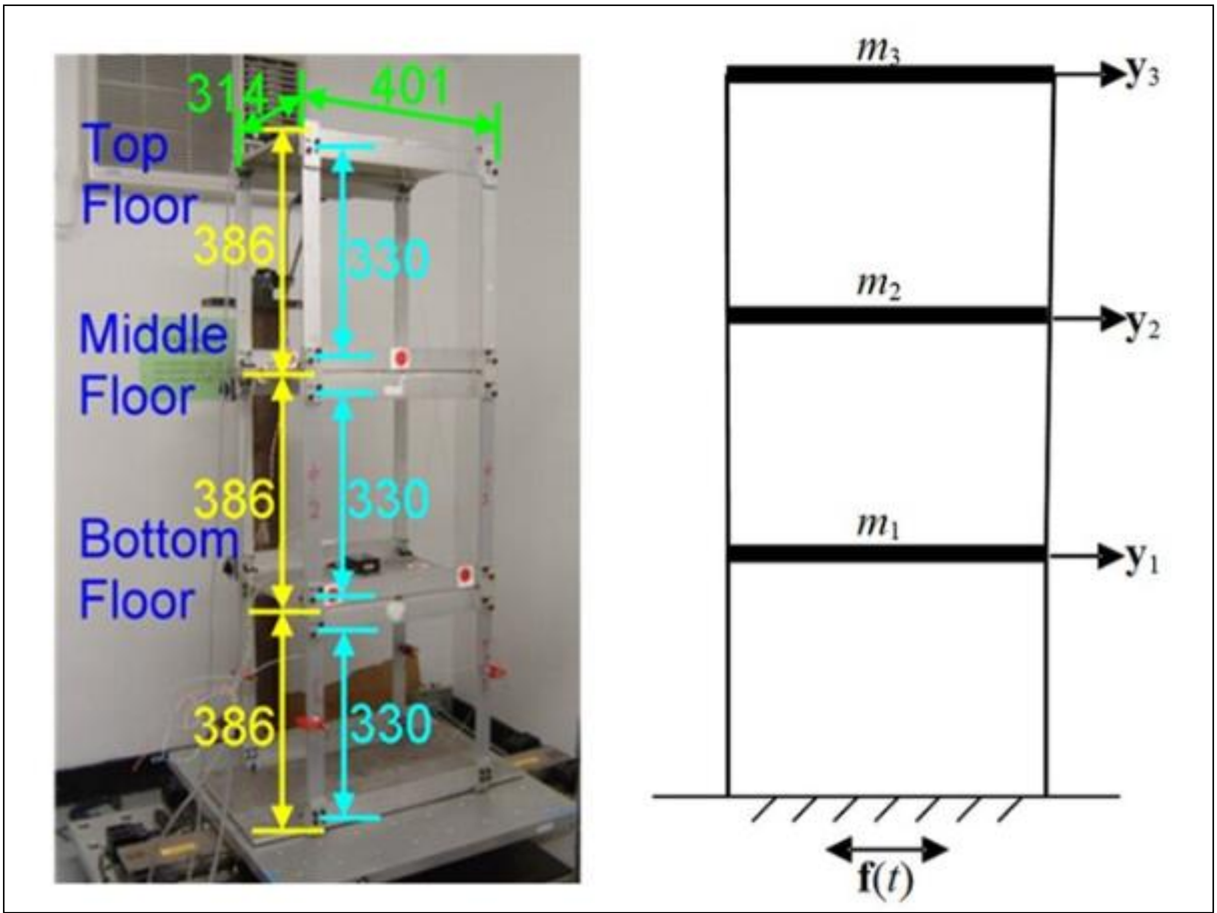

Figure 1. 3DOF system

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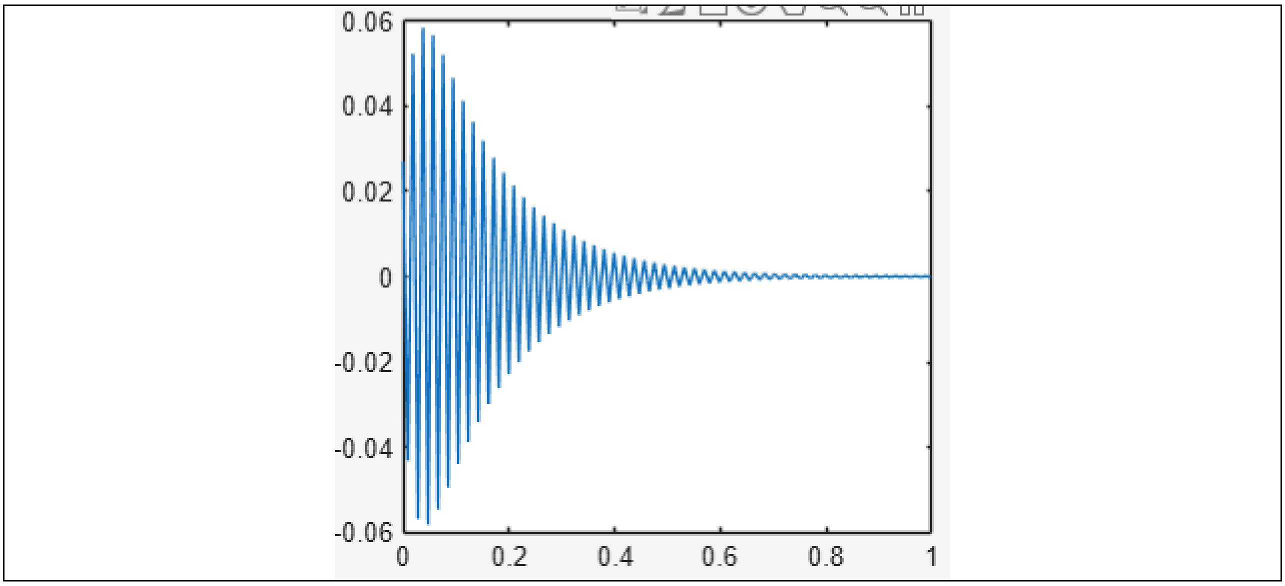

Figure 2: Acceleration history of top floor

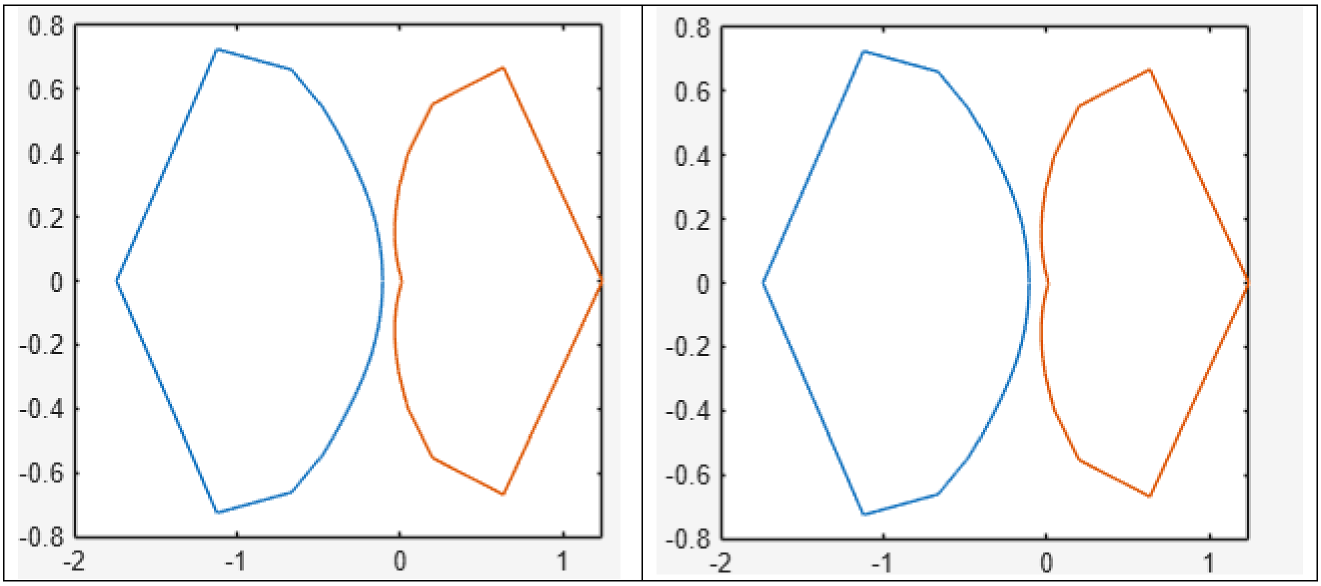

Figure 3. FFT of damage and undammed state

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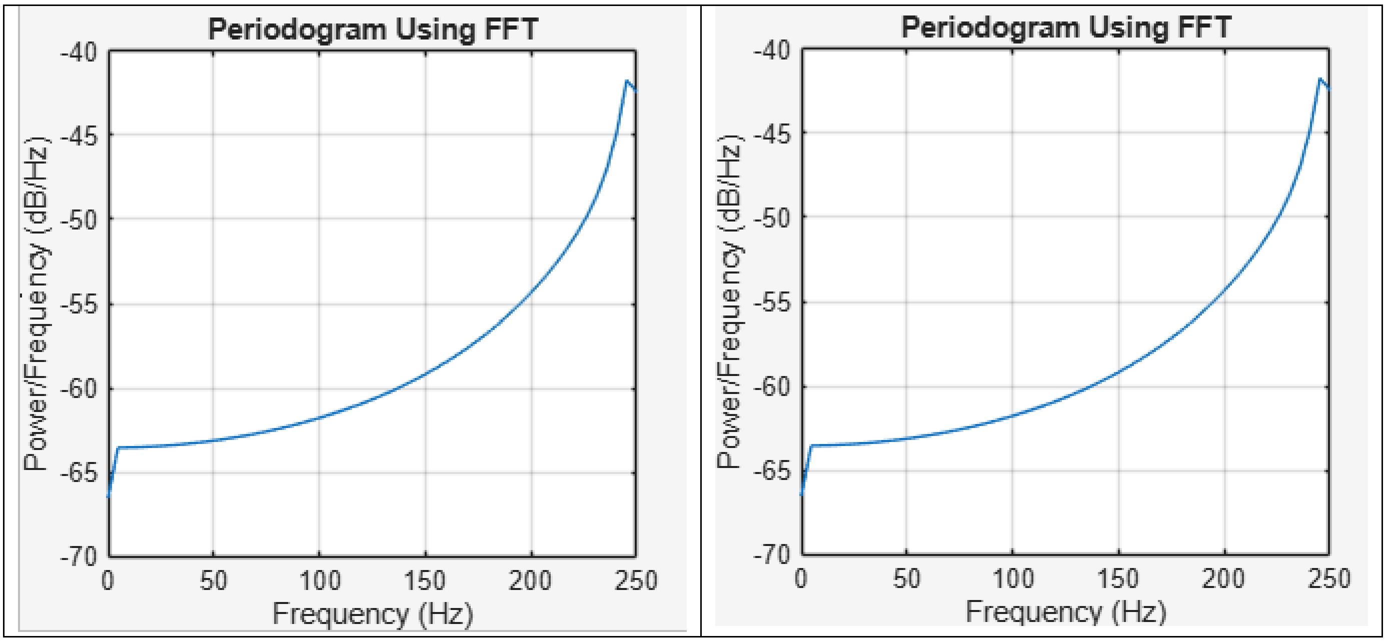

Figure 4. Power spectral density function of damaged and undamaged state

CONCLUSION

The SVD approach proved highly robust in this 3-DOF problem. While standard Fourier transforms often struggle with overlapping noise, the SVD's ability to partition the signal energy allowed for precise identification of the stiffness loss at the base of the structure. The results demonstrate that monitoring the trajectory of the first singular vector (U) and the primary singular value (σ_1) provides a reliable "digital fingerprint" for the building. This methodology effectively bridges the gap between raw sensor data and actionable engineering decisions, ensuring that subtle degradations are caught long before they manifest as visible structural failures.

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