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Structural Analysis of Composite Laminated Box-Beams under Various Types of Loading

Effect of Fiber Orientation Angle and Number of Layers on the Box-Beam Deformations

H. A. Elghazaly
Department of Civil Engineering
Faculty of Engineering, Fayoum University
Fayoum, Egypt

M. Abou Elmaaty Amin
Department of Civil Engineering
Faculty of Engineering, Fayoum University
Fayoum, Egypt

M. S. Gomaa
Department of Civil Engineering
Faculty of Engineering, Fayoum University
Fayoum, Egypt

Emad Omar Ali
Department of Civil Engineering
Faculty of Engineering, Fayoum University
Fayoum, Egypt

***Abstract*—Fiber reinforced polymer composite (FRP) is a new construction material, gradually gaining acceptance from civil engineers. In the past 15 years, experiments have been conducted to investigate the applicability of using FRP composite in bridge, and tunnel structures, including the applications of FRP composite beam, deck, and column. Beam is one of the most important structural elements in any structural system, so knowing the structural behavior of beams is very important. In this study an analytical solution for composite laminated beam with Box-section has been developed. The solution includes the structural characteristics which are often ignored in the most published studies such as axial and bending stiffness. Also, a finite element model has been developed using ANSYS software to validate the results obtained from the analytical solution and it has been seen a good agreement between results. Moreover, a parametric study has been conducted using the developed finite element model. The parametric study includes the effect of fiber orientation angle for symmetric angle ply Box beam on the axial, bending, and torsional deformations. Furthermore, the effect of changing the number of layers in both the web and flange laminates on the formerly mentioned deformations (i.e. axial, bending, torsional deformations) has been studied.**

Keywords—Composite laminated beams; Classical lamination theory; Fiber orientation angle; composite beam stiffness; Finite element method

1. INTRODUCTION

The fiber-reinforced composite materials are ideal for structural applications where high strength-to-weight and stiffness-to-weight ratios are required. Composite materials can be tailored to meet the particular requirements of stiffness and strength by altering lay-up and fiber orientations. The ability to tailor a composite material to its job is one of the most significant advantages of a composite material over an ordinary material. A number of researches have been made to develop numerous solution methods in the recent 25 years.

Zang, et al. studied the stress and strain distribution numerically in the thickness direction in the central region of symmetric composite laminates under uniaxial extension and in-plane pure shear loading [1]. Anido, et al. carried out an experimental evaluation of stiffness of laminated composite rectangular beam under flexure. Three point bending tests were performed on lay-up angle ply [±45]s beam elements made of AS-4/3501-6 carbon-epoxy [2]. Brown presented a combined analytical and experimental study of fiber-reinforced plastic composite bridges consisting of cellular box decks and wide flange I-beam as stringer. The study included design, modeling, and experimental/numerical study of fiber reinforced composite decks and deck-and-stringer bridge systems [3]. Aktas introduced a deflection function of an orthotropic cantilever beam subjected to point and distributed load using anisotropic elasticity. The deflections at the free end of the beam were calculated numerically using the obtained formulations for different fiber directions. It was found that the free end deflection of the beam increased for angles ranging from 0º to 90º for both load cases due to decreasing of stiffness [4]. Song, et al. presented analytical solutions for the static response of anisotropic composite I-beams loaded at their free-end, also the variation of the displacement quantities along the beam span was presented [5]. Zhou developed a systematic analysis procedure to investigate stiffness and strength characteristics of the multi-cellular FRP bridge deck systems consisted of pultruded FRP shapes [6]. Cardoso performed an optimal design of thin-walled composite beams against stress, displacement, natural frequencies, and critical load. The thickness of the laminates and ply orientations were considered as design variables. Equivalent beam stiffness and equivalent density properties were calculated [7]. Lee* and Lee1 presented a flexural-torsional analysis of I-shaped laminated composite beams. A general analytical model was developed to thin-walled I-section beams subjected to vertical and torsional load [8]. Hessabi studied the effect of stacking sequence on interlaminar stress distribution and the consequent change of the mode of failure experimentally [9]. Vo and Lee developed a general analytical model applicable to thin-walled box section composite beams subjected to vertical and torsional load. This model was based on the classical lamination theory, and accounts for the coupling of flexural and torsional responses for arbitrary laminate stacking sequence configurations, i.e. symmetric, as well as, asymmetric [10]. Gopal carried out finite element analysis to perform static analysis on a cross-ply laminated composite square plate based on the first order shear deformation theory. A finite element program (MATLAB) was used to obtain the finite element solutions for transverse displacements, normal stresses and transverse shear stresses [11]. Nik and Tahani introduced an analytical method to study the bending behavior of laminated composite plates. The method is capable of analyzing laminate plates with arbitrary lamination and boundary conditions [12]. Schmalberger used a finite element software ANSYS to simulate the behavior of four-layer symmetric laminate which was verified by showing the solutions for problems using the Classical Laminate Plate Theory (CLPT). The beam used in simulation was constructed as [θ/90]s lay-up for simplification. The response of the beam was investigated as a function of the orientation of fibers in outer layers [13]. Mokhtar, et al. carried out a survey on plate bending of cross-ply laminate by using the finite element method using ANSYS software. Two types of modeling were proposed: the first was modeling using a type of shell element, shell 99 and the second was an approach based on a type of solid element, solid 46 [14]. Jitech developed an analytical method for stress analysis of composite I-beam. This method included the structural response due to symmetrical of laminates, as well as, unsymmetrical I-beam cross section. The analytical expressions for the sectional properties such as centroid, axial and bending stiffness of composite I-beam were derived. A finite element model was created using ANSYS software to verify the results and excellent agreement was found with analytical results [15].

In this paper an analytical solution for composite laminated beam with Box-section has been developed based on the Classical Laminate Plate Theory (CLPT) and the analytical solution of composite laminated I-beam developed by Jitech [15]. The solution includes the structural characteristics which are often ignored in the most published studies such as axial and bending stiffness. Also, a finite element model has been developed using ANSYS software to validate the results obtained from the analytical solution and it has been seen a good agreement between results.

Moreover, a parametric study has been conducted using the developed finite element model. The parametric study includes the effect of fiber orientation angle for symmetric angle ply Box beam on the axial, bending, and torsional deformations. Furthermore, the effect of changing the number of layers in both the web and flange laminates on the formerly mentioned deformations (i.e. axial, bending, and torsional deformations) has been studied. Also for each loading condition, the optimum fiber orientation angle and the optimum number of layers in web and flange laminates have been determined.

2. CONSTITUTIVE EQUATION FOR LAMINATED COMPOSITE BEAMS

The resultant forces and moments per unit length in the x-y plane through the laminate thickness can be calculated from the following Equation:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ k_x \\ k_y \\ k_{xy} \end{bmatrix} \quad (1)$$

Where, [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices, respectively. Also [ε] and [k] are the mid-plane strains and curvatures of the laminate.

Equation (1) can be written as,

$$\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ k_x \\ k_y \\ k_{xy} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} & b_{11} & b_{12} & b_{16} \\ a_{12} & a_{22} & a_{26} & b_{12} & b_{22} & b_{26} \\ a_{16} & a_{26} & a_{66} & b_{16} & b_{26} & b_{66} \\ b_{11} & b_{12} & b_{16} & d_{11} & d_{12} & d_{16} \\ b_{12} & b_{22} & b_{26} & d_{12} & d_{22} & d_{26} \\ b_{16} & b_{26} & b_{66} & d_{16} & d_{26} & d_{66} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} \quad (2)$$

In structural analysis the beams are divided to wide and narrow beam depending on the width to depth ratio. For a narrow beam, there are no forces and moments in y-direction [15] hence, (2) can be modified as,

$$\begin{bmatrix} \varepsilon_x \\ k_x \end{bmatrix} = \begin{bmatrix} a_{11} & b_{11} \\ b_{11} & d_{11} \end{bmatrix} \begin{bmatrix} N_x \\ M_x \end{bmatrix} \quad (3)$$

Or

$$\begin{bmatrix} N_x \\ M_x \end{bmatrix} = \begin{bmatrix} A_1^* & B_1^* \\ B_1^* & D_1^* \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ k_x \end{bmatrix} \quad (4)$$

Where,

$$A_1^* = \frac{1}{a_{11} - \frac{b_{11}^2}{d_{11}}}$$

$$B_1^* = \frac{1}{b_{11} - \frac{a_{11} d_{11}}{b_{11}}}$$

$$D_1^* = \frac{1}{d_{11} - \frac{b_{11}^2}{a_{11}}} \quad (5)$$

A_1^*, B_1^*, and D_1^* refer to the axial, coupling and bending stiffnesses of the beam.

3. ANALYTICAL SOLUTION OF COMPOSITE LAMINATED BEAM WITH BOX-SECTION SUBJECTED TO AXIAL FORCE AND BENDING MOMENT

3.1 Geometry of Laminated Box-Section

The laminated composite Box-Section is divided into four sub-laminated, two flanges and two webs as shown in Fig. 1. The two flanges have different number of layers, thickness of layers, and laminate sequence, but the two webs have the same properties.

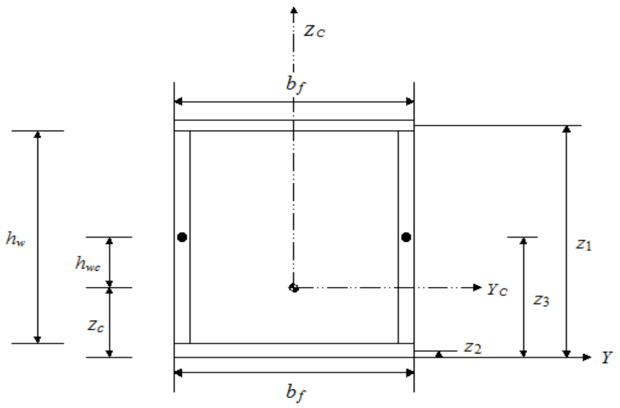

Fig. 1. Composite laminated box-section geometry.

3.2 Centroid of the Composite Laminated Box-Section

The centroid of the Box-section showed in Fig. 1**,** can be calculated as,

$$\bar{N}_x z_c = b_f N_{x1} z_1 + b_f N_{x2} z_2 + 2h_w N_{x3} z_3 \tag{6}$$

Where,

$$\bar{N}_x = b_f N_{x1} + b_f N_{x2} + 2h_w N_{x3} \tag{7}$$

N_{x1}, N_{x1}, and N_{x3} are the axial forces per unit width of the upper flange, lower flange, and the web along X-direction and $\bar{N}_x$ is the resultant axial force acting on the Box-section in the X-direction. Using (4) and (7) in (6) then, the centroid distance from Y-Axis can be calculated as,

$$z_c = \frac{b_f A^*_{1,f1} z_1 + b_f A^*_{1,f2} z_2 + 2h_w A^*_{1,w} z_3}{b_f A^*_{1,f1} + b_f A^*_{1,f2} + 2h_w A^*_{1,w}} \tag{8}$$

Where A^*_1 is defined in (5).

3.3 Equivalent Axial Stiffness, $\overline{EA}$

From (4) the axial force and bending moment per unit width for each sub-laminate in the cross section can be calculated as,

For the top flange laminate

$$N_{x1} = A^*_{1,f1} \varepsilon^o_{x,f1} + B^*_{1,f1} k_{x,f1} \tag{9}$$

$$M_{x1} = B^*_{1,f1} \varepsilon^o_{x,f1} + D^*_{1,f1} k_{x,f1} \tag{10}$$

For the top bottom laminate

$$N_{x2} = A^*_{1,f2} \varepsilon^o_{x,f2} + B^*_{1,f1} k_{x,f2} \tag{11}$$

$$M_{x2} = B^*_{1,f1} \varepsilon^o_{x,f2} + D^*_{1,f1} k_{x,f2} \tag{12}$$

For the web laminate

$$N_{xw} = A^*_{1,w} \varepsilon^o_{x,w} \tag{13}$$

$$M_{xw} = B^*_{1,w} \varepsilon^o_{x,w} \tag{14}$$

Because the load is applied at the centroid then there is no curvature for all flange and web laminates so,

$$k_{x,f1} = k_{x,f2} = k_{x,w} = 0 \tag{15}$$

Moreover, since the strain for all flange and web laminates are equal along the X-axis then,

$$\varepsilon^o_{x,f1} = \varepsilon^o_{x,f2} = \varepsilon^o_{x,w} = \varepsilon^c_x \tag{16}$$

Where, ε^c_x is the strain at the centroid in the X-direction.

Using Equations from (9) to (16) in (7), then the total force in X-direction can be modified as,

$$\bar{N}_x = \left(b_f A^*_{1,f1} + b_f A^*_{1,f2} + 2h_w A^*_{1,w}\right)\varepsilon^c_x \tag{17}$$

For another expression of the total force in X-direction using axial stiffness, then the resultant force can be written as,

$$\bar{N}_x = \overline{EA}\varepsilon^c_x \tag{18}$$

Using (17) in (18), then the axial stiffness can be calculated as,

$$\overline{EA} = b_f\left(A^*_{1,f1} + A^*_{1,f2}\right) + 2h_w A^*_{1,w} \tag{19}$$

If the top and bottom flanges have the same properties, then (19) can be modified as,

$$\overline{EA} = 2\left(b_f A^*_{1,f} + h_w A^*_{1,w}\right) \tag{20}$$

Where, the first term of (19) represents the axial stiffness of the beam flanges and the second term represents the axial stiffness of the beam webs.

3.4 Equivalent Bending Stiffness, $\overline{EI}$

If a moment is applied at the centroid of the Box-section in the X-direction then the applied moment can be related to the bending stiffness as,

$$\bar{M}_x = \overline{EI}\, K^c_x \tag{21}$$

Where, K^c_x is the radius of curvature at the centroid in X-direction.

Also the applied moment can be calculated from another expression as,

$$\bar{M}_x = \left(b_f M_{x1} + b_f N_{x1} z_{1c}\right) + \left(b_f M_{x2} + b_f N_{x2} z_{2c}\right) + 2 \int_{-\left(\frac{h_w}{2} - h_{wc}\right)}^{\left(\frac{h_w}{2} + h_{wc}\right)} z\, N_{x,w}\, dz \qquad (22)$$

Where, z_{1c}, z_{1c}, and h_{wc} are the distances between the mid-planes of upper flange, lower flange, and web laminates from the centroid as shown in Fig. 2.

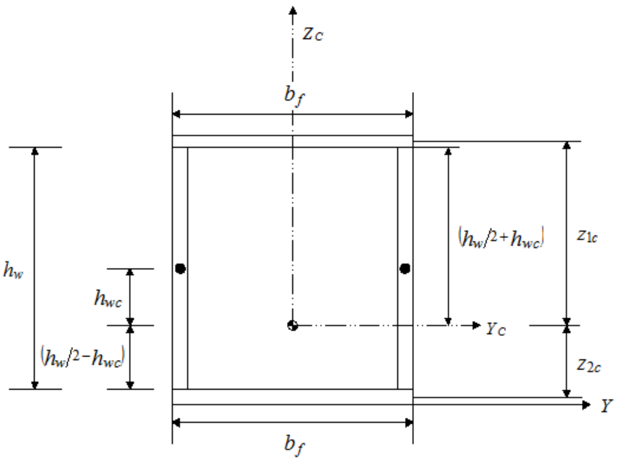

Fig. 2. Distances between mid-planes of sub-laminates from the centroid.

The first term of (22), for the upper flange laminate can be calculated as,

$$b_f M_{x1} + b_f N_{x1} z_{1c} = \begin{Bmatrix} b_f\left(B^*_{1,f1}\varepsilon^o_{x,f1} + D^*_{1,f1}k_{x,f1}\right) + \\ b_f\left(A^*_{1,f1}\varepsilon^o_{x,f1} + B^*_{1,f1}k_{x,f1}\right)z_{1c} \end{Bmatrix} \qquad (23)$$

And the mid-plane strain in X-direction for the upper flange can be written as,

$$k_{x,f1} = K^c_x$$

$$\varepsilon^o_{x,f1} = \varepsilon^c_x + z_{1c}k_{x,f1} \qquad (24)$$

Where,

$$\varepsilon^c_x = 0 \qquad (25)$$

Then (23) can be deduced as,

$$b_f M_{x1} + b_f N_{x1} z_{1c} = b_f\left(A^*_{1,f1}z^2_{1c} + 2B^*_{1,f1}z_{1c}D^*_{1,f1}\right)K^c_x \qquad (26)$$

Similarly, the second term of (22), for the lower flange laminate can be calculated as,

$$b_f M_{x2} + b_f N_{x2} z_{2c} = b_f\left(A^*_{1,f2}z^2_{2c} + 2B^*_{1,f2}z_{2c}D^*_{1,f2}\right)K^c_x \qquad (27)$$

For web laminate, the force in X-direction per unit width can be calculated as,

$$N_{x,w} = A^*_{1,w}\,\varepsilon^o_{x,w} = A^*_{1,w}\left(\varepsilon^c_x + zK^c_x\right) \qquad (28)$$

Using (25), then (28) can be deduced as,

$$N_{x,w} = A^*_{1,w}\, zK^c_x \qquad (29)$$

Then the third term of (22) can be calculated as,

$$2 \int_{-\left(\frac{h_w}{2} - h_{wc}\right)}^{\left(\frac{h_w}{2} + h_{wc}\right)} z\, N_{x,w}\, dz = 2 \int_{-\left(\frac{h_w}{2} - h_{wc}\right)}^{\left(\frac{h_w}{2} + h_{wc}\right)} z\, A^*_{1,w}\, zK^c_x\, dz \qquad (30)$$

$$2 \int_{-\left(\frac{h_w}{2} - h_{wc}\right)}^{\left(\frac{h_w}{2} + h_{wc}\right)} z\, N_{x,w}\, dz = 2A^*_{1,w}\left(\frac{1}{12}h^3_w + h_w h^2_{wc}\right)K^c_x \qquad (31)$$

Substituting (26), (27), and (31) in (22), then the moment can be written as,

$$\bar{M}_x = \begin{Bmatrix} b_f\left(A^*_{1,f1}z^2_{1c} + 2B^*_{1,f1}z_{1c} + D^*_{1,f1}\right) \\ + b_f\left(A^*_{1,f2}z^2_{2c} + 2B^*_{1,f2}z_{2c} + D^*_{1,f2}\right) \\ + 2A^*_{1,w}\left(\frac{1}{12}h^3_w + h_w h^2_{wc}\right) \end{Bmatrix} K^c_x \qquad (32)$$

Substituting (32) in (21), then the bending stiffness can be calculated as,

$$\overline{EI} = \begin{Bmatrix} b_f\left(A^*_{1,f1}z^2_{1c} + 2B^*_{1,f1}z_{1c} + D^*_{1,f1}\right) \\ + b_f\left(A^*_{1,f2}z^2_{2c} + 2B^*_{1,f2}z_{2c} + D^*_{1,f2}\right) \\ + 2A^*_{1,w}\left(\frac{1}{12}h^3_w + h_w h^2_{wc}\right) \end{Bmatrix} \qquad (33)$$

4. FINITE ELEMENT MODEL and VALIDATION

4.1 Finite Element Model

The finite element commercial package ANSYS 14.5 has been used for the finite element analysis using ANSYS SHELL99 element in the modeling. SHELL99 is a linear layered structural shell element providing up to 250 layers. The element has 8 nodes with six degree of freedom for each node as shown in Fig. 3.

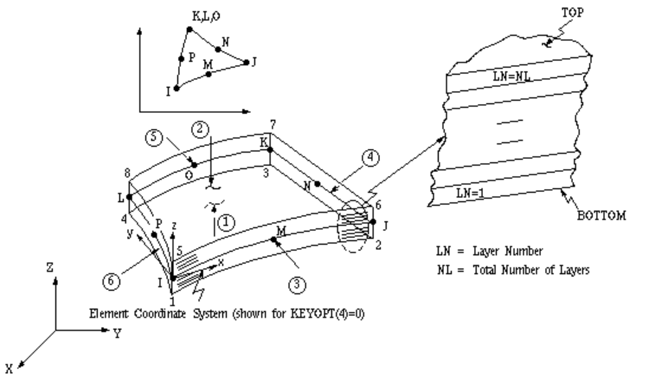

Fig. 3. Shell99 linear layered structural element geometry (ANSYS, 2009).

4.2 Validation of the Developed Analytical Expression

Example1: Fixed Cantilever Beam with Box-Section Subjected to Axial Force 20 t

A fixed cantilever composite beam with Box-section has length and cross section as shown in Fig. 4. The beam is subjected to an axial load equal 20 ton with stacking sequence [0/90/0] for all flange and web laminates. The material properties used in the analysis are shown in table1.

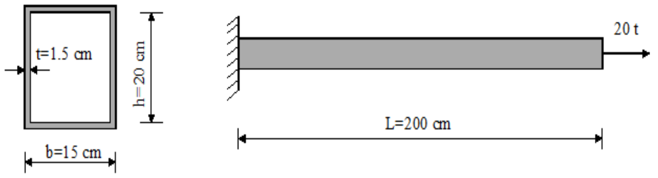

Fig. 4. Fixed Cantilever Composite Beam with Box-section Subjected to Axial Load 20 t.

TABLE1. ORTHOTROPIC PROPERTIES FOR UNIDIRECTIONAL GRAPHITE/EPOXY LAMINA [16]

Material Properties	E_1 (t/cm2)	E_2(t/cm2)	G_{12} (t/cm2)	ν_{12}
Value	1810	103	71.7	0.28

The axial displacement at the free end of the beam has been calculated from the developed analytical expression and obtained from the FEM. Also the stress in X-direction of the upper flange laminate has been calculated from the developed analytical expression and obtained from the FEM as following:

Results from the FEM

the FEM results are shown in Fig. 5.

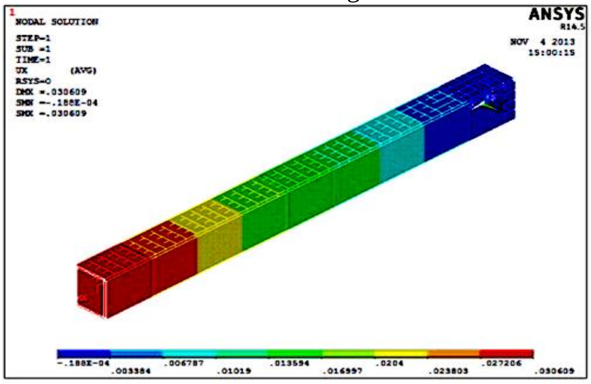

Fig. 5. Axial Displacement of Composite Beam with Box-section Subjected to Axial Force 20 t.

Comparison between Results

The results from the developed analytical expression and the FEM have been compared and it has been shown that the results are in good agreement as indicated in table2.

TABLE2. COMPARISON BETWEEN FEM AND THE DEVELOPED ANALYTICAL EXPRESSION FOR BOX-SECTION SUBJECTED TO AXIAL FORCE 20T

Results	*Analytical expression*	*FEM*	*Error (%)*
Axial Displacement (cm)	0.03059	0.03061	0

Example2: Fixed Cantilever Beam with Box-Section Subjected to Pure Moment 100 t.cm.

A fixed cantilever composite beam with Box-section has length and cross section as shown in Fig. 6. The beam is subjected to pure moment 100 t.cm with stacking sequence [0/90/0] for all flange and web laminates. The material properties used in the analysis are the same in example1.

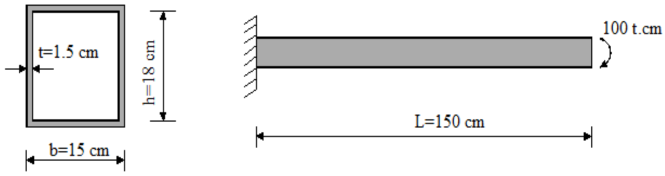

Fig. 6. Fixed Cantilever Composite Beam with Box-section Subjected to Pure Moment 100 t.cm.

The deflection at the free end of the beam and the strain of the upper flange have been calculated from the developed analytical expression and obtained from the FEM. Also the stress in X-direction of the upper flange laminate has been calculated from the developed analytical expression and obtained from the FEM as following:

Results from the FEM

the FEM results are shown in Figs. 7 and 8.

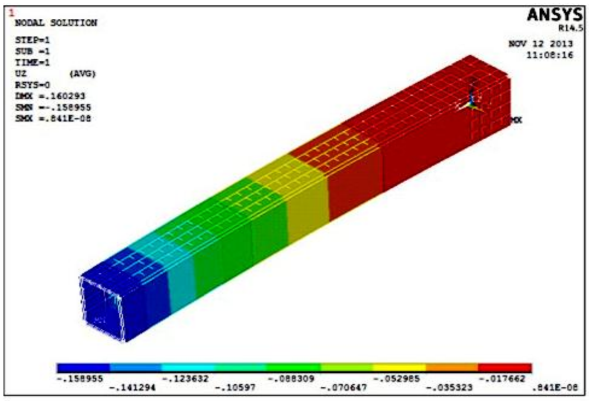

Fig. 7. Deflection of Composite Beam with Box-section Subjected to Pure Moment 100 t.cm.

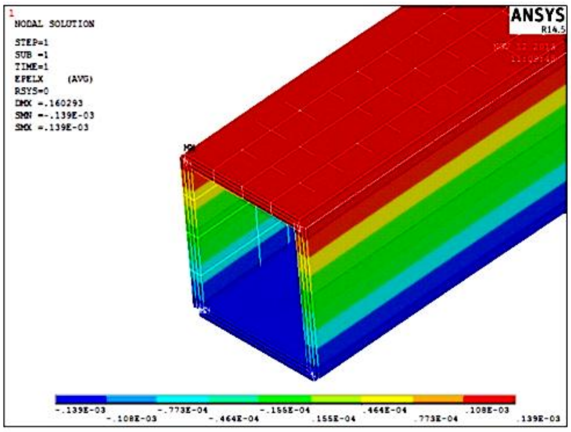

Fig. 8. Strain in X-direction of Composite Beam with Box-section Subjected to Pure Moment 100 t.cm.

Comparison between Results

The results from the developed analytical expression and the FEM have been compared and it has been shown that the results are in good agreement as indicated in table3.

TABLE 3. COMPARISON BETWEEN FEM AND THE DEVELOPED ANALYTICAL EXPRESSION FOR BOX-SECTION SUBJECTED TO PURE MOMENT 100 T.CM

Results	*Analytical Expression*	*FEM*	*Error (%)*
Deflection (cm)	0.1639	0.159	3
Strain in X-direction	$0.132x10^{-3}$	$0.136x10^{-3}$	2.9

5. PARAMETRIC STUDY

5.1 Studied Cases and Beams Configurations

All the studied cases and beams configurations are shown in Figs. 9 to 13. The beams configurations included the beam cross section, beam length, boundary conditions, and the loading type.

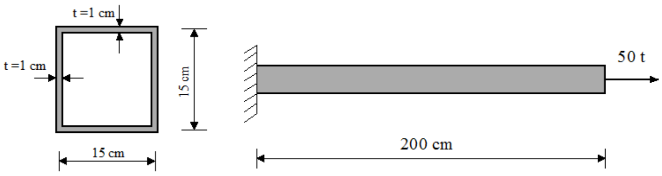

Fig. 9. Case (1) Fixed Cantilever Box-Beam Subjected to Axial Force.

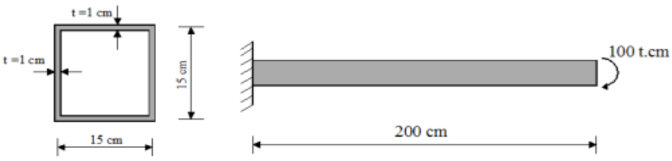

Fig. 10. Case (2) Fixed Cantilever Box-Beam Subjected to Pure Bending Moment.

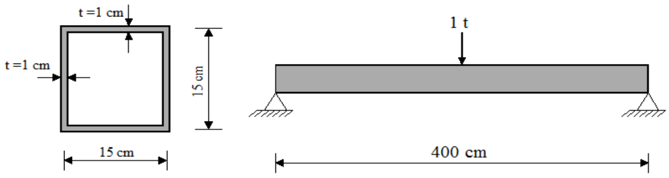

Fig. 11. Case (3)Simply Supported Box-Beam Subjected to Bending Moment and Shear Force due to Concentrated Load.

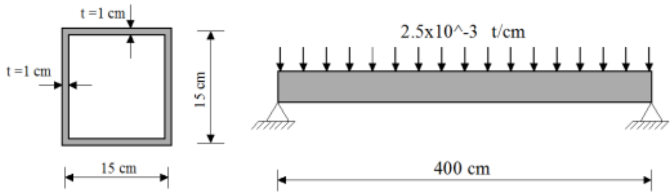

Fig. 12. Case (4) Simply Supported Box-Beam Subjected to Bending Moment and Shear Force due to distributed Load.

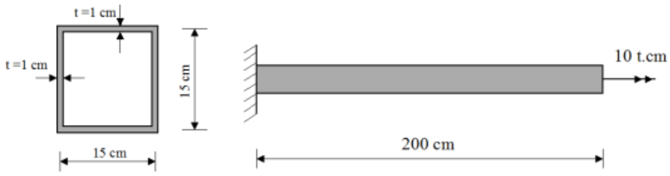

Fig. 13. Case (5) Fixed Cantilever Box-Beam Subjected to Pure Torsion Moment.

5.2 Material Properties of the Studied Cases

The material used in the analysis is T300/976 Graphite-Epoxy [4]. The T300/976 Graphite/Epoxy properties are shown in table4.

TABLE4. T300/976 GRAPHITE/EPOXY PROPERTIES

Material	*E1(t/cm2)*	*E2(t/cm2)*	*G12(t/cm2)*	*Nu12*
T300/976 Graphite/Epoxy	1560	130	70	0.23

5.3 Effect of Fiber Orientation Angle (FOA) on the Beam Deformations

In this section the effect of FOA on the Axial, bending, and torsional deformations of the beam has been studied. For each studied case, the number of layers in both web and flange laminates are four layers. Furthermore, the laminate stacking sequence has been taken as $[\theta_f/-\theta_f]_s$ for flange laminate and $[\theta_w/-\theta_w]_s$ for web laminate. The flange FOA (θ_f) has been increased from 0º to 90º with 10º step for each web FOA (θ_w) which also has been increased from 0º to 90º with 10º.

Case (1) Fixed Cantilever Box-Beam Subjected to Axial Load

In this case, the section has been subjected to axial force. The resulting axial displacements at the free end of the beam have been obtained for the studied angles and plotted together in chart as shown in Fig. 14.

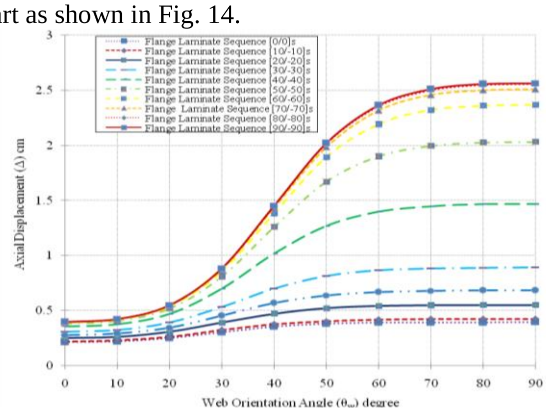

Fig. 14. The Relation between the Axial Displacement and the FOA for Box-Beam Subjected to Axial Force.

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For Fig. 14, it is shown that, by increasing the flanges FOA, the axial displacement increases. Also, by increasing the web FOA, the axial displacement increases. This increase is a gradual one when the web FOA ranges between 0º and 20º. On the contrary, a drastic change in the rate of increase is shown when the web FOA increases from 20º to 70º specially when the web FOA exceeds 30º, then the rate of increase changes to be a gradual one again. Moreover, it can be observed that the lowest axial displacement occurred at FOA equals 0º for the flanges and web laminates because the fiber direction in the same direction of the force.

Case (2) Fixed Cantilever Box-Beam Subjected to Pure Bending Moment

In this case, the section has been subjected to pure bending moment. The resulting deflections at the free end of the beam have been obtained for the studied angles and plotted together in chart as shown in Fig. 15.

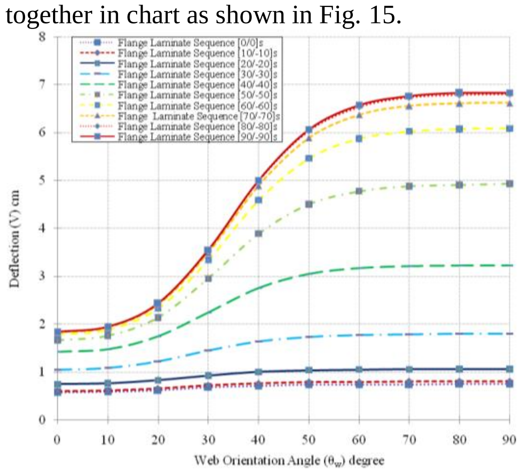

Fig. 15. The Relation between the Deflection and the FOA for Box-Beam Subjected to Pure Bending Moment.

For Fig. 15, it is shown that, the deflection increases by increasing the flanges FOA. Also, by increasing the web FOA, the deflection increases. Also it can be illustrated that the lowest deflection occurred at FOA equals 0º for the flanges and web laminates because the fiber direction in the same direction of the forces resulting from the normal stresses for both the flanges and web laminates.

Case (3) Simply Supported Box-Beam Subjected to Bending Moment and Shear Force due to Concentrated Load

In this case, the section has been subjected concentrated load at the beam mid-span. The resulting deflections at the mid-span of the beam have been obtained for the studied angles and plotted together in chart as shown in Fig. 16.

It is shown that from Fig. 16, the deflection increases by increasing the flanges FOA. Also, the deflection increases by increasing the web FOA except for flange FOA equals 0º and 10º, the deflection decreases when the web FOA increases from 0º to 20º then the deflection increases again for the other web FOA. It can be observed that the lowest deflection occurred at FOA equals 0º for the flange laminate and 20º for web laminate because the influence of the shear force on the web laminate.

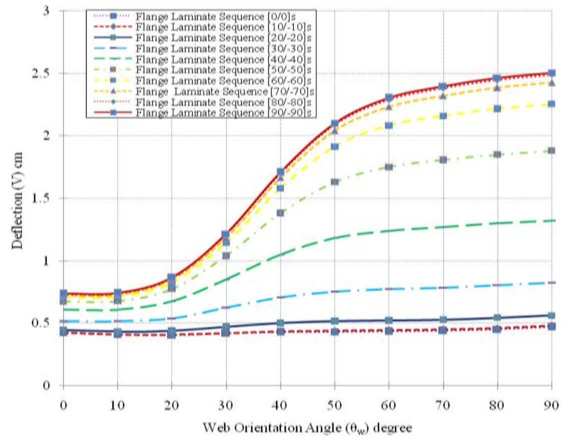

Fig. 16. The Relation between the Deflection and the FOA for Box-Beam Subjected to Bending Moment and Shear Force due to Concentrated Load.

Case (4) Simply Supported Box-Beam Subjected to Bending Moment and Shear Force due to Distributed Load

In this case, the section has been subjected distributed load. The resulting deflections at the mid-span of the beam have been obtained for the studied angles and plotted together in chart as shown in Fig. 17.

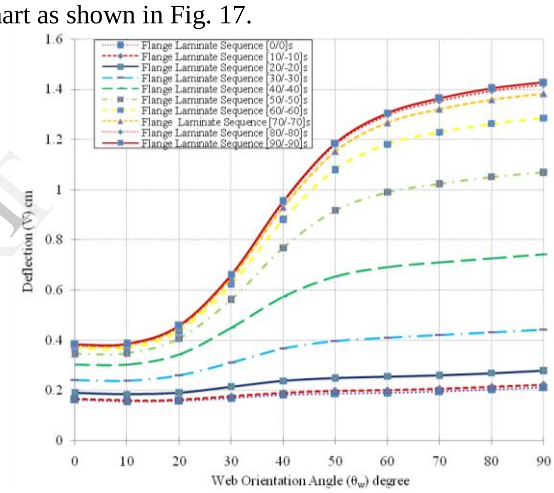

Fig. 17. The Relation between the Deflection and the FOA for Box-Beam Subjected to Bending Moment and Shear Force due to Distributed Load.

For Fig. 17, it is shown that the deflection increases by increasing the flanges FOA. Also, the deflection increases by increasing the web FOA except for flange FOA equals 0º,10º, and 20º, the deflection decreases when the web FOA increases from 0º to 10º then the deflection increases again for the other web FOA. It can be observed that the lowest deflection occurred at FOA equals 0º for the flange laminate and 10º for web laminate because the influence of the shear force on the web laminate which is less effectiveness than the concentrated load.

Case (5) Fixed Cantilever Box-Beam Subjected to Torsion Moment at Its Free End

In this case, the section has been subjected to torsion moment. The resulting rotation angles at the free end of the beam have been obtained for the studied angles and plotted together in chart as shown in Fig. 18.

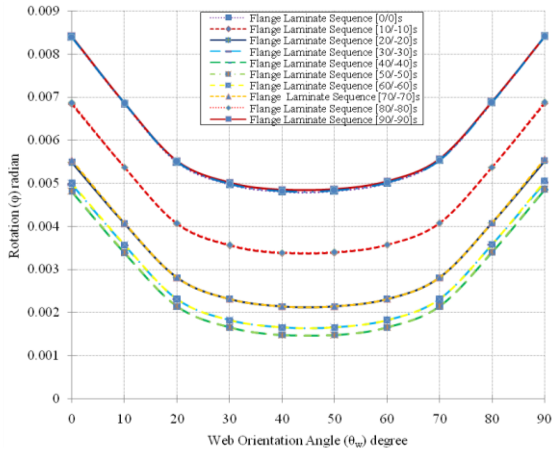

Fig. 18. The Relation between the Rotation Angle and the FOA for Box-Beam Subjected to Torsion Moment.

It is shown that from Fig. 18, the Rotation Angle decreases by increasing the FOA for both the flange and web laminates until FOA equals 45º then it increases when the FOA increases from 45º to 90º. Also, the rotation angels are symmetric around flange and web FOA equals 45º. It can be observed that the lowest Rotation angle occurred at FOA equals 45º for the flange and web laminates because of the shear stress resulting from the torsion moment.

5.4 Effect of Number of Layers on the Beam Deformations

In this section the effect of increasing number of layers for both flange and web laminates on the minimum deformations obtained from the previous section has been studied. Furthermore, the effect of increasing number of layers for both flange and web laminates on the optimum FOA obtained from the previous section has been studied. The number of layers has been increased from four layers to twelve layers for all studied cases. The relation between the beam deformations and the number of layers has been shown in the Figs. 19 to 23.

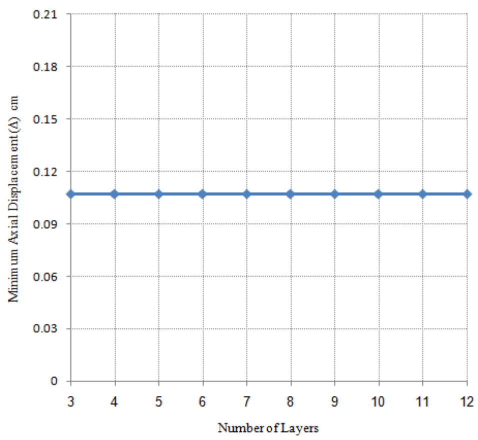

Fig. 19. The Relation between the Minimum Displacement and the Number of layers for Fixed Cantilever Box-Beam Subjected to Axial Force.

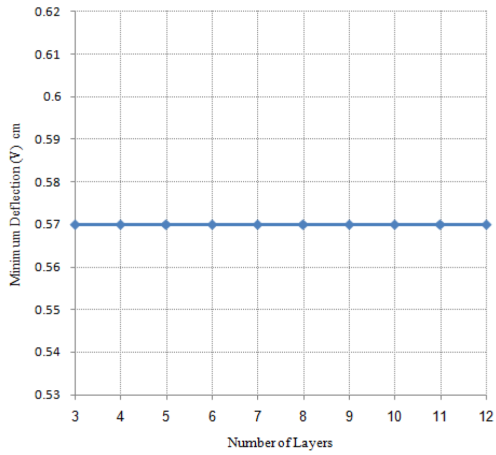

Fig. 20. The Relation between the Minimum Deflection and the Number of layers for Fixed Cantilever Box-Beam Subjected to Pure Moment.

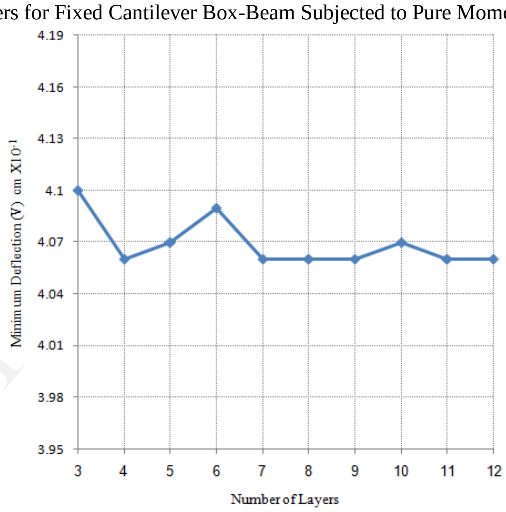

Fig. 21. The Relation between the Minimum Deflection and the Number of layers for Simply Supported Box-Beam Subjected to Concentrated Load.

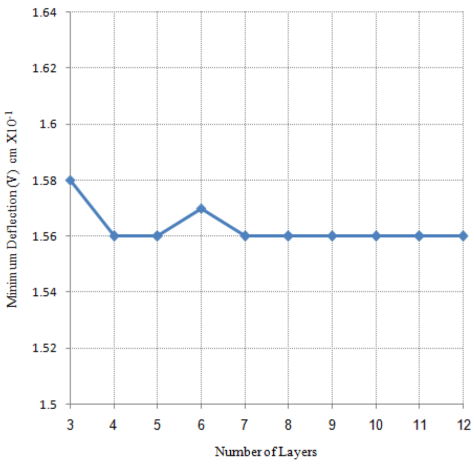

Fig. 22. The Relation between the Minimum Deflection and the Number of layers for Simply Supported Box-Beam Subjected to Distributed Load.

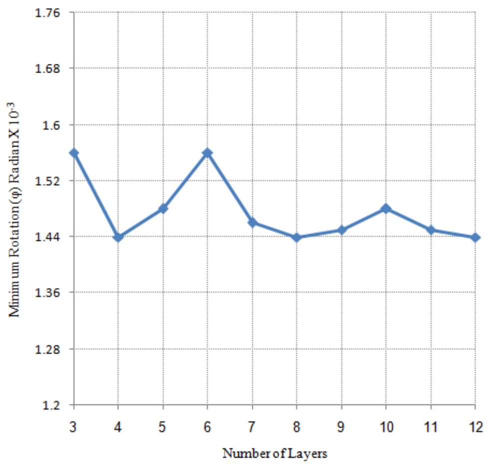

Fig. 23. The Relation between the Minimum Rotation angle and the Number of layers for Fixed Cantilever Box-Beam Subjected to Torsion Moment.

It is illustrated from Figs. 19 and 20, that the minimum axial displacements due to axial force and the minimum deflections due to pure bending moment are constants and not affected by increasing the number of layers so using three layers is enough to decrease the cost. But, for Figs. 21 and 22 it is shown that the minimum deflections due to concentrated or distributed load change as the number of layers change. Also, the difference ratio between the minimum deflections is less than 1.5% and the maximum value occurs at three layers so, using four layers is enough to decrease the cost.

For Fig. 23 it is observed that the minimum rotation angles due to torsion moment change as the number of layers change. Also, the difference ratio between the minimum rotation angles is less than 8.5% and the maximum value occurs at three and six layers. Furthermore, the minimum rotation angle occurs at number of layers equal 4n where, n = 1, 2, 3, ….etc. So, using four layers is enough to decrease the cost.

6. CONCLUSIONS

6.1 Summary

A simplified analytical method has been developed based on the Classical Laminate Plate Theory (CLPT) and the analytical solution of composite laminated I-beam developed by Jitech [15] to calculate sectional properties; equivalent axial stiffness, and equivalent bending stiffness. Also, a Finite Element Model (FEM) has been developed to validate the results obtained from the analytical solution and it has been shown that the results obtained from the analytical solution are in a good agreement with the results obtained from the FEM. Furthermore, the FEM has been used in the parametric study to know the effect of the fiber orientation angle and the number of layers on the Box-beam deformations.

6.2 Conclusion Points

The main conclusions obtained from the present study have been presented as following:

Effect of Fiber Orientation Angle (FOA)

- The minimum axial displacement occurs at Fiber Orientation Angle (FOA) equal 0º for both flange and web laminates.
- For symmetric angle ply Box-beam subjected to pure bending moment the minimum deflection occurs at flange FOA equal 0º and web FOA equal 0º.
- For symmetric angle ply Box-beam subjected to bending moment due to concentrated load at its mid-span, the minimum deflection occurs at flange FOA equal 0º and web FOA equal 20º.
- For symmetric angle ply Box-beam subjected to bending moment due to distributed load, the minimum deflection occurs at flange FOA equal 0º and web FOA equal 10º.
- For symmetric angle ply Box-beam subjected to pure torsion moment the minimum rotation at its free end occurs at flange FOA equal 45º and web FOA equal 45º.

Effect of Number of Layers

- Increasing the number of layers does not affect the minimum axial displacement or the optimum FOA for the flanges and web laminates, so using three layers is enough to decrease the cost.
- For symmetric angle ply Box-beam subjected to pure bending moment, Increasing the number of layers does not affect the minimum deflection or the optimum FOA for the flanges and web laminates, so using three layers is enough to decrease the cost.
- For symmetric angle ply Box-beam subjected to bending moment due to concentrated or distributed load, the minimum deflection changes as the number of layers changes and The difference between the minimum deflections is less than 1.5%, so using four layers is enough to decrease the cost.
- For symmetric angle ply Box-beam subjected to pure torsion moment the minimum rotation changes as the number of layers changes and the minimum rotation angle occurs when the number of layers equal 4n where, n = 1, 2, 3, ….etc. So, it is enough to use 4 layers only to decrease the cost. Also the difference ration between the minimum rotation angles is less than 8.5%.

ACKNOWLEDGMENT

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