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Strongly C*G- Continuous Maps In Topological Space

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Abstract

In this paper, we have introduced the concept of strongly c*g-continuous, perfectly c*g -continuous, c*g –locally closed, c*g –locally continuous in Topological space.

Key words: Strongly c*g-continuous, perfectly c*g -continuous, c*g-locally closed, c*g –locally continuous.

1. INTRODUCTION

Levine [3] introduced and investigated the concept of strong continuity in topological spaces. Sundaram [12] introduced strongly g – continuous maps and perfectly g – continuous maps in topological spaces. Pushapalatha [8] introduced strongly g*- continuous and perfectly g*- continuous maps in topological spaces. In this section we have introduced two strong forms of continuous maps in topological spaces, namely strongly c*g- continuous maps, perfectly c*g- continuous maps and study some of their properties .

The notation of a locally closed set in a topological space was introduced by Kurotowski & Sierpinski [13]. According to Bourbaki [20], a subset of a topological space X is locally closed in X if it is the intersection of an open set in X and closed set in X .Stone [14] has used the term FG for a locally closed subset . Locally closed sets are of some interest in the setting of local compactness,Stone-Cech Compactifications (or) Cech complete Spaces [15] . Sundaram [12] introduced the concept of generalized locally continuous function in topological space and investigated some of their properties. Pushpalatha [8] introduced strongly generalized locally continuous functions & some of their properties in topological spaces. In the chapter, we have introduced the concept of c*g- locally continuous functions and study some of their properties.

2. PRELIMINARIES

DEFINITION: 2.1

A map f: X→Y from a topological space X into a topological space Y is called

i) Strongly continuous if $f^{-1}(V)$ is both open and closed in X for each subset V in Y [3].

ii) Perfectly continuous if $f^{-1}(V)$ is both open and closed in X for each open subset V in Y [10].

iii) generalized continuous(g-continuous) if $f^{-1}(V)$ is g-open in X for each open set V in Y [12].

iv) Strongly g- continuous if $f^{-1}(V)$ is both open in X for each g-open set V in Y [12].

v) Perfectly g- continuous if $f^{-1}(V)$ is both open and closed in X for each g-open set V in Y [12].

iv) Strongly g*- continuous if $f^{-1}(V)$ is both open in X for each g*-open set V in Y [8].

v) Perfectly g*- continuous if $f^{-1}(V)$ is both open and closed in X for each g*-open set V in Y [8].

3. STRONGLY c*g- CONTINUOUS MAPS IN TOPOLOGICAL SPACE

Definition: 3.1

A map f : X →Y from a topological space X into a topological space Y is said to be strongly c*g- continuous if the inverse image of every c*g- open set in Y is open in X.

Theorem 3.2

If a map f: X → Y from a topological space X into a topological space Y in strongly c*g-continuous, then it is continuous but not conversely.

Proof:-Assume that f in strongly c*g-continuous. Let G be any open set in Y. Since, every open set in c*g- open, G is c*g open in Y. Since f is strongly c*g-continuous, $f^{-1}(G)$ is open in X. Therefore f is continuous.

The converse need not be true as seen from the following example.

Example 3.3: Let X = Y = {a,b,c} with the topologies τ_1 = {ϕ,X,{a}{a,b}} & τ_2 = {ϕ,Y,{a,b}}. Define a map f(X, τ_1) → (Y, τ_2) be the identity. Then f is continuous. But f is not strongly c*g continuous since, for the c*g open set G={b} in Y, f^{-1} (G) = {G} is not open in X.

Theorem 3.4: If f :X→Y from a topological space X into a topological space Y is strongly continuous then it is strongly c*g-continuous but not conversely.

Proof:- Assume that f is strongly continuous. Let G be any c*g open set in Y. Since f is strongly continuous, f^{-1} (G) open in X by the definition of strongly continuous. Therefore f is strongly c*g-continuous.

The converse need not be true as seen from the following example.

Example 3.5: Let X =Y={a,b,c) with topologies τ ={φ,x,{a},{b},{a,b}} and σ = { φ,y {a}}.Consider a map f: (x, τ) → (y , σ) is defined by f(a)=f(c)=c & f(b)=b. Then f is strongly c*g- continuous. But not strongly continuous. For the subset {a} of Y f^{-1} ({a}) = {a} is open in X, but is not closed in X.

Theorem 3.6: If f: X → Y is strongly c*g-continuous, then it is strongly g*-continuous but not conversely.

Proof: Assume that f is strongly c*g-continuous. Let G be any strongly g- open set in Y. Since every strongly g- open set is c*g- open, G is c*g- open in Y. Since f is strongly c*g- continuous, f^{-1} (G) is open in . Therefore f is strongly g*-continuous.

The converse need not be true as seen from the following example.

Example 3.7: Let X =Y={a,b,c} be topological spaces with the topologies τ = {φ, X, {a}, {c}, {a,b}, {a,c}} and σ = { φ,y {a,c}. Let f: (X, τ) → (y , σ) be the identity map. Then f is strongly g*-continuous, but not strongly c*g-continuous. For, {b} is a c*g- open in Y, but f ({b} = {b } is not open in X .

Therorem 3.8: A map f: (X → Y) from a topological spaces X into a topological space Y is strongly c*g- continuous if and if only if the inverse image of every c*g-closed set in Y is closed in X.

Proof:- Assume that f is strongly c*g-continuous. Let G be any c*g- closed set in Y. Then G^c is c*g- open in Y. Since f is strongly c*g- continuous $f^{-1}(G^c)$ open in X. But $f^{-1}(G^c)$ =X- f^{-1} (G) and so f^{-1} (G) is closed in X.

Conversely assume that the inverse image of every c*g- closed set in Y is closed in x. Let G be any c*g- open set in Y. Then G^c is c*g- closed in Y. By assumption, $f^{-1}(G^c)$ is closed in X. But $f^{-1}(G^c)$ = X - $f^{-1}(G^c)$ and so f^{-1} (G) is open in X. Therefore f is strongly c*g-continuous.

Remarks3.9: From the above observation we get the following diagram.

Strongly continuity

↓

Strongly c*g continuous

↓

Continuity.

In the above diagram none of the implications can be reversed.

Theorem 3.10: If a map f : X → Y is strongly c*g- continuous and a map g: Y → Z is c*g- continuous , then the composition g∘f: X → Z is continuous

Proof:- Let G be any open set in Z . Since g is c*g- continuous , g^{-1} (G) is c*g- open in Y. Since f is strongly c*g- continuous, f^{-1}

$[g^{-1}(G)]$ is open in X. But $(g\circ f)^{-1}(G) = f^{-1}[g^{-1}(G)]$. Therefore $g\circ f$ is continuous.

Definition 3.10 A map f : X → Y is said to be perfectly c*g- continuous if the inverse image of every c*g- open set in Y is both open and closed in X.

Theorem 3.11: A map f : X → Y from a topological space X into a topological space Y is perfectly c*g- continuous, then it is strongly c*g- continuous but not conversely.

Proof: Assume that f is perfectly c*g- continuous. Let G be any c*g- open set in Y. Since f is perfectly c*g- continuous, $f^{-1}(G)$ is open in X. Therefore f is strongly c*g- continuous.

The converse need not be true as seen from the following example.

Example 3.12: Let X =Y= {a,b,c} ,with topologies τ_1 ={φ,x,{a},b {a,b}} and τ_2 = { φ_1,y, {a,b}. Define f: (X, τ_1) → (y , τ_2) as the identity map. Then f is strongly c*g- continuous but not perfectly c*g- continuous. Since, for the c*g open set G={a} in Y, $f^{-1}(G)$ = {G} is open but not closed in X.

Theorem 3.13: If a map f : X → Y is perfectly c*g- continuous then it is perfectly g*- continuous but not conversely.

Proof: Assume that f is perfectly c*g- continuous. Let G be a a c*g- open set in Y. Then G is c*g- open in Y. Since f is perfectly c*g- continuous, $f^{-1}(G)$ is both open and closed in X . Therefore f is perfectly g*- continuous.

The converse need not be true as seen from the following example.

Example 3.14: Let X =Y={a,b,c) with topologies τ ={φ,X,{a},{b,c}} and σ = { φ,Y {a}}.Define a map f: (X, τ) → (Y , σ) as the identify function. Then f is perfectly g*- continuous, but not perfectly c*g- continuous, since for the c*g open set {b} in Y f^{-1} {(b) ={b} is not both open and closed in X.

Theorem 3.15: If a map f : X → Y from a topological space X into a topological space Y is perfectly c*g- continuous if and only if $f^{-1}(G)$ is both open and closed set in X for every c*g- closed set G in Y.

Proof: Assume that f is perfectly c*g- continuous. Let F be any c*g- closed set in Y. Then F^c is c*g- open set in Y. Since f is perfectly c*g- continuous, $f^{-1}(F^c)$ is both open & closed in X . But $f^{-1}(F^c) = X - f^{-1}(F)$ and also $f^{-1}(F)$ is both open and closed in X.

Conversely assume that the inverse image of every c*g- closed set in Y is both open and closed in X. Let G be any c*g- open set in Y . Then G^c is c*g- closed in Y. By assumption $f^{-1}(G^c)$ is both open and closed in Y. But $f^{-1}(G^c) = X - f^{-1}(G)$ and so $f^{-1}(G)$ is both open and closed in Y. Therefore f is perfectly c*g- continuous.

Remark 3.16: From the above observations we have the following implications and none of them are revereable.

Perfectly c*g-continuity

↓

Strongly c*g-continuity

↓

Strongly g*- continuous

↓

Continuity.

4. c*g – locally continuous function in topological spaces

Definition 4.1:

A subset S of X is called c*g- locally closed set [c*glc⁻-set] if S= A∩B, Where A is c*g- open in X and B is c*g- closed in X .C*GLC(X) denotes the class of all c*g- sets in X.

Theorem 4.2: If a subset S of X is locally closed then it is c*g- locally closed but not conversely.

Proof : Let S= P∩Q, Where P is open in X and Q is closed in X . Since every open set is c*g- open and every closed, S is c*g- locally closed in X.

The converse need not be true as seen from the following examples.

Example 4.3: Consider the topological space x = {a ,b, c} with topology τ = { φ,X,{a}}. Then the set {a,c} c*g - locally closed but is not locally closed.

Theorem 4.4: If a subset S of X is strongly generalized locally closed in X then S is c*g- locally closed but not conversely.

Proof :-Let S= P∩Q, where P is strongly g- open and q is strongly g- closed in X. Since strongly g- open implies c*g- open and strongly g- closed implies c*g- closed , S is c*g- locally closed set in X.

Example 4.4: Consider the topological space X = {a ,b, c} with topology τ = { φ,X,{b}}. Then the set {a,b} c*g - locally closed but is not strongly generalized locally closed.

Theorem 4.5: If a subset S of X is c*g- locally closed in X, then S is regular generalized locally closed but not conversely.

Proof :- Let S= P∩Q, Where P is c*g- locally closed and Q is c*g- locally closed in X.Since c*g- locally closed implies rg- closed and c*g- locally open implies rg- open. Therefore S is regular generalized locally closed.

Example 4.6: Let X={a,b,c,d}, τ={φ,X,{a}{b},{a,b}}.Then {d} is rg- locally closed but is not c*g- locally closed set in X.

Theorem 4.7: If A is c*g- locally closed in X and B is c*g- open (respectively closed) in X, then A∩ B is c*g- locally closed in X.

Proof :-There exist a c*g - open set P and a c*g- closed set Q such that A =P∩Q. Now, A∩B= (P∩Q)∩B = (P∩B)∩Q [respectively A∩B = P∩(Q∩B)]. Since P∩B is c*g- open [respectively Q∩B is c*g- closed], A∩B is c*g - locally closed in X.

Definition 4.8:

A subset S of a topological space X is called c*glc*- set if S= P∩Q where P is c*g- open in X and Q is closed in X.

Definition 4.9:

A subset S of a topological space X is called c*glc**- set if S= P∩Q where P is open in X and Q is c*g- closed in X.

Theorem 4.10:

i) If A is c*glc* –set in X and B is c*g- open (or closed), then A∩ B is c*glc*- set in X.

ii) If A is glc **- set in X and B is closed then A∩ B is c*glc**.

Proof :-

i) Since A is c*glc*- set ,there exist a c*g- open set P and a closed set Q .Such that A = P∩Q. Now A∩B = (P∩Q) ∩B = (P∩B) ∩Q. Since P∩Q is c*g- open and Q is closed, A∩B is c*glc*- set. In the case of B being a closed set, we have

A∩B = (P∩Q) ∩B = P∩ (Q∩B). Since P is c*g- open and Q∩B is closed, A∩B is c*glc*- set.

ii) Since A is c*glc**, there exist an open set P and a c*g- closed set Q such that A= (P∩Q). Now A∩B = (P∩Q) ∩B = P∩ (Q ∩B). Since Q is c*g -closed and B is closed , Q∩B is c*g –closed. Therefore , A∩B is c*glc**- set .

Theorem 4.11: A subset A of a topological space X is c*glc*-set if and only if there exists a c*g- open set P such that A= P∩cl(A).

Proof :-Assume that A is c*glc*-set. There exists a c*g- open set P and a closed set Q such that A = P∩Q. Since A⊂Q and Q is closed, A ⊂cl(A) ⊂ Q. Then A ⊂P and A⊂ cl(A), and hence A ⊂P∩cl(A). To prove the reverse inclusion let X ∈P∩cl(A).Then X ∈ P and X ∈ cl(A) ∩ Q and so X ∈ P∩Q =A. Hence P∩cl(A)⊆A. Therefore A=P∩cl(A).

Conversely assume that there exist a c*g-open set P such that A= P∩cl(A). Now P is c*g- open set and cl(A) is closed. Therefore A is c*glc* - set.

Theorem 4.12: If a subset A of a topological space X is c*glc** - set then there exists an open set P such that A = P∩cl*(A), where cl*(A) is the closure of A as defined by Dunham [19].

Proof :- By definition there exist an open set P and a c*g- closed set Q such that A= P∩Q. Then, since A ⊂ cl* (A) ⊂ Q, We have A⊂ P∩cl*(A). Conversely , if X ∈ P∩cl*(A),then X ∈ Q and X ∈ P. Then, X ∈ Q∩P = A and hence P∩cl*(A) ⊂A. Therefore A = P∩cl*(A).

Theorem 4.13: If A and B are c*glc*- set in a topological space X then A∩B is c*glc*-set in X.

Proof :-From the assumptions there exist c*g- open sets P and Q such that
A = P∩cl(A) and B = Q∩cl(B).Then A∩B = (P∩Q) ∩[(cl (A) ∩ cl(B)]. Since P∩Q is c*g-open and cl(A)∩cl(B) is closed , A∩B is c*glc* - set.

5. c*g - locally closed continuous functions

Notations: - LC(X) denotes the class of all locally closed sets in a topological space X and C*GLC(X) denotes the class of all c*glc- sets in X.

Similarly, C*GLC*(X) [respectively C*GLC**(X)] denotes the class of all c*glc*-sets [respectively c*glc** - sets]

Ganster and Reilly [2] have proved that

Continuity
⇓
LC- irresolute
⇓
LC- continuity

Pushpalatha [17] has proved that

LC – continuity
⇓
S*GLC- irresoluteness

⇓

S*GLC- continuity

⇓

GLC – continuous.

But none of these implications can be reversed. Also they observed that the composition of two S*GLC- irresolute functions is S*GLC- irresolute and the composition of a S*GLC – continuous function is S*GLC- continuous.

Definition 5.1.

A function f : X→Y from a space X into a space Y is called

(i) LC- irresolute [2] if f^{-1} (V) ∈ LC(X) for each V in LC(Y).

(ii) S*GLC- irresolute [17] if f^{-1} (V) ∈ S*GLC(X) for each V ∈ S*GLC(X).

(iii) LC-continuous [2] if f^{-1}(V)∈LC(X) for each open set V in Y .

(iv) S*GLC - continuous [17] if f^{-1} (V) ∈ LC(X) for each open set V in Y.

Definition 5.2.

A function f: x→y from a space X into a space Y is called

i) C*GLC-irresolute if f^{-1}(V) ∈ C*GLC(X) for each V ∈ C*GLC(X).

ii) C*GLC-continuous if f^{-1} (V) ∈ C*GLC(X) for each open set V in Y.

iii) C*GLC* – irresolute (respectively C*GLC**- irresolute) if f^{-1}(V) ∈ C*GLC*(X) (respectively f^{-1}(V) ∈ C*GLC**(X)) for each V ∈C*GLC*(Y) (respectively V ∈ C*GLC**(X)).

iv) C*GLC* - continuous (respectively C*GLC**- continuous) if f^{-1}(V) ∈ C*GLC*(X) (respectively f^{-1} (V) ∈ C*GLC**(X)) for each open set V inY.

Theorem 5.3: If a function f : X→Y from a space X into a space Y is LC- continuous then it is C*GLC- continuous but not conversely .

Proof: - Assume that f is LC -continuous. Let V be an open set in Y. Then f^{-1}(v) is locally closed in Y. But locally closed sets are c*g- locally closed sets. Therefore f^{-1}(V) ∈ C*GLC(X) and so f is C*GLC-continuous. The converse need not be true as seen from the following example.

Example 5.4: Let X = Y = {a,b,c}, τ = { φ,x,{a}} and σ be the discrete topology. Define f : (X,τ) →(Y,σ) as the identity function .Then f is not LC-continuous. Because {b} is open in Y but f^{-1}{b} = {b} is not locally closed in y, clearly f is C*GLC-continuous.

Theorem 5.5: If function f: X→Y from a space X into a space Y is C*GLC- irresolute then it is C*GLC- continuous.

Proof: - Let V be open in Y. Since every open set is c*g- open set and every c*g- set open set is c*g - locally closed , V ∈ C*GLC(Y). Since f is C*GLC- irresolute, $f^{-1}(V) \in$ C*GLC(X). Therefore f is C*GLC – continuous. Thus we have the following implications

i) Continuity

⇓

LC- irresolute

⇓

LC- continuity

⇓

C*GLC- continuous

ii)

C*GLC-irresoluteness

⇓

C*GLC – continuity

However none of the above implications can be reversed.

Theorem 5.6: If function f: X→Y from a space X into a space Y be C*GLC- continuous and A be a c*g- open subset of X (respectively closed). Then the restriction f/A: A→Y is C*GLC- continuous.

Proof: - Let V be open in Y.Let $f^{-1}(V)$ = W. Then W is c*glc in X.Since f is C*GLC- continuous .Let W= P∩Q where P is c*g - open in X and Q is c*g - closed in X. Now $(f/A)^{-1}$ (V) = W∩A = (P∩Q) ∩A = (P∩A)∩Q.

But P∩A [respectively A∩G] is c*g- closed by [18] is c*g open in X and so the restriction f/A is C*GLC- continuous.

Theorem 5.7: (i) Let f : X→Y be C*GLC- continuous and B be an open subset of Y containing f(X). Then f : X→B is C*GLC- continuous.

(ii) If f: X→Yand g : Y→Z are both C*GLC – irresolute then the composition g∘f: X→Z is C*GLC – irresolute.

(iii) If f : X→Y is C*GLC – continuous and g : Y→Z is continuous then the composition g∘f : X→Z is C*GLC- continuous.

Proof : - (i) Let V be open in B.Since B is open in Y, the set V is open in Y .Therefore $f^{-1}(V)$ ic c*glc in X. Hence f : X→B is C*GLC- continuous.

(ii) Let V be c*glc – set in Z. Since g is C*GLC- irresolute, g^{-1} (V) is c*glc in Y. Since f is C*GLC – irresolute, $f^{-1}(g^{-1}(V))$ is c*glc in X .But $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ and so g∘f is C*GLC- irresolute.

(iii) Let V be open in Z. Since g is continuous $g^{-1}(V)$ is open in Y. Since f is C*GLC- continuous, $f^{-1}(g^{-1}(V))$ is c*glc in X.But $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ and so g∘f is C*GLC – continuous.

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