Stress/Strength Models to Estimate Systems Reliability R(t)=P(x<y)

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***Abstract*—When determining the reliability of a component, product or the viability of a material, it is necessary to consider the stress conditions of the operating environment. That because the specification conditions and the used conditions it always be different. That is to say, uncertainty about the used stress conditions has to be encountered should be modeled as random. The stress-strength model takes in count that both stress and strength are random variables. In the stress-strength model the reliability is denote by the probability of a random variable X are suggest to a random variable Y, the if the stress are greater than the strength of the component this will fail. This paper presents, the stress-strength models for a different distribution with emphasis on the Weibull distribution, that´s because the importance of this distribution and the widely applications in reliability engineering.**

Keywords*— Probability; Failure; Stress-strength Models; Normal distribution; Exponential Distribution; Gumbel distribution; Log-Normal Distribution; Weibull Distribution; Reliability.

I. INTRODUCTION

All product will eventually fail in service or be discared because of wear or obsololense [1]. In part these failures can be attributed to a poor design or ignorance of factors that affect, the operation of the product or system can produce problems outside of the normal operating conditions (external noise), which was not possible by the manufacturer the elimination of this defects during the manufacturing process or inspection stage, the supplier has variation between batches of raw material producing noise, etcetera [2]. The consecuences of this failues are several [3], like injuries, fatalities, structural damage, loss of functionality and environmental damage [4]. As a result, the cost of the failure is a multidimensional quantity and variable [5]. Therefore, it is necessary to minimize the probability of failure, with this objective, based on significant knowledge has been developed to analyze such failures with the objetive to minimize their occurrence is named like reliability engineering [6]. Reliability is defined, as the ability of a system or component to perform its required functions under stated conditions for a specified period [7]; ie, reliability can be used as a measure of system success by providing its function properly [2]. Mathematically, the reliability R(t) is the probability that a system will succeed in the interval of time 0 at time t [8].

$$R(t)=P(T>t);\ \ t>0 \tag{1}$$

On the other hand, a difficulty in reliability lies in the sensitivity to the small changes of assumed models for the X and Y distributions, that is, relatively small perturbations can make the probability of failure much higher [9], this model is known as a stress-strength analysis [10]; the stress-strength models was originated from the classical non-parametric problem tests of inequality of two distribution functions, thus leading to expressions of type P(X <Y) [11]. Further, in stress-strength models, the variation in stress and strength produces a natural dispersion which results in a statistical distribution [12], in this way, when the two distributions interfered and the stress interference is greater than the strength, it produces a failure [13].

The purpose of this paper is to explain the importance of stress-strength models and to show the most used models, such as the Exponential, Normal and Weibull models, the last being the most important due to the flexibility of the Weibull distribution and because it does not exist a Weibull stress-strength model with a different shape parameter β for both the stress and the strength.

The structure of the paper is as follows; Section 2 presents the stress-strength generalities. Section 3, presents the Exponential distribution stress-strength model. In Section 4 the stress-strength normal are presents. Section 5 shows the Lognormal stress-strength. In section 6 the Weibull stress-strength model is showed and it is demonstrated the not closure solution for a different shape parameter β. Finally, in section 7, the conclusions are given.

II. STRESS-STRENGTH GENERALITIES

There are some applications where the product reliability depends on their physical inherent strength [14]. Thus, if a stress level higher than the stress is applied then they break down [15]. Therefore, if the random variable X represents the ‘stress’ and the random variable Y represents the ‘strength’ then, the stress-strength reliability is denoted by the probability that Y>X [R= P(Y<X)] [16]. In the stress-strength analysis, the term stress is referred to the load that produces the failure and the strength is referred to the ability component to sustain the load [17].

On the other hand, in the stress-strength model, the interference between the ‘stress’ variation and the ‘strength’ variation variables results in a statistical distribution [18]. Thus, a natural scatter occurs in these variables when the two

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 7 Issue 03, March-2018

distributions interfere each other [19]. And in particular, when the stress becomes higher than the strength, a failure occurs [9]. In other words, when the probability density functions of both stress and strength are known, the component reliability may be analytically determined by its interference (see Fig.1) [20]. Seeing this let Y and X be two random variables such that Y represents "strength" and X represents "stress" and let Y, X follow a joint probability density function pdf f(x,y) [21]. Then based on the f(x,y), the reliability of the component is estimated as

$$R = P(X < Y) = \iint_{-\infty}^{x} f(y,x)dx\,dy \tag{2}$$

where P(X<Y) is the probability that the strength exceeds the stress and f(y,x) is the joint pdf of Y and X [22].

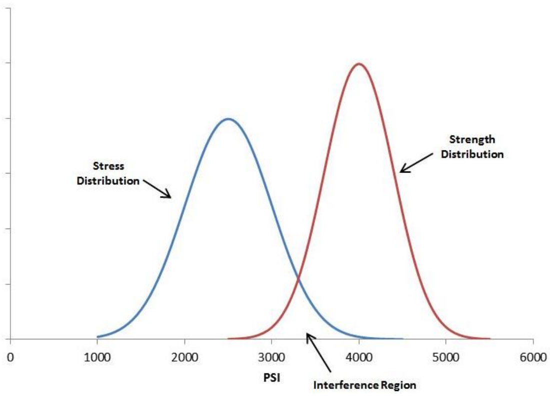

Figure 1: Stress and Strength Representation

The concept of stress-strength in engineering has been one of the deciding factors of failure of the devices [23]. It has been customary to define safety factors for longer lives of the systems [24]. This had being made in terms of the strength of the inherent component, and on the external stress being experienced by the systems [25]. The safety factor is given by

$$SF = \frac{strength}{stress} \tag{3}$$

However, in Eq. (3), the mean stress and mean strength values are used, then from figure 1, we can see that although the SF defined in Eq. (3) is greater than one (SF>1), then because the scatter of both the stress and the strength, a region where both distributions overlap exists [26]. This region is referred to as the "Interference Region", and it represents the failure probability [27].

The estimation of the stress-strength parameter plays a very important role in the reliability analysis [28]. For example, if a variable X represent the strength and a variable Y is the stress which the system is subject, then the parameter R(t) measures the system performance [29]; Moreover the parameter R(t) represent the probability of a system failure if the applied stress is greater that the strength of the system [30]. In other words, the stress-strength model has been widely used, like an evaluation of various engineering domains [31]. However, the reliability inference it's still very complex [7]; because t the expression of reliability is presented as an implicit integral form and incorporates many parameters in the systems of resistance to stress [32].

III. GENERALIZED EXPONENTIAL DISTRIBUTION STRESS-STRENGTH MODEL

Let X be the strength of the component which is subjected to Stress Y with parameter α and β, respectively [33]. The probability density functions of X and Y is given by:

$$f(x;\alpha) = \frac{1}{\alpha} e^{-\frac{x}{\alpha}}, \quad x > 0, \qquad \alpha > 0 \tag{4}$$

$$f(y;\beta) = \frac{1}{\beta} \mathrm{e}^{-\frac{y}{\beta}}, \quad y > 0, \qquad \beta > 0 \tag{5}$$

The cumulative distribution function is given by

$$F(x;\alpha) = (1 - \exp -\tfrac{x}{\alpha}) \tag{6}$$

$$F(y;\alpha) = (1 - \exp -\tfrac{y}{\alpha}) \tag{7}$$

The exponential distribution has very wide application in life testing issues [34]; the situation when X,Y are the Stress and Strength respectively and fallows an exponential distribution, has been studied by many authors like, [35] and [36], how estimated the probability of R=(X>Y) based on the complete sample observation [37]. In addition, the stress-strength exponential model has been also studied by [38]; [39]; [40]; [41], [42].

On the other hand, suppose x_1…x_n and y_1…y_n are two independent samples from X and Y [33], then the stress-strength reliability can be expressed as fallows [43].

Let Y~GE(α,λ),X~GE(α,λ) where X and Y are independently distributed [44], then:

$$R = P[Y < X] = \int_0^{\infty}\int_0^{x} \alpha\beta\lambda^2\, \mathrm{e}^{-\lambda(y+x)}\left(1 - e^{\lambda x}\right)^{\beta-1}\left(1 - e^{\lambda y}\right)^{\alpha-1} dydx$$

$$= \int_0^{x} \beta\lambda\, \mathrm{e}^{-\lambda(x)}\left(1 - e^{\lambda x}\right)^{\alpha+\beta-1} dx \tag{8}$$

$$= \frac{\beta}{\lambda+\beta} \tag{9}$$

According to the problem of estimating P(Y<X) for the generalized exponential distribution by integrated Eq. (8) it is observed in Eq.(9) that the maximum likelihood estimator works quite well [45], [46].

Now let's consider the stress/strength model reliability estimation in the normal case.

IV. NORMAL DISTRIBUTION STRESS-STRENGTH MODEL

The normal distribution is the most widely used distribution [22], it describes well the distribution of random variables that arise in practice [47]. The mean μ and the standard deviation σ, are the two parameter of the normal distribution [48]; the location parameter is described by the mean μ and the shape

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 7 Issue 03, March-2018

parameter is described by the standard deviation σ [49]. The notation of a normally distributed variable X is X~N(μ,σ) [50].

The pdf of a normal distribution is expressed as

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (10)$$

And the cdf is given by

$$F_x(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \quad (11)$$

The pdf and cdf of a random variable X~N(1,0), is called the standardized normal distribution (see Fig.2) [51].

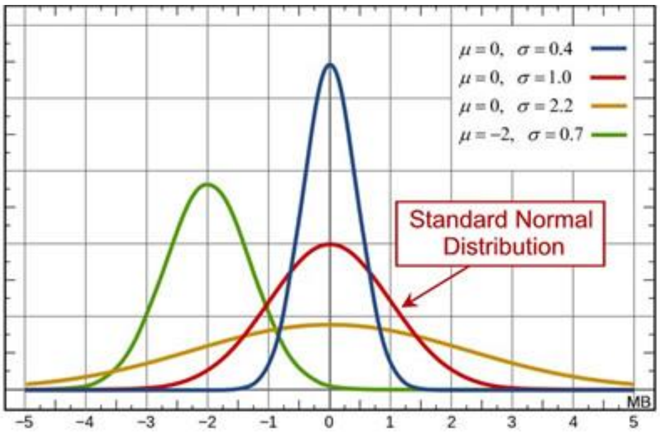

Figure 2: Normal Distribution Representation

A random variable X~(μ,σ) can be transformed into a standard normal variable Z [52] with the equation

$$Z = \frac{x-\mu}{\sigma} \quad (12)$$

On the other hand, in testing reliability in the stress/strength setting when the stress and the strength of a unit or a system has a cumulative cdf, F(x) and is subjected to a stress F(y), and both are normal distributed [53], is of considerable importance in engineering [54]. Then, the reliability estimation R=P(X>Y), when X and Y are distributed as N~(μ,σ) respectively [55], is given as

$$R = P(X > Y) = \Phi\left(\frac{\mu_1-\mu_2}{\sqrt{\sigma_1^2+\sigma_2^2}}\right) \quad (13)$$

Where Φ is the standard normal cdf.

The testing problem H_0:R<R_0 versus H_1:R>R_0 is equal to testing H_0:θ<θ_0 versus H_1:θ>θ_0 where

$$\theta = \left(\frac{\mu_1-\mu_2}{\sqrt{\sigma_1^2+\sigma_2^2}}\right) \quad (14)$$

And θ=Φ^(-1) R_0 [56].

This is because the normal distribution has the closure property

Note, that when both the stress and the strength fit a normal distribution, is not necessary to solve the double integral [57]. [58], and because of this, the difference of two variables X and Y are normally distributed [59].

Now let's present the stress/strength model reliability estimation in the Log-normal Distribution case.

V. LOG-NORMAL DISTRIBUTION STRESS-STRENGTH MODEL

Lognormal distribution has been widely applied in many different aspects of life sciences [60]. The lognormal distribution is defined as the distribution of a random variable whose logarithm is normally distributed [61], and usually is formulated with two variables X ~GLN(σ_x,μ_x) [62]. The lognormal distribution is given by

$$f(c;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2} \quad (15)$$

The expectation value and the variance are given by

$$E(x) = e^{\mu+\frac{\sigma 2}{2}} \quad (16)$$

$$E(x) = e^{\mu+\frac{\sigma 2}{2}} \quad (17)$$

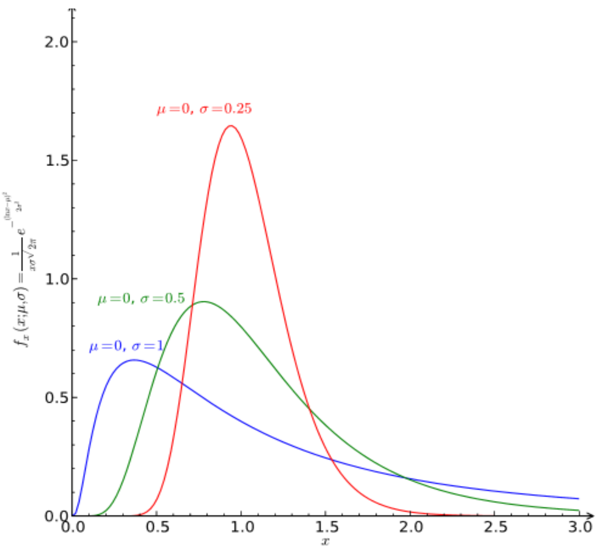

Figure 3: Lognormal Distribution Representation

On the other hand, the stress-strength model, with lognormal for both the stress and the strength have applications in a wide engineering domains see [12]. Originally, the normal distribution was used in engineering stress-strength models, however, it has been increasingly replaced by the lognormal distribution model [63], due to the more realistic properties as the positiveness of its values and the positive skewness of its shape [64]. Let X be the strength of a product or system and R has a lognormal distribution, i.e., X=ln X is normal with mean μ_x and standard deviation σ_x, it is to say, X~ln 〖(μ_x,σ_x)〗 [65].

The density of R is [66]:

$$f(r) = \frac{1}{r\sigma_x\sqrt{2\pi}} e^{-[(\ln r-\mu_x)^2/(2\sigma_x^2)]} \quad (18)$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 7 Issue 03, March-2018

Then

$$\mu_X = e^{\mu_x+\sigma_x^2/2)} \quad (19)$$

$$\sigma_X^2 = e^{(2\mu_x+\sigma_x^2/2)}\left[e^{(\sigma_x^2-1)}\right] \quad (20)$$

Same relations hold for the stress Y applied to a product or system and its logarithm Y=ln⁡y, the notation is Y~ln⁡〖(μ_x,σ_x)〗 [67]. Assuming that the strength and the stress are both independent, the reliability R(t) of the system is therefore R(t)=P(X-Y>0)=P(X/Y>1) [68]. Using the properties of the normal distribution variables X and Y it can be denoted by R(t)=Φ(β) where Φ is the cumulative distribution function of the standard normal distribution and β is the safety index [69], with

$$\beta = \frac{\mu_x-\mu_y}{\sqrt{\sigma_x^2+\sigma_y^2}} \quad (21)$$

It is to be noted that the same expression for R(t) holds for independently distributed normal stress and strength, i.e., for X~ln⁡〖(μ_x,σ_x)〗 and Y~ln⁡〖(μ_x,σ_x)〗 [70]. Also, the lognormal stress-strength model, applies approximately, when several independent variables of stress, or strength, have a multiplicative effect each on the resulting system stress, or strength [71].

Now let's present the stress/strength model reliability estimation in the Weibull Distribution case.

VI. WEIBULL DISTRIBUTION STRESS-STRENGTH MODEL

The Weibull distribution is widely used in reliability and life data analysis due to its versatility [72]. Depending on the values of the parameters, the Weibull distribution can be used to model a variety of life behaviors [73]. An important aspect of the Weibull distribution is how the values of the shape parameter, β, and the scale parameter, η, affect the distribution characteristics [74]. The Weibull shape parameter, β, is also known as the Weibull slope [75]. This is because the value of β is equal to the slope of the line in a probability plot [62]. Different values of the shape parameter can have high effects on the behavior of the distribution [76]. In fact, some values of the shape parameter will cause the distribution equations to reduce to those of other distributions [77]. For example, when β = 1, the pdf of the three-parameter Weibull reduces to that of the two-parameter exponential distribution [78]. The parameter β is a pure number i.e., it is dimensionless [79]. The following figure shows the effect of different values of the shape parameter, β, on the shape of the pdf [80]. One can see that the shape of the pdf can take a variety of forms based on the value of β [81].

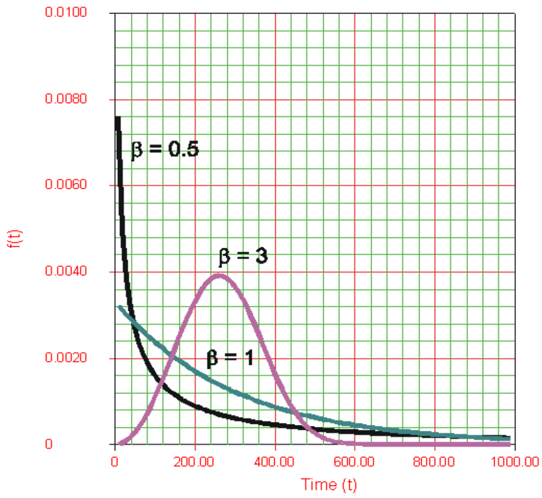

Figure 4: Effect of different values of the shape parameter, β

The use of Weibull distribution in reliability and quality control has been advocated by Kao (1959). The distribution is often suitable where the conditions of 'strict randomness' of the Exponential distribution are not satisfied [82]. Considering the problem of finding the strength reliability of an item functioning until first failure, when both strength (Y) and stress (X) follow the Weibull distribution was considered by [83]. with the following Weibull pdf form:

$$f(x) = \beta\lambda x^{\beta-1}\, e^{\lambda x^{\beta}} \quad (22)$$

$$g(y) = \beta\lambda y^{\beta-1}\, e^{\lambda y^{\beta}} \quad (23)$$

Then, substituting Eq.(22) and (23) [84]:

Suppose X and Y follows a Weibull distribution W_x~(η_1,β_1) and W_y~(η_2,β_2) respectively and they are independent [85].

$$P(X > Y) = \int_1^{\infty}\int_0^{\infty} y(g)yf(vy)dydv \quad (24)$$

The integral involved in Eq.(24) cannot be solved explicitly, in general [86]. However by writing the expansion of e^((-λy^β)) we get

$$P(X > Y) = \sum_{i=0}^{\infty} -1^i \left[\frac{\Gamma\, i\left(\frac{c}{k}\right)+1}{\Gamma(i+1)}\right]\left(\frac{\lambda}{\theta^{\frac{c}{k}}}\right)^i \quad (25)$$

Finally, if both β_1 and β_2 are equal to one R(t) is given by

$$R(t) = \frac{\eta_1}{\eta_1+\eta_2} \quad (26)$$

Then, the Weibull stress-strength analysis for these case can be performed [86]. Here note that it is equivalent to say that the Weibull-Weibull stress-strength model is the same as that of the Exponential-Exponential stress-strength model [87], so long as the two Weibull distributions have the same shape parameter [88]. However, since the Weibull distribution does not have the additive property (known as Weibull closure property) [89], then when the stress and strength variables present different shape parameter β the Weibull stress-strength analysis is not defined.

VII. CONCLUSION

In this paper, different cases of stress-strength models for reliability analysis have been considered. Stress-strength can be described as an assessment of reliability of a product or system in terms of random variables X representing stress experienced by the product and Y representing the strength of the product capability to overcome the stress. According to this, if the stress exceeds the strength (X>Y) the system or product will fail, then the reliability is defined as the probability of not failing, i.e., R(t)=P(X<Y). On the other hand, a stress-strength models have been proposed to deal with different variables behavior, including distribution as the Normal, Lognormal, Exponential and Weibull. In this paper, we presented the stress-strength models for the models mentioned above and the importance of these models to estimate the reliability of a component which is subject to a stress. Finally, as a conclusion, it is important to note that for the Weibull distribution model there is not a defined form, because, the Weibull distribution doesn't have the closure property, i.e., the stress-strength models are an algebraic sum, and Weibull variables cannot be added.

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