

Stress Analysis Of Helical Gear By FEM Techniques With Variation In Face Width And Helix Angle

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Abstract

Gears are the most important members of mechanical power transmission systems. For power transmission helical gears have become the subject of attention. The main factors responsible for the failure of a gear set are bending stress and surface strength of a gear tooth. Therefore stress analysis becomes an important area of research which deals with minimization or reduction of the stresses and also with optimal design of gears. This paper explains about the stress analysis techniques used for helical gears.

This paper presents a detailed study of different techniques proposed and used by various researchers to optimize and to calculate the stresses involved in the helical gear design. Several three dimensional solid models of gears of different specification have been developed by various researchers for analysis. They then used various analysis tools like Ansys, Computer aided FEM; Pro-e software etc for analysis. In this work various parameters that can affect the gear tooth, i.e. variation in face width, helix angle etc. for the complex design problems is also discussed. After going through the literature it is suggested that Matlab Simulink could be used as one of the tools to effectively solve the problems which are taken up as future work.

1. Introduction

Gears are one of the most critical components in mechanical power transmission systems. For reducing the noise level and power transmissions, the helical gears are used. The bending and surface strength of the gear tooth are considered to be one of the main parameters for the failure of the gears. For optimal design of gears and to reduce their failures, the area of research on stresses is becoming increasingly important.

For determining the bending stresses, various three-dimensional solid models of gears of different specifications such as number of teeth, module and helix angle are generated by Pro-Engineer software, which is a powerful and modern solid modeling software and the final numerical solution would be done using ANSYS, which is a finite element analysis software package. All the analytical investigations would be carried out on the basis of Lewis stress formula.

Face width and helix angle are very important geometrical parameters in the design of gears. These are used to determine the state of stresses during the design of gears. Thus, in this work a study is conducted on various works in which the effect of varying face width and helix angle on the bending stress of helical gear are studied. A critical study is also conducted on the tools or methods like FEM, TCA etc. that are utilized in carrying out studies on face width and helix angle to identify if the existing methods are the optimal methods or any suitable methods can be suggested to improve the existing methods or support them in making the methods more users friendly.

1. Methods of stress analysis

TSAY and H.FONG [1] in their work, computer simulation and stress analysis of helical gears with pinion circular arc teeth and gear involute teeth have applied the tooth contact analysis technique and finite element method to gear contact and stress analysis. In their research, they assumed a mathematical model for helical gears with pinion circular arc teeth and gear involute teeth. They have performed computer simulation of the conditions of gear meshing including the axes misalignment and center distance variation. They also identified that tooth contact analysis (TCA) is used for determining the locations of bearing contact and contact pattern of mating tooth surfaces. The location and direction of applied Loadings for the computer-aided FEM stress analysis have been found

from the tooth contact analysis. In order to facilitate the computer-aided stress analysis model for a new pair of gearing, they applied the given mathematical model and the TCA techniques together. They have also investigated a three-dimensional stress analysis for this type of gearing expressed by Von Mises stress contour distributions.

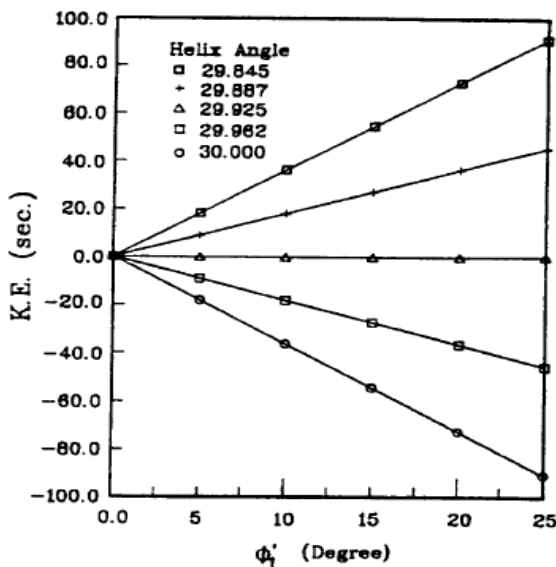


Fig. 1 Kinematic errors due to crossed axes misalignment with compensation.

Fig. (1) Explains that how TCA technique is incorporated with computer aided FEM for gear stress analysis.

From their investigations they identified that the contact pattern found is a point contact instead of a line contact. They also identified that in comparison with the ideal case of meshing there was no significant change found in the contact positions of this type of gearing when there exists a misaligned angle of rotating shaft. In this particular work lower assembly precision was permitted. As far as the center distance variation was concerned there was no change in contact position on the pinion tooth surface, while there was a change in gear tooth surface. From the obtained results they concluded that the bending stress may be increased in this case because of the dislocation movement of the bearing contact on the gear tooth surface. There was an improvement in dislocation of the bearing contact and reduction in the kinematic errors of the gear train by regrinding the pinion with a new modified helix angle.

S. Vijayaragan and N. Ganesan [2] carried out a static analysis of composite helical gears using three dimensional finite element methods to study the displacements and stresses at various points on a helical

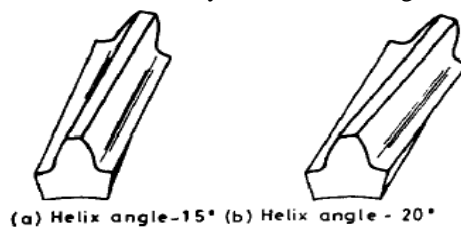
gear tooth. In their work, they have compared the performance of Kevlar/e proxy and graphite /epoxy material helical gears with that of the carbon steel helical gears using a three- dimensional finite element method. The obtained results of FEM were tested by the root stress for C-45 steel material gear and with the results obtained from the conventional gear design procedure.

From their investigations they inferred that there was a similarity in behavior of helical gears made up of orthopedic material and carbon steel helical gears. The variation of stresses and the displacement were found to be along the involute tooth profile and across the tooth thickness. They also identified that the behavior of the composite material helical gears was similar to that of the carbon steel helical gears and also the behavior of graphite/epoxy helical gear was very similar to that of carbon steel helical gears, except for the larger displacements at the tip.

From the obtained results, they concluded that the helical gears made up of composite material behave similarly like carbon steel and graphite helical gears and they can be used for the power transmission purposes with a constraint of suitably increased face width. It is observed from the results that composite materials can be used safely for power transmission helical gears but the face width has to be suitably increased.

Huston [3] et al. in their work, A basis for solid modeling of gear teeth with application in design and manufacture have suggested a new approach for modeling of gear tooth surfaces. They simulated tooth fabrication processes by using a computer graphics solid modeling procedure. This new approach was based on the ideology of differential geometry that concern to envelopes of curves and surfaces. The approach was interpreted with the modeling of spur, bevel, spiral bevel, helical and hypoid gear teeth. From their studies they have discussed various applications of the new proposed approach in the field of design and manufacturing.

Rao and Muthuveerappan [4] in their research finite element modeling and stress analysis of helical gear teeth have explained the geometry of helical gears by simple mathematical equations. They also explained the load distribution for various positions of the contact line and the stress analysis of the helical gears by using



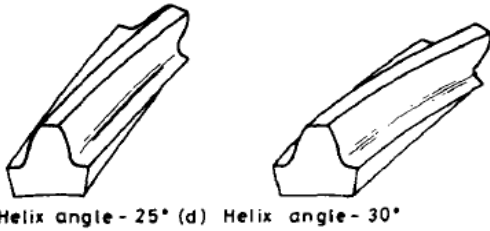


Fig. 2 : various teeth models with different helix angles.

three dimensional finite element methods. Fig .(2) represents the various teeth models having different helix angle.

To carry out their analysis they developed a computer program for the stress analysis of the gears. They evaluated root stresses for different positions of the contact line when it moves from the root to the tip. To verify the legality of the present program, the changes in the tendency of the maximum root stress values at various places of the tooth along the face width have been compared with the experimental outcomes.

In their research, a parametric study was made to study the effect on the root stresses of helical gears by varying the face width and the helix angle. From their investigations they have clarified the effect of helix angle and face width on the root stresses of helical gears for different positions of the contact line.

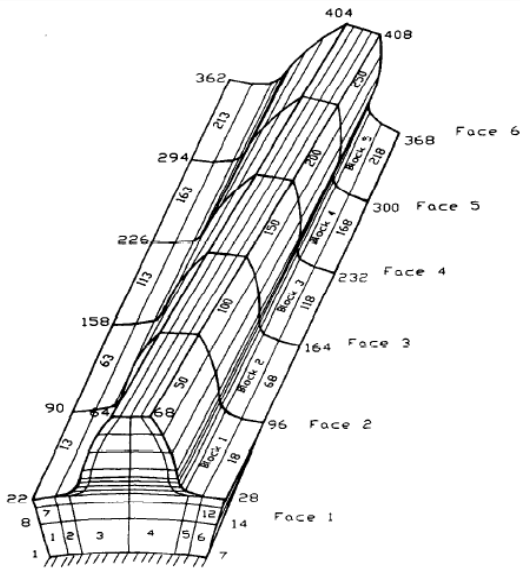


Fig.(3) Finite element method

In this work the helical gear tooth sector was discretized into 250, eight noded isoparametric Brick element with 408 elements in total. The discretization of the helical gear tooth model is shown in fig (3).

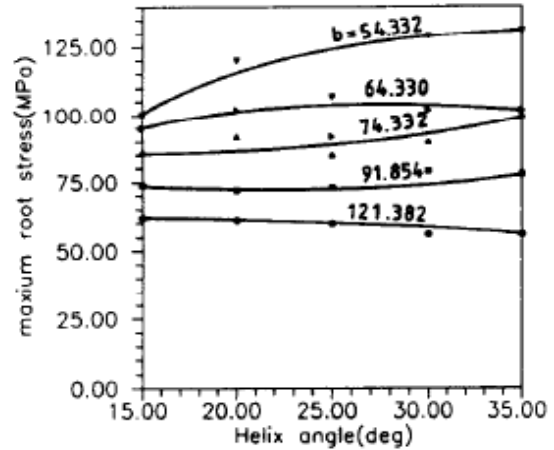


Fig. (4) Combined effect of face width and helix angle on maximum root stress along the face width.

From their study they concluded that as the helix angle is larger the maximum root stress occurs near the first block and the maximum root stress will be much higher on a gear of lower face width with a higher helix angle. Fig (4) shows the combined effect of face width and helix angle on maximum root stress along the face width. They have also suggested that if the space is the only criterion, for a gear with lower face width, an helix angle approximately equal to 15 was suggested and if, material strength value is the criterion then a gear with any desired helix angle with a relatively larger face width was preferred.

J. Lu et al. [5] in their research load share and finite element stress analysis for double circular arc helical gears have analyzed the tooth surface contact and stresses for double circular-arc helical gear drives. They introduced approach was based on the application of computerized simulation of meshing and contact loaded gear drives by using finite element method.

They have investigated the conditions of load share under a load and determined the real contact ratio for aligned and misaligned gear drives, respectively. To carry out their analysis they used finite element method to analyze the elastic deformation of teeth and the stress analysis of the double circular-arc helical gears. The finite element models for the gear and the pinion were constructed. They performed stress analysis for aligned and misaligned gear drives, respectively, and the numerical results were compared with those obtained by other approaches.

From the obtained results they inferred that the results of calculation are close to those obtained by experimental formula. They have also concluded that the pressure on contact ellipse has been calculated accurately.

Jianfeng L. et al [6] in their work Finite element analysis of Cylindrical Gears, Communication in numerical methods in engineering, have proposed a method, namely the normal stiffness matrix along contact line for analysing gears. They have established three-dimensional finite element solid models for spur and helical gears.

From their studies they suggested that the different parameters and materials can be analyzed using these models. They have also presented various results such as load distribution along the contact lines, deformations and stiffness at any contact position, and contact stresses. They also concluded that the calculated results show that the trend of gear tooth deformation coincides with the tested ones using the dynamic speckle photography method.

Jianfeng Li et al. [7] in their work finite element analysis of instantaneous mesh stiffness of cylindrical gears (with and without flexible gear body) have carried out finite element analysis of instantaneous mesh stiffness of cylindrical gears. They have established three dimensional highly accurate solid models of gear teeth on the basis of analysis of the profile formulation principle of internal and external mesh gears of hobbling and slotting, which take the accurate fillet curve and non-simplification tooth profile into consideration.

From their investigations they inferred that the contact range and the whole finite element model could be automatically adjusted with the change of meshing process. Also Instantaneous mesh stiffness of spur and helical gears with and without flexible gear body, have been obtained.

Zhang et. al [8] in their research on Analysis with varying mesh Stiffness, have proposed a new approach to analysis of the loading and stressing distribution for spur and helical gears accounting for the varying meshing stiffness, geometric modification and elastic deflection of the engaged gears. The design of proper modification of tooth flanks or mismatch of a pair of gears relies on an accurate computation of the loading and stressing distribution over the gearing surfaces at any phase of mesh.

From their study they concluded that by combining a discretized gear model with finite element analysis (FEA) a good computational accuracy and efficiency can be obtained.

Y Zhang et. al [9] in their work, Analysis of tooth contact and load distribution of helical gears with crossed axes, have proposed an approach for the analysis of tooth contact and load distribution of helical gears with crossed axis. The proposed approach was based on a tooth contact model that accommodates the influence of tooth profile modifications, gear manufacturing errors and tooth surface deformation on gear mesh quality. In their approach the tooth contact load was assumed to be distributed along the tooth surface line that coincides with the relative principal direction of the contacting tooth surfaces. In comparison with the existing tooth contact (TCA) analysis model that assumes rigidity for the contacting surfaces, the model in this paper provides a more sensible analysis on gear transmission errors, contact patterns and the distribution of contact load.

From their investigations it was found from the analysis that helical gears with small crossing angles have meshing characteristics and load distribution similar to those of parallel-axis gears. Also the approach may be extended to other types of gearing. They concluded that the proposed approach accommodates the tooth deformation in the tooth contact analysis and provides a quantitative measurement on the transmission errors, contact patterns and load distribution on the tooth surfaces of crossed axis helical gear drives. Also the analysis by the computer program that implements the approach shows that tooth profile modification improves the meshing quality and load distribution of crossed axis gearing.

Yi-Cheng Chen et al. [10] in their study stress analysis of a helical gear set with localized bearing contact have investigated the contact and the bending stresses of helical gear set with localized bearing contact by using finite element analysis. In their work they have proposed a helical gear set which comprises an involute pinion and double crowned gear. They have derived the required Mathematical models of the complete teeth geometry of the pinion and the gear on the basis of theory of gearing. Consequently a mesh-generation program was also developed for finite element stress analysis. In this work, commercial finite element analysis package ABAQUS standard was used for the investigation of the gear stress distribution.

From the obtained results they identified that the proposed helical gear set exhibits localized bearing contacts due to double crowning on the gear's tooth surfaces. They also identified that the contact stress calculated by FEA was close to the Hertzian contact stress obtained from the Hertzian stress formulae and curvature analysis.

Also the tensile and compressive bending stresses along the pinion's fillets under different contact positions were investigated. The maximum fillet stress occurs near the middle section of the tooth flank (below the contact points).

They concluded that the proposed FEA method could accurately calculate the contact and bending stresses. Also this model can be extended further to investigate the load share and transmission errors under load.

Litvin et al [11] in their work new version of Novikov–Wildhaber helical gears computerized design, simulation of meshing a stress analysis they have considered tooth contact analysis, stress analysis, design and generation of Novikov–Wildhaber helical gear drive. They have inferred that the advantages of their developed gear drive in comparison with previous ones are the possibility of grinding and application of hardened materials, reduction of stresses and the possibility of grinding and application of hardened materials. For that they have applied new geometry, double crowning of pinion and parabolic types of transmission error. They have confirmed the advantages of their proposed drive by simulation of meshing and contact stress analysis and also by investigation of formation of gear drives.

For the manufacturing of gears they had applied gear grinding or cutting disks, and grinding or cutting worms. They have developed computer programs that cover computerized design, TCA, and automatic development of finite element models of new version of Novikov–Wildhaber gear drives.

From the obtained result they inferred the sensitivity of the gear drive to errors of alignment is reduced due to application of a predesigned parabolic function of transmission errors. Transmission errors normally caused by misalignment are absorbed by that function. They have also inferred by the application of predesigned parabolic function of transmission errors the predesigned parabolic function of transmission errors.

Loutrids [12] in his work gear failure prediction using multi scale local statics recommended multi scale local statistics as a tool for gear failure prediction. He examined the experimental data from gears with localized defects in the form of bending fatigue cracks. His work concluded that second order central moments increased with defect magnitude.

His estimated procedure was carried out in the range of scale where the defects were major. He has recognized empirical law that relates variance at various scales to crack magnitude.

From his results he concluded that the multi scale local statics only need data from the damaged pair, as

compared to methods that are based on comparison between signals from healthy and damaged conditions. He also concluded that other than the existence of a local probability distribution around a point; Multi scale statistics make no a-priori assumptions about the nature of vibration time series. He has also concluded that his proposed method is computationally capable because his method uses a noncritical weighting function and is based on the concept of convolution. He also concluded that his proposed method is substitute to more complex methods, such as those based on time–frequency analysis.

Worden et.al [13] in their study Natural computing for mechanical systems research: A tutorial overview inferred about development of computational algorithms over the past half-century been encouraged by biological system or processes like artificial neural networks. These algorithms are frequently grouped together under the terms soft or natural computing. According to them the general property of all the algorithms that they allow exploration of, or learning from, data. And also the property of these algorithms has proven enormously valuable in the solution of lots of diverse problems in science and engineering. In their work they have explained a tutorial overview of the basic theory of some of the most common methods of natural computing as they have applied in the perspective of mechanical systems research. They have represented the case studies to show the function of some of the main algorithms.

From their conclusions they have explained how natural computing algorithms have already taken a high-flying place in the mechanical systems literature. They have provided a number of applications of many algorithms with thorough case studies. They have also contingent as many of the algorithms which have proved precious have emerged over time from the machine learning committee.

Saravanan et al.[14] in their work on fault diagnosis of spur bevel gear box using artificial neural network they have shown that the condition of operating machine can be guessed by the vibration signals dig out from rotating parts of machine which carries lot many information within them. They have also concluded that processing of these raw vibration signatures measured at a convenient location of the machine unravels the condition of the component or assembly under study. Their work deals with the usefulness of wavelet-based features for fault diagnosis of a gear box using artificial neural network and proximal support vector machines.

The statistical feature vectors from Morlet wavelet coefficients are classified using J48 algorithm and the predominant features were feed as input for training

and testing ANN and PSVM and their relative efficiency in classifying the faults in the bevel gear box was compared.

Their research concluded that they have made a comparative study of different classifying techniques and the ability of Morlet wavelet in feature extraction for bevel gear box fault detection was carried out.

Park and Yoo [15] in their research on deformation overlap in the design of spur and helical gear pair have investigated the deformation overlap by study of the elastic deflection of gear teeth. They have made displacement analysis of the deformation overlap, which was the quantity calculated numerically, and that was the piled region of a contact tooth pair due to the elastic deformation.

For calculating the contact force and teeth deflection they used FEA. They have calculated the contact forces between teeth from the transmitted torque, and the contact problem was found out as a QP problem. They have used the contact forces as boundary conditions for calculating the deformation overlap. They have used the deformation overlap of a spur gear pair for the basis of the tooth tip relief, for a profile shifted gear pair made analysis of deformation characteristics, and also considered teeth deflection of profile shift.

From the results which they have obtained showed that for the design of a profile shifted gear pair and the tooth tip relief calculation deformation overlap were applicable. They recommended the standard amount of removable material for the tip relief and also they have obtained deformation characteristics and optimal shift for long and short addendum system by using deformation overlap. They have also suggested that deformation overlap is applicable for helical gear pair and three dimensional problems, and also it showed good impact on three dimensional problems. They have also extends that the other types of problems which arises with gear pair were also solved with deformation overlap.

Atan and Ozdemir [16] in their study Intelligence modeling of the transient asperity temperatures inferred that for design of gear rise in temperature at the contact zone of meshing gears is a serious problem. According to them if the temperature of lubricating surface increases then surfaces may result in the significant decrease on the material strength and lubricant viscosity which reduces the film thickness, causing solid to solid contact.

They used the equations and evaluations of temperature rise and also data which were given in [Pro. VDI Berichte 2 (1665) (2002) 615–626] for establishing an artificial intelligence model where a multi layer feed forward neural network has been employed.

Their recommended model accepts surface roughness, gear ratio, horsepower and the number of teeth as input variables, and outputs calculated pinion surface asperity temperatures. They have also intended that the advantage of the model is the simplification, and the lessening of the iterative efforts. From the obtained results they concluded the analytical method requires many iterations between the viscosity and the temperature and through the iterations, many data have to be entered. But for the ANN method none of these difficulties exist.

Wang et al. [17] carried out accurate bending strength analysis of the asymmetric gear using the novel ES-PIM with triangular mesh. They have introduced a method viz. the edge-based smoothed point interpolation method to the bending strength analysis of asymmetric gear with complex outlines. They have used a special designed rack cutter for generation of five sets of asymmetric gears with the pressure angles of $20^\circ/20^\circ$, $25^\circ/20^\circ$, $30^\circ/20^\circ$, $35^\circ/20^\circ$, $40^\circ/20^\circ$ respectively. They have kept four key factors for those five models were accuracy, convergence; convergence rate and computational efficiency of the present ES-PIM are checked in great detail.

They have also used the finite element method for calculating the above factors for comparing the results with ES-PIM. They have showed that the ES-PIM method using the background mesh of triangular cells is used to conduct the asymmetric gear bending stress analysis.

From the obtained result they inferred that ES-PIM could provide more efficient and accurate solutions in stress field than the finite element method, and also they suggested ES-PIM is more suitable for stress analysis of the complicated asymmetric gears. They have also inferred their proposed method can better solve the stress concentration problem, and also the ES-PIM method convenient for bending strength analysis of complicated asymmetric gear problem which involves complex outlines, stress concentration at the fillet and concentration load at the HPSTC.

Sfakiotakis and Anifantis [18] in their research on Finite element modelling of spur gearing fractures introduced a numerical procedure of solution for the study of stress singularities in gear teeth fractures incorporating kinematic rotations and varying contact conditions. They have used the global design variables for parametric description of both the geometry and the operation, and also they have used the finite element method for approximation of elastic deformations which enables the development of a computational design procedure which simulates the operation of spur gear drives.

They have considered the members of the drive as individual sub domains which could elastically deform and rotate; the engagement of operating surfaces at successive points enables the simulation of conjugate action. They have evaluated the stress intensity factor by displacement formulas and in the presence of cracks; the behaviour of stress singularities is approximated by quarter point elements.

From the obtained results they have inferred that the model which they developed enables the design engineers with a powerful computational tool, which eliminates any modelling requirements on load sharing. And also their approach may be further extended to describe three-dimensional problems arising in other types of gearing and defects.

Songa and Imb [19] in their study the applicability of process design system for forward extrusion of spur gears showed through experiment the applicability of the automated process design system developed for cold forward extrusion of solid or hollow spur gears was confirmed. They have used the developed system for designing the extrusion die set.

They considered an elastic analysis of dies for shrink fit and carried out to determine level of hoop and effective stresses depending on its ratios. In their investigation they were extruded hollow and solid gears that depend on types of lubrication and types of materials.

From their results which they obtained from experiments they have inferred that filling status of the hollow gear was improved in comparison with that of the solid gear irrespective of forming conditions and the predicted forming load obtained from the developed design system matched well with the experimental load requirements. They have also highlighted that solid and hollow spur gear was easily extruded without any forming defects through the die design set-up determined from their developed method.

2. Conclusion:

Exhaustive research work has been carried out by many researchers in the field of helical gear stress analysis.

A parametric study was also made to study the effect on the root stresses of helical gears by varying the face width and the helix angle. Many authors agree that if material strength is the criterion then a gear with any desired helix angle with a relatively larger face width is preferred.

It is understood from these studies, as discussed earlier, that various methods such as TCA, FEM analysis, mathematical modeling etc. are available for bending stress analysis of helical gears. From the

literature it is clear that static analysis of composite helical gears has also been carried out. Various methods of solid modeling have been introduced in the past. Another method namely normal stiffness matrix along contact line for analyzing gears was also studied.

In most of the approaches FEM was used as a tool for the analysis. From the various results obtained it can be easily realized that the results obtained by using FEM are very close to the results of other methods. The main advantage of the FEM is that it can give the solution of very complex design problems. However, Modeling and simulation of the design problem takes much longer time through FEM.

For this, it is suggested that a method based on MATLAB SIMULINK technique can be used for the stress analysis of helical gears. In this method, the input data sets are given to Simulink program and after processing the input information it gets itself ready for predicting the new output for a given new input.

The main advantage of this method is that it is less time consuming and less complex and prediction can be made very easily for a new data set of helical gears. Results can be predicted without going through the laborious design and analysis process and if the predicted results are out of range, the same could be checked and corrected inputs may very easily be given. Once the predicted outputs are in range, then the design could be simulated and one can go for the analysis part in whichever method like FEM or any other one desires. This reduces the iterations during the design process. Simulink actually could be used as a supporting tool to reduce the effort and time in the design and simulation process using FEM.

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