Strength Prediction Of Composite Laminate

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Abstract

The use of composites in modern structures is steadily increased. The use of these materials was encountered in first place in the aerospace applications. In order to predict failure load, requires information about stresses and strains in a structure. In the present investigation, the stress analyses of CFRP composite laminates with and without cut-outs have been carried out by using both analytical and finite element approaches. In analytical approach, a matlab code has been developed for a flat panel using Classical Laminated Plate Theory (CLPT) and different composite failure theories. MSC.NASTRAN finite element analysis code is used for carrying out finite element analysis. Comparison of stress and strain values obtained from both analytical and finite element methods shows that they are in good agreement for flat panel. Failure load of the flat composite laminate (without cut-out) is determined using four different failure criteria such as maximum stress, maximum strain, Tsai-Hill and Tsai-Wu criteria. The predicted values are compared with experimental results. It is found that the most appropriate theory is Tsai-Wu failure criterion, since the predicted value based on this theory is very closure to experimental failure loads. The results are compared with experimental failure loads available in the literature. The comparison shows that they are in very good agreement. Tsai-Wu failure criterion best predicts the failure load of a composite laminate with and without cut-outs.

Key words: Composite laminate, CFRP, Static Analysis of laminate, Analytical calculation, Numerical calculation

1. Introduction

Composite materials occupy an important place in every important field such as aerospace, defence, automobiles, civil infrastructure, biomaterials as well as sports and leisure. These materials originally developed for the use in aerospace applications have now become a part of daily life. The scope of application of composites being unlimited, these materials will dominate the materials field for a long period in the years to come.

Fibre Reinforced Polymer (FRP) composites are extensively used for primary structural components such as wing, empennage and fuselage; and substructures such as wing ribs and intermediate spars in new generation aircraft as they give rise to high stiffness and strength to weight ratio. In this present investigation the failure load predictions of composites are extremely important in order to ascertain the flight safety during its service periods. The stress analysis is a part of failure prediction process.

Composite panels of size $35\text{mm}\times100\text{mm}\times2.4$ mm, under uniform static tensile loading with and without cutouts are considered for study. Cut-outs considered in this study are respectively of circular and elliptical shape. The panel consists of 16 layers of carbon fiber and epoxy matrix, each layer has 0.15mm thickness with a stacking sequence of [45/-45/0/45/0/-45/0/90]_s. Stress analyses of those panels will be carried out and failure loads will be predicted using failure theories. The results will be correlated with experimentally obtained failure loads.

2. Literature

Improvement in flight performance is one of the most important criteria in the design of aerospace structures. Weight reduction measures, combined with compliance to strength, stiffness and stability requirements are important.

Y.X. Zhang and **C.H. Yang** [2] presented a review of the recent development of the finite element analysis for laminated composite plates. The first-ply failure analysis and the failure were presented clearly.

T.Y Kam and F.M Lai [3] studied the Experimental and theoretical methods for the first ply failure strength of laminated composite plates under different loading conditions.

D. Bruno, G. Spadea and R. Zinno [5] adopts the first-ply failure criterion by application of a polynomial function and the finite element procedure.

M Yasar Kaltake [8] investigated the tensile and compression stress concentration and failure

criteria for anisotropic composite plates with circular cut outs.

X W Xu and H C. Man [12] presented a strength prediction technique for the composite plate with elliptical holes.

3. Stress Analysis of Composite Laminate

An engineering problem can be solved by Analytical, Numerical & Experimental method.

3.1 Nomenclature Used in Laminates

Firstly the coordinate system must be defined through the thickness, length and width of the laminate for the purpose of analysis.

Composite Laminate Properties, Composite laminate is made up of 16 layers of CFRP T300/914C, lamina of thickness 0.15mm following stacking sequence $[+45/-45/0/+45/0/-45/0/90]_{\text{S}}$. The properties are obtained from the literature [16] is tabulated in the table 1.

	Engineering constants for lamina and laminate								
Material	E ₁ × 10 ³ (<i>MPa</i>)	E ₂ × 10 ³ (MPa)	E ₃ × 10 ³ (MPa)	G ₁₂ × 10 ³ (MPa)	G ₂₃ × 10 ³ (MPa)	G ₃₁ × 10 ³ (MPa)	v ₁₂	V ₂₃	v ₂₁
Lamina[16]	130	10	10	5.0	3.27	5.0	0.35	0.5	0.027

Table 1 Material Constants for Lamina and Laminate

The Matlab codes are developed using Classical Lamination Plate Theory and local stress-strains are determined.

4. Finite Element Analysis of Composite Laminate

In the present work MSC NASTRAN is used for carrying out the stress analysis of composite laminate. The finite element model i.e. finite element mesh, boundary conditions and material properties is generated using pre-processor PATRAN. The post-processing is done in PATRAN. Finite element mesh for 560 numbers of elements has been shown in Figure. 1. The mesh details have been tabulated in Table 2.

Finite Element Meshes	Element types	Number of Elements	No. of nodes
Mesh-1	Quad-4	40	55
Mesh-2	Quad-4	140	168
Mesh-3	Quad-4	225	260
Mesh-4	Quad-4	396	442
Mesh-5	Quad-4	560	615
Mesh-6	Quad-4	900	969

Table 2 Convergence Study Mesh Details for flat panel



Figure 1 Finite Element Model of the Flat Panel

5. Results and Discussion

5.1 Stress Analysis of a Flat Panel

Tabulation of Analytical Results

Global strains for all laminas are found to be same because mid-plane curvature terms are zero. Since only extension force is and not moment.

The laminate is the combination of +45,-45, 0 and 90 degree orientation laminas, values of same laminas are tabulated.

(ϵ_x)	([€]	°) ([∈]	x) ([∈] x	(5.2)
€y	} ={€;	,{ _}{∈	y{ =}€y	<pre>/ = {-2.3</pre>	× 10 ⁻³
(γ_{xy})	$(\gamma_{x_{1}})_{(+45)}$	$\left(\gamma_{x}\right)_{(-45)}$	$_{y})_{(0)} (\gamma_{xy})$,) ₍₉₀₎ (0)

Global stresses are calculated by using global strains and stiffness matrix of each lamina. These values are shown in Table 3.

Ply orientation	$\sigma_x(MPa)$	$\sigma_y(MPa)$	$\tau_{xy}(MPa)$
+45	147.4358	72.6528	89.8321
-45	147.4358	72.6528	-89.8321
0	677.3916	-4.3241	0
90	44.7486	-277.6388	0

Table 3 Global Stresses in Flat Panel by Analytical Method

The global stresses i.e. Stresses in laminate coordinate system is transformed to the principal material coordinate system. These values are tabulated in Table 4.

Ply orient ation	σ_1 (<i>MPa</i>)	σ ₂ (MPa)	$ au_{12}$ (MPa)	ε_1	ϵ_{2}	Y 12
+45	199.8	20.2	-37.3	0.001	0.001	-0.003
-45	199.8	20.2	37.3	0.001	0.001	0.003
0	677.3	-4.32	0.00	0.005	-0.002	0
90	-277.6	44.7	0.00	-0.002	0.005	0

 Table 4 Local Stress-Strains in Flat Panel by Analytical

5.2 Mesh details of Flat panel

The finite element model with boundary condition is shown in Figure 1. Local coordinate is created with xaxis along length, y-axis along width and z-axis along the thickness of laminate. All laminas are oriented with respect to the x-axis i.e. 0^0 lamina along x-axis. Uniformly distributed load of 800N/mm is applied at the top edge

The mesh is converged and the stress values are varied based on number of elements, finally where the stress value is constant, that mesh is considered for further analysis and noted in below table 5.

4	SI NO	Mesh details	Mesh
\sim	1	Element type	CQuad 4
	2	No. of elements	560
	3	No. nodes	615

Table 5 Converged Mesh Details for Flat Panel

Fabulation	of FEA	Results
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Ply orient ation	σ_1 (<i>MPa</i>)	σ_2 (MPa)	$ au_{12}$ (MPa)	$\epsilon_{\scriptscriptstyle 1}$	ϵ_{2}	γ ₁₂
+45	205.42	20.36	-36.68	0.0015	0.0015	- 0.0036
-45	205.42	20.36	36.68	0.0015	0.0015	0.0036
0	674.3	-4.78	0	0.0051	- 0.0021	0
90	- 263.45	45.50	0	-0.0021	0.0051	0

Table 6 Local Stress-Strain in Flat Panel by FEA

5.3 Stress Analysis of Panel with Circular Hole

Composite panel of dimensions $100 \times 35 \times 2.4$ mm with a circular hole of 3mm diameter is modelled using MSC.PATRAN. The bottom edge is constrained with U_x, U_y, U_z, R_x, R_y, and R_z values zeros. Uniformly distributed load of 550N/mm is applied at the top edge and details of mesh are shown in Table 7.

SI NO	Mesh details	Mesh
1	Element type	Quad 4
2	No. of elements	2016
3	No. nodes	2100

Table 7 Mesh Details for Flat Panel with Circular Hole

The mesh refinement around hole is performed by taking $1/10^{\text{th}}$ of the element length of mesh of flat panel (same as element sizes located far from hole). Figure 2 shows the finite element model with boundary conditions for panel with circular hole.



Figure 2 Finite Element Model of Panel with Circular Hole

The Average Local Stress-Strains values in Panel with Circular Hole for Different Lamina Orientations are shown in below Table 8.

Ply orient ation	$(MPa)^{\sigma_1}$	σ ₂ (MPa)	$ au_{12}$ (MPa)	ϵ_{i}	ϵ_{2}	Y ₁₂
+45	137.62	13.92	-26.35	0.0010	0.0010	-0.00264
-45	137.62	13.92	26.35	0.0010	0.0010	0.002635
0	474.14	-3.37	0	0.0036	-0.001	0
90	-198.90	31.21	0	-0.001	0.0036	0

 Table 8 The average local stress and strain values for panel with circular hole

The below figure 3 shows the average stress variations across laminate thickness along fiber direction and perpendicular to fiber direction. The average value has been obtained by taking the average value of all respective stresses developed in the elements.



Figure 3 Stress variations along Thickness of Panel with circular hole

5.5 Comparison of Analytical and FEA Results for the Flat Panel

The below tables gives the comparison of analytical and FEA results for flat panel.

	Ply	$\sigma_1 (M)$	%	
	Orientation	Analytical	FEA	Error
7	+45	199.88	205.42	2.698
	-45	199.88	205.42	2.698
	0	677.39	674.29	0.4579
	90	-277.63	- 263.45	5.11

Table 9 Comparison of Principal Fiber DirectionStresses for Flat Panel

Ply	$\sigma_1 (MP)$	0/ Emer	
Orientation	Analytical	FEA	% E1101
+45	20.21	20.36	0.710913
-45	20.21	20.36	0.710913
0	-4.32	-4.79	9.679373
90	44.75	45.50	1.654325

Table 10 Comparison of Stresses Perpendicular toFiber Direction for Flat Panel

Ply	$\tau_{12}(MP)$	% Error	
orientation	Analytical	FEA	70 EITOI
+45	-37.39	- 36.69	1.92363
-45	37.39	36.69	1.92363
0	0	0	0
90	0	0	0

Table 11 Comparison of Shear Stress for Flat Panel

Ply	E	% Error	
orientation	Analytical	FEA	70 L1101
+45	0.0015	0.00153	1.960784
-45	0.0015	0.00153	1.960784
0	0.0052	0.005199	0.01923
90	-0.0023	-0.00214	7.47664

Table 12 Comparison of Principal Fiber DirectionStrain for Flat Panel

Ply	E	% Error	
orientation	Analytical	FEA	70 L1101
+45	0.0015	0.00153	1.960784
-45	0.0015	0.00153	1.960784
0	-0.0023	-0.00214	6.9565
90	0.0052	0.005199	0.01923

Table 13 Comparison of in -Plane Strain for Flat Panel

Ply	γ 12	% Error	
orientation	Analytical	FEA	70 L1101
+45	-0.0037	- 0.00367	0.81744
-45	0.0037	0.00367	0.81744
0	0	0	0
90	0	0	0

 Table 14 Comparison of shear strain for Flat Panel

The percentage of error is very much less for the comparison of FEA v/s Analytical, so the comparison graph is almost same. Comparison of stress and strain values obtained by analytical and finite element

methods are in good agreement for flat panel. Local stresses and strains developed along fiber direction is maximum in 0^0 lamina and that of the 90^0 lamina is minimum for all the three panels (Flat, with circular and elliptical cut-outs). Local stress and strain developed perpendicular to fiber direction is maximum in 90^0 lamina and that of 0^0 lamina is minimum. The shear stress and strain in 0^0 and 90^0 lamina is zero and +45 and -45⁰ laminas is maximum. The stress analysis results are used for strength prediction of different laminates.

5.6 Strength Prediction of Composite Laminate

Failure criteria for composite materials are significantly more complex than yield criteria for metals because composite materials can be strongly anisotropic and tend to fail in a number of different modes depending on their loading state.

First Analytical method is used to predict the strength of panel Using Maximum Stress, Maximum Strain, Tsai-Hill and Tsai-Wu with Hoffman's coefficient and compared with Experimental data. Most suitable failure theory is selected and used for Finite Element Analysis of panel with and without cutout.

5.6.1 Strength Prediction of Flat Panel by Analytical Method

Stress values obtained in chapter 4 for flat panel are used in different failure theories and Failure Index (FI) and strength ratio (SR) is determined. The values are tabulated in Table 15.

Lamina	Max. s The	stress ory	Max. strai	n Theory	Tsai Th	i Hill eory	Tsai-Wu	1 Theory
orientation	FI	SR	FI	SR	FI	SR	FI	SR
45	0.5753	1.738	0.3707	2.6971	0.6112	1.2791	0.7534	1.3272
-45	0.5753	1.738	0.3707	2.6971	0.6112	1.2791	0.7534	1.3272
0	0.5644	1.772	0.5657	1.7676	0.3218	1.7626	0.211	4.731
90	1.1187	0.894	1.3056	0.7659	1.341	0.8635	1.308	0.7245

Table 15 FI and SR for Flat Panel from Analytical Method

From the above values it is observe that 90^{0} lamina as got minimum strength ratio compare to +45,-45 and 0^{0} lamina. So we can conclude that 90^{0} lamina will fail first under axial tensile load.

Now neglect the 90^{0} lamina and following the same procedure to calculate the local stress strain and tabulated in Table 16. It shows that 90^{0} lamina will not play any role in caring load.

Lamina orientation	σ_1 (MPa)	σ ₂ (MPa)	$ au_{12}$ (MPa)	$\epsilon_{\scriptscriptstyle 1}$	\mathbb{C}_2	γ ₁₂
+45	116	11.7	-48.7	8.62E-04	8.62E-04	-4.87E-03
-45	116	11.7	48.7	8.62E-04	8.62E-04	-4.87E-03
0	739	-20.2	0	5.74E-03	-4.01E-03	0
90	-	-	-	-	-	-

Further using the above Local Stress-Strains, Failure index and Strength ratio is calculated. Table 19 shows the Failure index and strength ratio from different failure theories after first ply failure from analytical method. From the Table 17 it is observe that+45 and -45° lamina as got minimum strength ratio compare to 0° lamina. So we can conclude that +45 and -45° laminas will fail leaving only 0° Lamina.

Now neglect the +45 and -45^{0} laminas and follow the same procedure to calculate the stress strain values after calculating the stress strain values, it shows that 90^{0} lamina and 45/-45^{0} lamina will not play any role in carrying load. FI & SR for Second ply failure for flat panel also predicted.

Lamina orientation	Max. Th	. stress eory	Max. stra	ain Theory	Tsai The	Hill eory	Tsai-W	u Theory
	FI	SR	FI	SR	FI	SR	FI	SR
45	0.7518	1.33	0.2154	4.6424	0.6570	1.2337	0.8	1.25
-45	0.7518	1.33	0.2154	4.6424	0.6570	1.2337	0.8	1.25
0	0.6155	1.625	0.6214	1.6092	0.4006	1.5799	.0181	9.5
90	-	-	- /	-	-	-	-	-

 Table 17 FI and SR after First Ply Failure for Flat Panel by Analytical Method

The strength ratio obtained from different failure theory is multiplied with corresponding applied load gives the Failure load and is tabulated in Table 18.

Failure load KN	Max. stress Theory	Max. strain Theory	Tsai Hill Theory	Tsai-Wu Theory
First ply	25	21.44	24.179	21.392
Second ply	45.5	45.05	34.54	34.6584
Last ply	37.8	37.8	37.8	54.8716

Table 18 : Failure Load for Flat Panel by Analytical Method

5.6.2 Strength Prediction of flat Panel by FEA

From analytical method, most suitable failure theory is found i.e. Tsai-Wu theory with Hoffman's coefficient which predicts the failure load closure to experimental data. So in Finite element analysis only Tsai-Wu criterion is used to predict the strength. In order to get the failure index (FI) of lamina, average FI (sum of all elements FI/NO. of elements) is tabulated and shown in Table 19.This results are good agreement with analytical results shows in Table 15.Further for calculation of FI same procedure is adopted for panel with cut-out.

Lamina	Tsai-Wu Theory		
orientation	FI	SR	
45	0.7691	1.3002	
-45	0.7691	1.3002	
0	0.188	5.3186	
90	1.317	0.7593	

Table 19 Failure Index and Strength Ratio by FEA



Figure 3 FI Plot for Flat Panel +45⁰ Lamina



Figure 4 FI Plot for Flat Panel -45⁰ Lamina



Figure 5 FI Plot for Flat Panel 0⁰ Lamina



Figure 6 FI Plot for Flat Panel 90⁰ Lamina

5.6.3 FI and SR for Flat Panel after First Ply Failure

by FEA

Lamina	Tsai-Wu Theory		
orientation	FI	SR	
45	0.7887	1.2679	
-45	0.78887	1.2679	
0	0.00463	215.9	
90	-	-	

Table 20 Failure Index and Strength Ratio by FEA

5.6.4 FI and SR for Flat Panel after Second Ply Failure by FEA

Lamina	Tsai-Wu Theory		
orientation	FI	SR	
45	-	-	
-45	-	-	
0	0.5448	1.84	
90	-	-	

Table 21 Failure Index and Strength Ratio by FEA

5.6.5 Failure Load for Flat Panel by FEA

Flat Panel	Failure load (KN)
First ply	21.26
Second ply	35.5
Last ply	51.394

Table 22 Failure Load for Flat Panel by FEA

5.7 Strength Prediction of Panel with Circular Hole by FEA

Tsai-Wu theory predicted failure load is closely match with the experimental data for the flat panel. Therefore for further analysis only Tsai-Wu with Hoffman's coefficient failure theory is used. Table 23 shows the failure index and strength ratio for panel with circular hole before any lamina fails.

Lamina	Tsai-Wu		
orientation	FI SR		
45	0.7854	1.27	
-45	0.7854	1.27	
0	0.0763	13.1	
90	0.8549	1.17	

 Table 23 FI and SR for Panel with Circular Hole by

 FEA



Figure 7 FI Plot for Panel with Circular Hole +45⁰ Lamina



Figure 8 FI Plot for Panel with Circular Hole -45⁰ Lamina



Figure 9 FI Plot for Panel with Circular Hole 0⁰ Lamina



Figure 10 FI Plot for Panel with Circular Hole 90⁰ Lamina

5.7.1 FI and SR for Panel with Circular Hole after First Ply Failure by FEA

Lamina	Tsai-Wu		
orientation	FI	SR	
45	0.7936	1.25	
-45	0.7936	1.25	
0	0.0975322	10.25	
90	-	-	

Table 24 FI and SR for Panel with Circular Hole after
First Ply Failure by FEA

5.7.2 FI and SR for Panel with Circular Hole after Second Ply Failure by FEA

Lamina	Tsai-Wu		
orientation	FI	SR	
45	-	-	
-45	-	-	
0	0.70319	1.422	
90	-	-	

 Table 25 FI and SR for Panel with Circular Hole after Second Ply Failure by FEA

5.7.3 Failure Load of a Panel with circular hole by FEA

Flat Panel	Failure load (KN)
First ply	20.1
Second ply	24.0625
Last ply	27.37

Table 26 Failure Load for Flat Panel by FEA

The strength ratio obtained from first ply, second ply and last ply failure is multiplied with the applied load gives the corresponding failure load and tabulated in Table 26.

6.0 Validation Study

Analytical and finite element method results are validated by experimental method. The experimental data is shown in Table 27 for flat panel and panel with circular hole. The universal test machines are used for testing CFC panels.

Specimen number	Failure Load for Flat panel(<i>KN</i>)	Failure Load for FP with circular hole(<i>KN</i>)
1	56.98	28.5
2	54.93	29.66
3	50.45	28.72
4	58.50	-
Average	55.22	28.97

 Table 27 Experimental Failure Load for Panel and Panel with Circular Hole

Last ply Failure load (KN)	Percentage error between Analytical and FEA	Percentage error between Analytical and Experimental method	Percentage error between FEA and Experimental method
Flat panel	4.37	0.6338	4.96
FP with circular hole	-	-	5.52

 Table 28 Comparison of Failure Loads between

 Analytical, FEA and Experimental Method

Comparison of failure loads between analytical, finite element method and experimental method is tabulated in Table 28 in terms of percentage error. Results show good agreement with each other. Allowable stressstains are calculated for each case and tabulated in Table 29.

	Flat Panel		Panel with circular Hole	
	$\sigma_{ m allow}$ (MPa)	ε_{allow} ×10 ⁻³	σ _{allow} (MPa)	ϵ_{allow} ×10 ⁻³
First ply failure	253.1	3.947	239.2	3.795
Second ply failure	422.6	7.172	275	5.031
Third ply failure	611.8	12.545	325.8 33	6.779

 Table 29 Allowable Stress-Strain Curve for Flat Panel,

 with Circular Hole and Elliptical Hole by FEA

7 Conclusions and Scope of Future Work

Stress analysis of a flat composite panel is carried out using both analytical and finite element methods. Convergence study has been carried out for this composite panel considering 6 different meshes. Local stresses and strains developed along fiber direction is maximum in 0^0 lamina and that of the 90^0 lamina is minimum for both cases. Local stress and strain developed perpendicular to fiber direction is maximum in 90° lamina and that of 0° lamina is minimum. The shear stress and strain in 0° and 90° lamina is zero and +45 and -45° laminas is maximum. The stress analysis results are used for strength prediction of different laminates. Failure load for flat composite panel has been predicted by both analytical method and finite element methods. These values are in good agreement. The failure loads are obtained using different failure theories such as maximum stress, maximum strain, Tsai-Hill and Tsai-Wu failure criteria. On comparison of these values with experimental failure loads, it is observed that failure load obtained using Tsai-Wu failure criterion is in very good agreement with experimental failure load. This observation concludes that Tsai-Wu failure criterion is appropriate criterion for predicting the failure of laminated composite panels. For further analysis of composite panel with a circular cut-out in order to predict failure load, Tsai-Wu criterion is used. This failure criterion predicts the failure loads of composite panels with circular cut-out very closely with experimental failure loads. It is concluded that Tsai-Wu failure criterion predicts the failure loads of composite panels with and without cutouts more accurately.

Scope of Future Work

In the present work composite laminate is modeled using 2-D quad-4 elements. 3-D finite element analysis may be carried out to obtain more realistic results. The curved composite panels have not been covered in the present study. The same can be taken up in future investigation. The non-linearity aspect has not been considered in the present study. The same may be considered in future.

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