Stochastic Modelling for Annual Rainfall at Jaipur Region

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***Abstract*— Rainfall is an important weather parameter for estimation of crop water requirement. As the rainfall vary with space and time. Still the prediction of rainfall is an unsolved mystery. Markov chains are the statistic approach which is used to predict the rainfall on short term basis. This paper deals with stochastic process to generate the variation of annual rainfall in Jaipur region based on Markov Chain models. For this purpose rainfall data for 41 years (1970–2010) of Jaipur region is collected. The performed statistical tests indicated that the series of the annual rainfall data is trend free. So frequency distribution table is formed. For calculating the yearly rainfall variations the class interval is treated as states, transition probability matrix and Cumulative Probablity Transtion Matrix are formed. Then Uniform random number is generated using the uniform random states. This type of simulation is very useful to generate intervals of rainfall when our instrument is malfunctioned during short period of time.**

Keywords*— *Stochastic;, Markov Chain; Rainfall;, Probability Transition Matrix;

I. Introduction

Jaipur has a semiarid climate under the Köppen climate classification, receiving over 650 millimetres (26 in) of rainfall annually but most rains occur in the monsoon months between June and September. The variability and pattern of rainfall is very important for agricultural as well as the economy of region. It is well established that the rainfall is changing on both the global and the regional scales (Hulme et al. 1998, Dore 2005, Kayano and Sans´ıgolo 2008) due to global warming. The development of a rainfall model is increasingly in demand, not only for data-generation purposes, but also to provide some useful information in various applications, including water resource management and the hydrological and agricultural sectors (Barkotullo 2010). There are four techniques used in rainfall prediction. Statistical technique, Stochastic method, Artificial Neural Network and Numerical weather prediction (Tamil Selvi S. and Samuel Selvaraj R., 2011). Stochastic process is used in this study to describe the variation of annual rainfall of Jaipur region. In this study the modelling is done using the first order Markov process is done. The Markov models are frequently proposed to quickly obtain forecasts of weather "states" at some future time using the information given by current states (Dash Priyankranjan R,. 2012). The markov chain models have 2 advantages:

- The forecasts are available immediately after the observations
- They require minimal computations after the climatologically data have been processed.

A first order Markov chain only require one variable (like temperature, rainfall, fog, frost, cloudiness, wind) to forecast its component at some later time. Recently, (Thyer and Kuczera 1999, 2000) developed a hidden state Markov model to account the long term persistence in annual rainfall. The annual rainfall from 1970 to 2010 is used and the frequency distribution table is formed. The class interval is treated as states and then uncertainty under various states occupied by formation of transition probability matrix. The main purpose of this study is to show the use of first order Markov process to analyse the annual rainfall over Jaipur region.

II. Study Area

The study area comes under the semi arid region of eastern plain of Rajasthan, and is situated at 26^0 55" N latitude , 76^0 50" E longitude and at an altitude of 431.9 m above the sea level.

Fig. !. Study area showing station location

III. Data Used

Annual rainfall of study area in millimetres was obtained from the Department of Water Resources, Jaipur for the year 1970 to 2010. From the fig. 2 it is seen that fluctuation of annual rainfall of Jaipur does not follow any pattern. 41 year rainfall data is converted into the rainfall states as shown in frequency distribution table 1. In table 1 each class interval represents a state of rainfall and denoted by A, B, C, D, E, F and G. Table 2 shows properties like average rainfall, variance , mean, coefficient of variation and kurtosis of 41 years of annual rainfall. The positive kurtosis indicates heavy tails and peakedness relative to the normal distribution, whereas negative kurtosis indicates light tails and flatness.

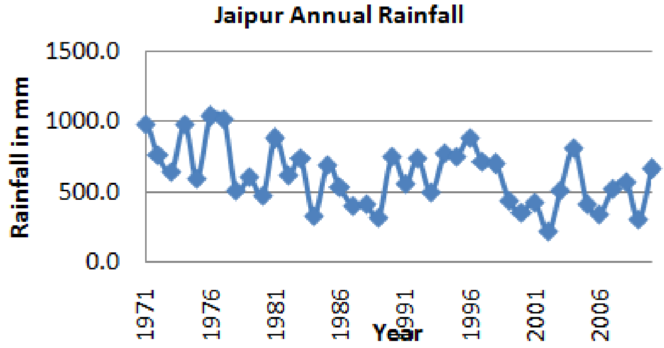

Fig. 2. Fluctuation of Annual Rainfall

TABLE 1: Frequency distribution table for annual rainfall

Class Interval	States	Frequency
222.4-338.62	A	5
338.62- 454.85	B	6
454.85-571.08	C	7
571.08-687.31	D	6
687.31-803.54	E	9
803.54-919.77	F	4
919.77-1036	G	4

TABLE 2 :Summary statistics of rainfall data

Statistics	Value
Sample size	41
avearge	616.3
Variance	45598.0
Standard deviation	213.5
Coefficient of variation	34.64
skewness	0.2
Kutosos	-0.8

IV. Methodology

According to Markov process, a sequence or chain of discrete states in time for which the probability of transition from one state to any given state in the next step in the chain depends on the condition during the previous step (Zohadie Bardaie, M. and Ahmad Che Abdul Salam., 1981). A first order Markov chain is a stochastic process having the property that the value of the process at time t, X_t , depends only on its value at time t-1, X_{t-1}, and not on the sequence of values that the process passed through in arriving at X_{t-1}.

In general, the number of states at each time instant assumed as n. Hence, there will be n x n transitions between two successive time instances. It is then possible to find the number of transition probabilities, p_{ij} from a state at time t to another state at time t+1, and accordingly, the following transition probability matrix, $P_{t,t+1}$ can be prepared from observed rainfall data. The structure of the transition probability matrix would be

$$P_{t,t+1} = \begin{pmatrix} P_{11} & P_{12} & P_{13} \ldots P_{1n} \\ P_{21} & P_{22} & P_{23} \ldots P_{2n} \\ \cdot & \cdot & \cdot \quad \cdot \\ \cdot & \cdot & \cdot \quad \cdot \\ \cdot & \cdot & \cdot \quad \cdot \\ P_{n1} & P_{n2} & P_{n3} \ldots P_{nn} \end{pmatrix}$$

And it can be estimated by (**Siriwardena *et al.*, 2002**):

$$P_{ij} = \frac{fij}{\sum_{j=1}^{c} fij}$$

Where f_{ij} = historical frequency of transition from state i to state j and C = the maximum number of state. With the yearly extreme rainfall data maximum this matrix shows the transition probabilities, p_{ij}: of yearly rainfall in state i at time t to state j at time t +1 given n rainfall state the following properties of the transition matrix are valid by definition. Any state probability varies between zero and one. Notationally,

$$0 < p_{ij} < 1.0 \qquad \text{where i, j= 1,2..., C}$$

$$\sum_{j=1}^{n} pij = 1$$

Where i = 1,2..., C

V. Results and Discussions

In the following matrix, the yearly rainfall is presented with seven states in the form of population probability transition matrix as

$$P_{T,T+1} = \begin{pmatrix} 0 & 0.6 & 0 & 0.2 & 0.2 & 0 & 0 \\ 0 & 0.5 & 0.167 & 0 & 0.167 & 0.167 & 0 \\ 0.285 & 0 & 0 & 0.143 & 0.43 & 0 & 0.143 \\ 0.167 & 0 & 0.333 & 0 & 0.167 & 0.167 & 0.167 \\ 0.222 & 0 & 0.222 & 0.111 & 0.222 & 0.111 & 0.111 \\ 0 & 0 & 0.5 & 0 & 0.25 & 0.25 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0.25 & 0.25 \end{pmatrix}$$

This matrix provides the basis of future likely synthetic rainfall states generations. It the transition probability in the ith row at the kth state is Pik, then the cumulative probability, Pik can be expressed as

$$P_{ik} = \sum_{j=1}^{k} P_{ij}$$

The following steps are necessary for the generation of extreme rainfall states.

1. Calculate the cumulative probability transition matrix, Pc in which each row ends with 1. Hence, cumulative summation within each row leads to

P_c =

2. The primary state is adopted randomly and then by using a uniform random number generator a random value ε is generated within the range of 0 and 1.

3. The next state is obtained where this random value is greater than the cumulative probability of the previous state but less than or equal to the cumulative probability of the following state. In this way, any desired number of rainfall states can be generated.

$$\begin{pmatrix} 0 & 0.6 & 0.6 & 0.8 & 1 & 1 & 1 \\ 0 & 0.5 & 0.67 & 0.67 & 0.83 & 1 & 1 \\ 0.28 & 0.28 & 0.28 & 0.42 & 0.85 & 0.85 & 1 \\ 0.17 & 0.17 & 0.5 & 0.5 & 0.67 & 0.83 & 1 \\ 0.22 & 0.22 & 0.44 & 0.55 & 0.78 & 0.89 & 1 \\ 0 & 0 & 0.5 & 0.5 & 0.75 & 1 & 1 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0.75 & 1 \end{pmatrix}$$

Here in this study the states A, B, C, D, E, F and G are the states and they are the class interval value as it appeared in Table- 1, so when we generate the synthetic series for these states we can get the states of rainfall not the exact value (amount in mm) of rainfall. The synthetic series is generated using uniform random number when second row is considered initially. The states will be decomposed according to following rule,

$$\text{State} = \begin{cases} A & \text{if } u < 0.22 \\ \text{B is ignored, probability of occurrence is zero} & \\ C & \text{if } 0.22 \le u < 0.44 \\ D & \text{if } 0.44 \le u < 0.55 \\ E & \text{if } 0.55 \le u < 0.78 \\ F & \text{if } 0.78 \le u < 0.89 \\ G & \text{if } 0.89 \le u < 1.00 \end{cases}$$

Where u is generated uniform random number.

TABLE 3: Generation of synthetic series of rainfall

Uniform Random Number	0.176	0.748	0.983	0.763	0.451	0.93	0.114
Random States	A	F	G	E	D	G	A

The Table-3 is clearly shown how the states change with changing the probability where each state (English alphabets) is attached with some class interval. This type of simulation is very needful to generate interval of rainfall when our instrument malfunctioned during some short period of time. It is also beneficial to Aviation Company to make short term probabilistic prediction for climatic conditions.

VI. CONCLUSION

Obtaining the information on rainfall plays a significant role in all allied fields like insurance, industry, agriculture, etc. Once the rainfall process is adequately and appropriately modeled, then the model can be used in agricultural planning, may be able to aid in draught, soil erosion and flood predictions, impact of climate change studies, rainfall runoff modeling, crop growth studies and other important fields. In this study the transition probability matrix represents the weather model in which the trend of the following year is estimated. Due to single variable dependency long term forecasting based on this model does not give more accuracy. Markov modeling might be the most significant tool that can assist the planners to forecast the rainfall.

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