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Stochastic Modeling of Markov Chain in Insurance Companies

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Abstract- **Stochastic choice modeling captures the decision-making behavior of individuals or market segments under uncertainty. By integrating probabilistic elements, discrete choice models estimate the likelihood of selecting a particular alternative based on multiple attributes. This study applies discrete stochastic models to identify the factors influencing individuals' selection of insurance companies.**

A Markov chain model is employed to analyze the dynamic operations of insurance companies, focusing on earning patterns and subscriber enrollment trends. As insurance firms play a critical yet often overlooked role in developing economies, understanding their long-term performance is essential for financial stability and growth. In high-risk investment environments, especially those involving public savings, accurate forecasting of company profitability and client participation becomes vital.

The study demonstrates that stochastic modeling provides a robust framework for evaluating and predicting insurance company performance. Long-run probabilities are estimated for various models, and comparative analysis identifies the most suitable model for the data. The findings contribute to both the economic understanding of insurance operations and the methodological advancement of stochastic modeling in Bangladesh.

Keywords- **Markov Chain, Monte Carlo Simulation, MCMC Simulation, Metropolis-Hasting (M-H) Algorithm, Transition probability matrix (TPM), Limiting Probabilities (LP) and Goodness-of-fit.**

1. INTRODUCTION

Stochastic modeling, particularly through the application of Markov chains, has played a crucial role in the evolution of insurance mathematics and actuarial science. Insurance companies operate under uncertainty—policyholder behavior, claim occurrence, and market fluctuations are inherently random. Markov chain models provide a rigorous mathematical framework to represent and analyze these uncertainties, allowing insurers to predict long-term risks, optimize premium structures, and ensure financial solvency. The following review traces the chronological development of stochastic modeling in insurance from 1967 to 2016, highlighting major contributions in the field.

The roots of stochastic modeling in insurance are grounded in early statistical theory. Chakravarti, Laha, and Roy (1967) introduced the Kolmogorov–Smirnov Goodness-of-Fit Test, a fundamental statistical tool used to verify the suitability of theoretical distributions—such as exponential, Poisson, or normal laws—to observed claim data. Their contribution established the statistical basis for validating stochastic and Markovian models that later became standard in insurance risk analysis.

Kulp and Hall (1968), in their work Casualty Insurance, provided one of the earliest practical discussions on uncertainty and risk in insurance operations. While not explicitly Markovian, their approach was stochastic in nature, recognizing that insurance risk depends on probabilistic variations in claim frequency and severity. Their insights set the stage for using stochastic processes to model real-world insurance phenomena.

A significant theoretical advancement came with Peskun (1973), who explored Optimum Monte Carlo Sampling Using Markov Chains. His study introduced an efficient computational method—later developed into Markov Chain Monte Carlo (MCMC)—that has become a key simulation tool in modern actuarial science. Insurance companies use MCMC algorithms to simulate claim processes, estimate transition probabilities, and assess solvency under uncertainty.

Jagers (1975) further advanced stochastic theory with his seminal text Branching Processes with Biological Applications. The branching process, which describes random systems where elements reproduce or die probabilistically, found applications in insurance portfolios and reinsurance models. This framework parallels the way claims or policy events propagate across time and policies in an insurer's book of business.

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The 1980s marked the formal introduction of stochastic processes into applied insurance modeling. Bruss (1984) proposed Extinction Criteria for Bisexual Galton–Watson Processes, extending classical branching models to more complex interdependent systems. In an insurance context, this concept relates to the "survival" or "ruin" of a company—whether it continues to sustain its reserves or becomes insolvent due to recurring claims.

Bhat (1984), in Elements of Applied Stochastic Processes, provided one of the earliest comprehensive introductions to continuous-time Markov chains, renewal processes, and queueing theory. His formulations became directly applicable to insurance risk modeling, where claims arrive randomly over time and affect company reserves. Bhat's work influenced subsequent actuarial studies that use Markov chains to model transitions between financial states, such as solvency, deficit, and ruin.

By the 1990s, stochastic modeling in insurance had evolved from theoretical formulations to applied decision-making frameworks. Medhi (1994), in his textbook Stochastic Processes, elaborated on birth–death processes, renewal processes, and semi-Markov systems. These concepts directly apply to insurance operations: the birth–death process models inflows (premium income) and outflows (claim payments), allowing actuaries to compute ruin probabilities and expected reserve levels over time.

Ross (1995) expanded these ideas in Stochastic Processes by integrating discrete- and continuous-time Markov models into business and actuarial contexts. His analysis of Poisson and Markov jump processes became foundational in modeling claim arrivals and policyholder transitions. These methods help insurers forecast claim counts, optimize reinsurance strategies, and evaluate the time until insolvency.

Hamdy Taha (1999) introduced stochastic decision models in Operations Research: An Introduction. His framework allowed insurance companies to optimize pricing, investment, and reinsurance strategies under stochastic conditions. The combination of Markovian models and operations research principles laid the foundation for modern actuarial optimization.

Latouche and Ramaswami (1999) made a landmark contribution with Matrix Analytic Methods in Modeling. Their techniques enable efficient computation of large-scale Markov chains used in complex insurance models—such as those modeling multiple claim types, varying policyholder states, and interdependent risks. Matrix-analytic methods are now integral in the actuarial modeling of policyholder transitions and queue-like systems in insurance claim processing.

The early 2000s brought a surge in practical applications of stochastic modeling. Bhat (2000), in Stochastic Models, consolidated theoretical foundations with case-based applications, demonstrating how Markov and renewal models describe claim dynamics and risk evolution in insurance.

Papoulis and Pillai (2002) provided deep theoretical insight into Probability, Random Variables, and Stochastic Processes, emphasizing the role of Markov chains and Poisson processes in real-world applications. Their formulations underpin much of the probabilistic modeling now used in actuarial science for life, health, and general insurance.

Gujarati (2003) and Hines et al. (2003) reinforced the statistical underpinnings of stochastic analysis, providing econometric and engineering perspectives for parameter estimation and model fitting. In insurance, these methodologies are essential for estimating transition intensities and claim probabilities based on observed data.

Biswas (2004) and M.K. Roy (2004) expanded applied stochastic modeling into practical actuarial contexts. Biswas's Applied Stochastic Process explicitly analyzed claim occurrence and inter-arrival times using Markovian assumptions, while Roy's work provided essential probability theory for constructing reliable risk models.

By the mid-2000s, the intersection between stochastic modeling and insurance economics gained prominence. Hollis and Strauss (2007) examined asymmetric information and risk in automobile insurance markets, employing stochastic frameworks to explain variations in claim likelihoods. Strauss and Hollis (2007) extended this work to market-level analysis, showing how information uncertainty—modeled through stochastic transitions—affects insurer pricing and competition.

Gupta and Kapoor (2008) contributed a statistical foundation through Fundamentals of Mathematical Statistics, emphasizing hypothesis testing and parameter estimation vital for validating Markov-based insurance models. Meanwhile, Barbu and Limnios (2008) significantly advanced the field with Semi-Markov and Hidden Semi-Markov Models. These models account for unobserved (hidden) risk factors and variable sojourn times, providing a realistic representation of complex insurance systems, such as health deterioration or delayed claims.

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The study by Ghosh, Roy, and Biswas (2016) illustrates the data-driven evolution of stochastic modeling. Though focused on rainfall data, their method of identifying best-fit probability distributions demonstrates how modern insurance companies employ empirical stochastic modeling to predict risk patterns, particularly for weather-related and agricultural insurance. Their work signifies the integration of classical Markovian principles with real-world data analytics.

From the foundational probability studies of Chakravarti et al. (1967) to the data-driven stochastic analyses of Ghosh et al. (2016), the literature reflects a continuous evolution in the application of Markov chain models to insurance problems. Over five decades, these models have advanced from abstract mathematical constructs to essential tools in actuarial science—used for predicting claim dynamics, assessing solvency, and optimizing premium strategies. The integration of semi-Markov, matrix-analytic, and Monte Carlo approaches has further enhanced insurers' ability to manage uncertainty. Today, stochastic modeling based on Markov chains stands as a cornerstone of risk management and decision-making within the insurance industry.

We illustrate our theoretical knowledge of stochastic modeling in two most renowned companies' data: Green Delta Life Insurance Company (GDLIC) and National Life Insurance Company (NLIC). The Markov Chain model is established for the annual profit of two companies. We found only 4 years (2008-2011) annual profit for GDLIC and 2 years (2010-2011) annual profit for NLIC. This information is not adequate for estimating Markov chain. So simulation is carried out for generating the annual profit of GDLIC and NLIC in two phases for each company. That's why we have simulated profit considering the original mean and standard deviation by Markov Chain (MC) at the first phase and at the second phase by Markov Chain Monte Carlo (MCMC) simulation using Metropolis-Hasting (MH) algorithm.

Table 1: Annual profit of Green Delta Life Insurance Company (GDLIC) during 2008-2011.

Year	Profit (Taka in millions)
2011	143.43
2010	562.4
2009	277.7
2008	295.1

Table 2: Annual profit of National Life Insurance Company (NLIC) during 20010-2011.

Year	Profit (Taka in millions)
2011	1466.70
2010	3591.28

2: ESTIMATING MARKOV CHAIN WITH GENERATED PROFIT

Data are generated for the annual profit of GDLIC and NLIC in two phases for each company. At the first phase, Markov Chain is simulated by generating random numbers using Monte Carlo simulation and Markov Chain Monte Carlo (MCMC) simulation is done by using Metropolis-Hasting algorithm in the second stage. As the profit follows normal distribution with mean μ and Variance σ^2. Hence, we will generate the profit of GDLIC with mean (319.6575) and standard deviation (175.4457) and the profit of NLIC with mean (2528.99) and standard deviation (1502.305) in the first stage. In the second stage, the profits of two companies are generated by MCMC using Metropolis-Hasting (MH) Algorithm.

2.1: Markov Chain Estimation using Monte Carlo Simulation

Before simulating the profit of two companies, we need to classify different states of this variable. Since the minimum profit of GDLIC is 6 million taka and maximum is 598 million taka, whereas the minimum profit is -156 million taka and maximum profit is 4401.2 for NLIC. So the states range is different for two companies.

Table 2.1.1: Classification of States in the profit of GDLIC for Markov Chain model during 2008-2011

Generated profit range (Taka in millions)	State
6-154	1
154-302	2
302-450	3
450-598	4
≥598	5

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Transition matrix for the generated profit of GDLIC in the first phase simulation is given below:

$$T = \begin{bmatrix} 4 & 17 & 11 & 5 & 1 \\ 9 & 19 & 21 & 13 & 4 \\ 16 & 19 & 19 & 8 & 3 \\ 6 & 8 & 12 & 4 & 0 \\ 2 & 3 & 3 & 0 & 0 \end{bmatrix} \quad (1)$$

As a result, Transition probability (TP) matrix for GDLIC using MC simulation during 2008-2011 is

$$p = \begin{bmatrix} 0.1052632 & 0.4473684 & 0.2894737 & 0.1315789 & 0.02631579 \\ 0.1363636 & 0.2878788 & 0.3181818 & 0.1969697 & 0.06060606 \\ 0.2461538 & 0.2923077 & 0.2923077 & 0.12230769 & 0.04615385 \\ 0.2000000 & 0.2666667 & 0.4000000 & 0.1333333 & 0.00000000 \\ 0.2500000 & 0.3750000 & 0.3750000 & 0.0000000 & 0.0000000 \end{bmatrix} \quad (2)$$

From this first order TP matrix in equation (2) we observed that the profit of GDLIC starts with 6 to 154 million in the first year will be retain in the same range in the next year with probability .105, there is 45% chance in increase the profit to154 - 302 millions, 29% likelihood for increasing the profit to 302-450 millions, 13% possibility to raise in 450-598 million and only 2% chance remaining to earn 598 million taka or more than 598 million taka in the next year.

When the profit of GDLIC starts with 154-302 million taka in the first year it decreases to 6-154 million taka in the next year with probability .14, there is 29% chance in remaining the same profit range, 32% likelihood for increasing the profit to 302-450 million taka, 20% possibility to raise in 450-598 million taka and only 6% chance remaining to earn 598 million taka or more than 598 million taka in the next year.

When the profit of GDLIC starts with 302-405 million taka in the first year it decreases to 6-154 millions in the next year with probability .25, there is 29% chance to decrease the profit range 154-302 million taka, there is 29% chance in remaining the same profit range, 12% possibility to raise in 450-598 million taka and only 4% chance remaining to earn 598 million taka or more than 598 million taka in the next year.

When the profit of GDLIC starts with 450-598 million taka in the first year it decreases to 6-154 millions in the next year with probability .20, there is 27% chance to decrease the profit range 154-302 million taka, 40% possibility to raise in 302-450 million taka, there is 29% chance in remaining the same profit range and only 0% chance remaining to earn 598 million taka or more than 598 million taka in the next year.

When the profit of GDLIC starts with 598 million taka or more than 598 million taka in the first year it decreases to 6-154 million taka in the next year with probability .250, there is 38% chance in decreasing the profit range to 154-302 million taka, 38% possibility to decrease in 302-450 million taka, 0% likelihood for decreasing the profit to 450-598 million taka, and only 0% chance remaining to earn in the same rage in the next year.

$$p^2 = \begin{bmatrix} 0.1762349 & 0.3054508 & 0.3199308 & 0.1551398 & 0.04324364 \\ 0.1864774 & 0.3121385 & 0.3255936 & 0.1400694 & 0.03572103 \\ 0.1738776 & 0.3298426 & 0.3162443 & 0.1423510 & 0.03768447 \\ 0.1825445 & 0.3187200 & 0.3129996 & 0.1458496 & 0.03988631 \\ 0.1697598 & 0.3294120 & 0.3013020 & 0.1529122 & 0.04661391 \end{bmatrix} \quad (3)$$

From the second order TP matrix in equation (3) we observed that the profit of GDLIC starts with 6 to 154 million taka in the first year will be retain in the same range in the next year with probability .18, there is 30% chance in increase the profit to154 - 302 million taka, 32% likelihood for increasing the profit to 302-450 million taka, 16% possibility to raise in 450-598 million taka and only 4% chance remaining to earn 598 million taka or more than 598 million taka in the next year.

When the profit of GDLIC starts with 154-302 million taka in the first year it decreases to 6-154 million taka in the next year with probability .19, there is 31% chance in remaining the same profit range, 33% likelihood for increasing the profit to 302-450 million taka, 14% possibility to raise in 450-598 million taka and only 3% chance to earn 598 million taka or more than 598 million taka in the next year.

When the profit of GDLIC starts with 302-405 millions in the first year it decreases to 6-154 millions in the next year with probability .17, there is 33% chance to decrease the profit range to 154-302 million taka, 32% chance in remaining in the same profit range, 14% possibility to raise in 450-598 million and only 4% chance to earn 598 million taka or more than 598 million taka in the next year.

When the profit of GDLIC starts with 450-598 millions in the first year it decreases to 6-154 millions in the next year with probability .18, there is 32% chance in decreasing the profit to 154-302, 31% likelihood for decreasing the profit to 302-450 millions, 15% possibility for remaining in the same profit range and only 3% chance to earn 598 million taka or more than 598 million taka in the next year.

When the profit of GDLIC starts with 598 million taka or more than 598 million taka in the first year it decreases to 6-154 millions in the next year with probability .17, there is 33% chance in decreasing the profit to 154-302, 30% likelihood for decreasing the profit to 302-450 million taka,

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15% chance to decrease the profit range 450-598 million taka and only 4% possibility for remaining in the same profit range in the next year.

$$p^3 = \begin{bmatrix} 0.1807945 & 0.3178800 & 0.3199948 & 0.1434148 & 0.03791597 \\ 0.1792838 & 0.3192029 & 0.3178938 & 0.1447673 & 0.03885218 \\ 0.1790175 & 0.3172746 & 0.3187956 & 0.1457501 & 0.03916208 \\ 0.1788646 & 0.3187601 & 0.3190421 & 0.1447668 & 0.03856630 \\ 0.1791918 & 0.3171056 & 0.3206719 & 0.1446926 & 0.03833798 \end{bmatrix} \quad (4)$$

Similarly we can get p^4, p^5, p^6, p^7, p^8, p^9, And finally we get the transition probability matrix which is convergence in the matrix of order 10 and for higher order than 10.The convergence matrix is-

$$p^{10} = \begin{bmatrix} 0.1794057 & 0.3182054 & 0.3188322 & 0.1448351 & 0.03872171 \\ 0.1794057 & 0.3182054 & 0.3188322 & 0.1448351 & 0.03872171 \\ 0.1794057 & 0.3182054 & 0.3188322 & 0.1448351 & 0.03872171 \\ 0.1794057 & 0.3182054 & 0.3188322 & 0.1448351 & 0.03872171 \\ 0.1794057 & 0.3182054 & 0.3188322 & 0.1448351 & 0.03872171 \end{bmatrix} \quad (5)$$

From this convergence transition probability matrix in equation (5) we observe that the highest long run probability 32% is associated with the profit range 154-302 and 302-450 million taka, followed by 18% with 6-154 million taka, 14% with 450-598 million taka and the minimum probability 4% associated with profit 598 million taka or more million taka.

Table 2.1.2: Classification of States in the profit of NLIC for Markov Chain (MC) model during 2010-2011

Generated profit range (Taka in millions)	State
(-156)-983.4	1
983.4-2122.8	2
2122.8-3262.22	3
3262.22-4401.2	4
≥4401.2	5

Transition matrix for the generated profit of NLIC in the first phase simulation is given below

As a result, Transition probability (TP) matrix for NLIC using MC simulation during 2010-2011 is

$$T = \begin{bmatrix} 2 & 3 & 4 & 5 & 2 \\ 3 & 4 & 9 & 4 & 1 \\ 5 & 5 & 13 & 7 & 5 \\ 5 & 9 & 3 & 3 & 2 \\ 1 & 0 & 5 & 3 & 0 \end{bmatrix} \quad (6)$$

As a result, Transition probability (TP) matrix for NLIC using MC simulation during 2010-2011 is

$$p = \begin{bmatrix} 0.1250000 & 0.1875000 & 0.2500000 & 0.3125000 & 0.12500000 \\ 0.1428571 & 0.1904762 & 0.4285714 & 0.1904762 & 0.04761905 \\ 0.1428571 & 0.1428571 & 0.3714286 & 0.2000000 & 0.14285714 \\ 0.2272727 & 0.4090909 & 0.1363636 & 0.1363636 & 0.09090909 \\ 0.1111111 & 0.0000000 & 0.5555556 & 0.3333333 & 0.00000000 \end{bmatrix} \quad (7)$$

From this first order TP matrix in equation (7) we observed that the profit of NLIC starts with (-156)-983.4 million taka in the first year will be retain in the same range in the next year with probability .125, there is 19% chance in increase the profit to million taka 983.4-2122.8, 25% likelihood for increasing the profit to 2122.8-3262.223 million taka, 31% possibility to raise in 3262.223-4401.2 million taka and only 14% chance to earn 4401.2 or more than 4401.2 million taka in the next year.

When the profit of NLIC starts with 983.4-2122.8 millions in the first year it decreases to (-156)-983.4 million taka in the next year with probability .142, there is 19% chance in remaining the same profit range, 42% likelihood for increasing the profit to 2122.8-3262.223 million taka, 19% possibility to raise in 3262.223-4401.2 million taka and only 4% chance remaining to earn 4401.2 million taka or more than 4401.2 million taka in the next year.

When the profit of NLIC starts with 2122.8-3262.223 million taka in the first year it decreases to (-156)-983.4 million taka in the next year with probability .142, there is 14% chance in decreasing the profit range to 983.4-2122.8 million taka, 37% chance in remaining the same profit range, 20% possibility to raise in 3262.223-4401.2 million taka and only 14% chance to earn 4401.2 or more than 4401.2 million taka in the next year.

When the profit of NLIC starts with 3262.223-4401.2 million taka in the first year it decreases to (-156)-983.4 millions in the next year with probability .227, there is 41% chance in decreasing the profit range to 983.4-2122.8 million taka, 14% possibility to decrease in 2122.8-3262.223 million taka, 14% chance in remaining the same profit range, and only 9% chance to earn 4401.2 or more than 4401.2 million taka in the next year.

When the profit of NLIC starts with 4401.2 million taka or more than 4401.2 million in the first year it decreases to (-156)-983.4 millions in the next year with probability .111, there is 0% chance in decreasing the profit range to 983.4-2122.8 million taka, 56% possibility to decrease in 2122.8-3262.223 million taka, 33% likelihood for decreasing the profit to 3262.223-4401.2 million taka, and only 0% chance to remain in the same profit range in the next year

$$p^2 = \begin{bmatrix} 0.1630366 & 0.2227070 & 0.3165224 & 0.2090571 & 0.08867695 \\ 0.1548736 & 0.2022135 & 0.3289597 & 0.2084854 & 0.10546794 \\ 0.1526541 & 0.1888760 & 0.3415358 & 0.2210321 & 0.09590291 \\ 0.1474239 & 0.1958014 & 0.3518923 & 0.2251156 & 0.07976682 \\ 0.1690115 & 0.2365620 & 0.2795815 & 0.1912879 & 0.12355700 \end{bmatrix} \quad (8)$$

From the second order TP matrix in equation (8) we observed that the profit of NLIC starts with (-156)-983.4 million taka in the first year will be retain in the same range in the next year with probability .163, there is 22% chance in increase the profit to million taka 983.4-2122.8, 31% likelihood for increasing the profit to 2122.8-3262.223 million taka, 20% possibility to raise in 3262.223-4401.2 million taka and only 8% chance to earn 4401.2 or more than 4401.2 million taka in the next year.

When the profit of NLIC starts with 983.4-2122.8 millions in the first year it decreases to (-156)-983.4 million taka in the next year with probability .154, there is 20% chance in remaining the same profit range, 33% likelihood for increasing the profit to 2122.8-3262.223 million taka, 21% possibility to raise in 3262.223-4401.2 million taka and only 11% chance remaining to earn 4401.2 million taka or more than 4401.2 million taka in the next year.

When the profit of NLIC starts with 2122.8-3262.223 million taka in the first year it decreases to (-156)-983.4 million taka in the next year with probability .152, there is 19% chance in decreasing the profit range to 983.4-2122.8 million taka, 34% chance in remaining the same profit range, 22% possibility to raise in 3262.223-4401.2 million taka and only 9% chance to earn 4401.2 or more than 4401.2 million taka in the next year.

When the profit of NLIC starts with 3262.223-4401.2 million taka in the first year it decreases to (-156)-983.4 millions in the next year with probability .147, there is 19% chance in decreasing the profit range to 983.4-2122.8 million taka, 35% possibility to decrease in 2122.8-3262.223 million taka, 23% chance in remaining the same profit range, and only 8% chance to earn 4401.2 or more than 4401.2 million taka in the next year.

When the profit of NLIC starts with 4401.2 million taka or more than 4401.2 million in the first year it decreases to (-156)-983.4 millions in the next year with probability .169, there is 24% chance in decreasing the profit range to 983.4-2122.8 million taka, 28% possibility to decrease in 2122.8-3262.223 million taka, 19% likelihood for decreasing the profit to 3262.223-4401.2 million taka, and only 12% chance to remain in the same profit range in the next year.

$$p^3 = \begin{bmatrix} 0.1547783 & 0.2037306 & 0.3315432 & 0.2147406 & 0.09520734 \\ 0.1543428 & 0.1998393 & 0.3345894 & 0.2162926 & 0.09493586 \\ 0.1557451 & 0.2038117 & 0.3293865 & 0.2140962 & 0.09696043 \\ 0.1566956 & 0.2073006 & 0.3264862 & 0.2110305 & 0.09848725 \\ 0.1520643 & 0.1949435 & 0.3422087 & 0.2210622 & 0.08972133 \end{bmatrix} \quad (9)$$

Similarly we can get $p^5, p^6, p^7, p^8, p^9, p^{10}, p^{11}, p^{12}, p^{13}$ and finally in the matrix of order 14 and higher than 14 it will be convergence and the convergence matrix is as follows-

$$p^{14} = \begin{bmatrix} 0.1551616 & 0.2028915 & 0.331384 & 0.2146519 & 0.09591107 \\ 0.1551616 & 0.2028915 & 0.331384 & 0.2146519 & 0.09591107 \\ 0.1551616 & 0.2028915 & 0.331384 & 0.2146519 & 0.09591107 \\ 0.1551616 & 0.2028915 & 0.331384 & 0.2146519 & 0.09591107 \\ 0.1551616 & 0.2028915 & 0.331384 & 0.2146519 & 0.09591107 \end{bmatrix} \quad (10)$$

From this convergence transition probability matrix in equation (10) we can see that the highest long run probability 33% is associated with the profit range 2122.8-3262.223 million taka, followed by 21% with 3262.223-4401.2 million taka, 20% with 983.4-2122.8 million taka, 16% with 983.4-2122.8 million taka and the minimum probability 9% associated with profit 4401.2 million taka or more million taka

2.2: Markov Chain Estimation using MCMC Simulation using Metropolis-Hasting (M-H) Algorithm

Before simulating the profit of two companies, we need to classify different states of this variable. Since the minimum profit of GDLIC is 2.54million taka and maximum is 555.98 million taka, whereas the minimum profit is -581.73 million taka and maximum profit is 4408.219 for NLIC. So the states range is different for two companies.

Table 2.2.1: Classification of States in the profit of GDLIC for Markov Chain Monte Carlo (MCMC) model during 2008-2011

Generated profit range (Taka in millions)	State
(2.54)-140.90	1
140.90-279.26	2
279.26-417.62	3
417.62-555.98	4
≥555.98	5

Transition matrix for the generated profit of GDLIC in the second phase simulation is given below

$$T = \begin{bmatrix} 4 & 8 & 8 & 10 & 1 \\ 4 & 10 & 15 & 14 & 6 \\ 12 & 19 & 17 & 9 & 3 \\ 10 & 9 & 16 & 12 & 5 \\ 0 & 4 & 4 & 7 & 0 \end{bmatrix} \quad (11)$$

As a result, Transition probability (TP) matrix for GDLIC using MCMC simulation during 2008-2011 is

$$p = \begin{bmatrix} 0.12903226 & 0.2580645 & 0.2580645 & 0.3225806 & 0.03225806 \\ 0.08163265 & 0.204816 & 0.3061224 & 0.2857143 & 0.12244898 \\ 0.20000000 & 0.3166667 & 0.2833333 & 0.1500000 & 0.05000000 \\ 0.19230769 & 0.1730769 & 0.3076923 & 0.2307692 & 0.09615385 \\ 0.00000000 & 0.2666667 & 0.2666667 & 0.4666667 & 0.00000000 \end{bmatrix} \quad (12)$$

From this first order TP matrix in equation (12) we observed that the profit of GDLIC starts with 2.54 to 140.90 million taka in the first year will be retain in the same range in the next year with probability .129, there is 26% chance in increase the profit to140.90-279.26 million taka, 26% likelihood for increasing the profit to 279.26-417.62 million taka, 32% possibility to raise in 417.62-555.98 million taka and only 3% chance remaining to earn 555.98 or more than 555.98 million taka in the next year.

When the profit of GDLIC starts with140.90-279.26 millions in the first year it decreases to 2.54 to 140.90 million taka in the next year with probability .08, there is 20% chance in remaining the same profit range, 31% likelihood for increasing the profit to 279.26-417.62 million taka, 29% possibility to raise in 417.62-555.98 million taka and only 12% chance remaining to earn 555.98 million taka or more than 555.98 million taka in the next year.

When the profit of GDLIC starts with 279.26-417.62 million taka in the first year it decreases to (2.54)-140.90 million taka in the next year with probability .20, there is 32% chance in decreasing the profit range to 140.90-279.26 million taka, 28% chance in remaining the same profit range, 15% possibility to raise in 417.62-555.98 million taka and only 5% chance remaining to earn 555.98 or more than 555.98 million taka in the next year.

When the profit of GDLIC starts with 417.62-555.98 million taka in the first year it decreases to (2.54)-140.90 millions in the next year with probability .192, there is 17% chance in decreasing the profit range to 140.90-279.26 million taka, 15% possibility to decrease in 279.26-417.62 million taka, 23% chance in remaining the same profit range, and only 9% chance remaining to earn 555.98 or more than 555.98 million taka in the next year.

When the profit of GDLIC starts with 555.98 million taka or more than 555.98 million in the first year it decreases to (2.54)-140.90 millions in the next year with probability .000, there is 27% chance in decreasing the profit range to 140.90-279.26 million taka, 27% possibility to decrease in 279.26-417.62 million taka, 47% likelihood for decreasing the profit to 417.62-555.98 million taka, and only 0% chance remaining to earn 555.98 or more than 555.98 million taka in the next year.

$$p^2 = \begin{bmatrix} 0.1513635 & 0.2321187 & 0.2932740 & 0.2435612 & 0.07968266 \\ 0.1433625 & 0.2417582 & 0.2908403 & 0.2536374 & 0.0704055 \\ 0.1371696 & 0.2452558 & 0.2883166 & 0.2554410 & 0.07381687 \\ 0.1448598 & 0.2479674 & 0.2864370 & 0.2557654 & 0.06497053 \\ 0.1648456 & 0.2196354 & 0.3007780 & 0.2238828 & 0.09085819 \end{bmatrix} \quad (13)$$

From the second order TP matrix in equation (13) we observed that the profit of GDLIC starts with 2.54 to 140.90 million taka in the first year will be retain in the same range in the next year with probability .151, there is 23% chance to increase the profit range 140.90-279.26 million taka, 29% likelihood for increasing the profit to 279.26-417.62 million taka, 24% possibility to raise in 417.62-555.98 million taka and only 7% chances to earn 555.98 or more than 555.98 million taka in the next year.

When the profit of GDLIC starts with140.90-279.26 millions in the first year it decreases to 2.54 to 140.90 million taka in the next year with probability .143, there is 24% chance in remaining the same profit range, 29% likelihood for increasing the profit to 279.26-417.62 million taka, 25% possibility to raise in 417.62-555.98 million taka and only 7% chance remaining to earn 555.98 million taka or more than 555.98 million taka in the next year.

When the profit of GDLIC starts with 279.26-417.62 million taka in the first year it decreases to (2.54)-140.90 million taka in the next year with probability .137, there is 25% chance in decreasing the profit range to 140.90-279.26 million taka, 29% chance in remaining the same profit range, 26% possibility to raise in 417.62-555.98 million taka and only 7% chance remaining to earn 555.98 or more than 555.98 million taka in the next year.

When the profit of GDLIC starts with 417.62-555.98 million taka in the first year it decreases to (2.54)-140.90 millions in the next year with probability .144, there is 25% chance in decreasing the profit range to 140.90-279.26 million taka, 29% possibility to decrease in 279.26-417.62 million taka, 26% chance in remaining the same profit range, and only 6% chance remaining to earn 555.98 or more than 555.98 million taka in the next year.

When the profit of GDLIC starts with 555.98 million taka or more than 555.98 million in the first year it decreases to (2.54)-140.90 millions in the next year with probability .164, there is 22% chance in decreasing the profit range to 140.90-279.26 million taka, 30% possibility to decrease in 279.26-417.62 million taka, 22% likelihood for decreasing the profit to 417.62-555.98 million taka, and only 9% chance remaining in the same profit range in the next year.

$$p^3 = \begin{bmatrix} 0.0.1439727 & 0.2427063 & 0.2894032 & 0.2525293 & 0.07138843 \\ 0.1451782 & 0.2411071 & 0.2902252 & 0.2503316 & 0.07315787 \\ 0.1445068 & 0.2406465 & 0.2904484 & 0.2509647 & 0.07343362 \\ 0.1454069 & 0.2402863 & 0.2904712 & 0.2498847 & 0.2509647 \\ 0.1424098 & 0.2455885 & 0.2881124 & 0.2551114 & 0.06877783 \end{bmatrix} \quad (14)$$

Similarly we can get $p^4, p^5, p^6, p^7, p^8, p^9, p^{10}, p^{11}$ and finally in the matrix of order 12 and for higher order than 12 it will be convergence and the convergence matrix is as follows-

$$p^{12} = \begin{bmatrix} 0.1446648 & 0.2413253 & 0.2900788 & 0.2510692 & 0.07286186 \\ 0.1446648 & 0.2413253 & 0.2900788 & 0.2510692 & 0.07286186 \\ 0.1446648 & 0.2413253 & 0.2900788 & 0.2510692 & 0.07286186 \\ 0.1446648 & 0.2413253 & 0.2900788 & 0.2510692 & 0.07286186 \\ 0.1446648 & 0.2413253 & 0.2900788 & 0.2510692 & 0.07286186 \end{bmatrix} \quad (15)$$

From this convergence transition probability matrix in equation (15), we get the highest long run probability 29% is associated with the profit range 279.26-417.62 million taka, followed by 25% with 417.62-555.98 million taka, 24% with 140.90-279.26, 14% with2.54-140.90 million taka and the minimum probability 7% associated with profit 555.98 million taka or more million taka.

Table 2.2.2: Classification of States in the profit of NLIC for Markov Chain Monte Carlo (MCMC) during 2010-2011

Generated profit range (Taka in millions)	State
(-581.73) - (-1183.74)	1
(-1183.7404) - (214.2496)	2
214.2496 - 1612.2396	3
1612.2396 - 3010.2296	4
≥4408.219	5

Transition matrix for the generated profit of NLIC in the second phase simulation is given below

$$\boldsymbol{T} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 7 \\ 0 & 0 & 0 & 7 & 7 \\ 1 & 1 & 7 & 8 & 14 \\ 0 & 8 & 5 & 15 & 19 \end{bmatrix} \quad (16)$$

As a result, Transition probability (TP) matrix for NLIC using MCMC simulation during 2010-2011 is

$$p = \begin{bmatrix} 0.00000000 & 0.00000000 & 1.0000000 & 0.0000000 & 0.0000000 \\ 0.00000000 & 0.10000000 & 0.10000000 & 0.1000000 & 0.7000000 \\ 0.00000000 & 0.00000000 & 0.00000000 & 0.5000000 & 0.5000000 \\ 0.03225806 & 0.03225806 & 0.2258065 & 0.2580645 & 0.4516129 \\ 0.00000000 & 0.17021277 & 0.1063830 & 0.3191489 & 0.4042553 \end{bmatrix} \quad (17)$$

From this first TP matrix in equation (17) we observed that the profit of NLIC starts with (-581.73)-(-1183.74) million taka in the first year will be retain in the same range in the next year with probability .000, there is 0% chance in increase the profit to (-1183.7404)-214.2496 million taka, 100% likelihood for increasing the profit to 214.2496-1612.2396 million taka, 0% possibility to raise in 1612.2396-3010.2296 million taka and only 0% chance to earn 4408.219 or more than 4408.219 million taka in the next year.

When the profit of NLIC starts with (-1183.7404)-214.2496 millions in the first year it decreases to (-581.73)-(-1183.74) million taka in the next year with probability .000, there is 10% chance in remaining the same profit range, 10% likelihood for increasing the profit to 214.2496-1612.2396 million taka, 10% possibility to raise in 1612.2396-3010.2296 million taka and only 70% chance remaining to earn 4408.219 million taka or more than 4408.219 million taka in the next year.

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When the profit of NLIC starts with 214.2496-1612.2396 million taka in the first year it decreases to (-581.73)-(-1183.74) million taka in the next year with probability .000, there is 0% chance in decreasing the profit range to (-1183.7404)-214.2496 million taka, 0% chance in remaining the same profit range, 50% possibility to raise in 1612.2396-3010.2296 million taka and only 50% chance to earn 4408.219 or more than 4408.219 million taka in the next year.

When the profit of NLIC starts with 1612.2396-3010.2296 million taka in the first year it decreases to (-581.73)-(-1183.74) millions in the next year with probability .03, there is 3% chance in decreasing the profit range to (-1183.7404)-214.2496 million taka, 23% possibility to decrease in 214.2496-1612.2396 million taka, 26% chance in remaining the same profit range, and only 45% chance to earn 4408.219 or more than 4408.219 million taka in the next year.

When the profit of NLIC starts with 4408.219 million taka or more than 4408.219 millions in the first year it decreases to (-581.73)-(-1183.74) millions in the next year with probability .000, there is 17% chance in decreasing the profit range to (-1183.7404)-214.2496 million taka, 11% possibility to decrease in 214.2496-1612.2396 million taka, 32% likelihood for decreasing the profit to 1612.2396-3010.2296 million taka, and only 40% chance to remain in the same profit range in the next year.

$$p^2 = \begin{bmatrix} 0.000000000 & 0.00000000 & 0.0000000 & 0.5000000 & 0.5000000 \\ 0.003225806 & 0.13237474 & 0.1070487 & 0.3092107 & 0.4481400 \\ 0.016129032 & 0.10123542 & 0.166097 & 0.2886067 & 0.4279341 \\ 0.008324662 & 0.08842075 & 0.1418004 & 0.3268581 & 0.4345961 \\ 0.010295127 & 0.09612582 & 0.1320931 & 0.2815914 & 0.4798946 \end{bmatrix} \quad (18)$$

From the second order TP matrix in equation (18) we observed that the profit of NLIC starts with (-581.73)-(-1183.74) million taka in the first year will be retain in the same range in the next year with probability .000, there is 0% chance in increase the profit to (-1183.7404)-214.2496 million taka, 0% likelihood for increasing the profit to 214.2496-1612.2396 million taka, 50% possibility to raise in 1612.2396-3010.2296 million taka and only 50% chance to earn 4408 or more than 4408.219 million taka in the next year.

When the profit of NLIC starts with (-1183.7404)-214.2496 millions in the first year it decreases to (-581.73)-(-1183.74) million taka in the next year with probability .003, there is 13% chance in remaining the same profit range, 11% likelihood for increasing the profit to 214.2496-1612.2396 million taka, 31% possibility to raise in 1612.2396-3010.2296 million taka and only 45% chance remaining to earn 4408.219 million taka or more than 4408.219 million taka in the next year.

When the profit of NLIC starts with 214.2496-1612.2396 million taka in the first year it decreases to (-581.73)-(-1183.74) million taka in the next year with probability .016, there is 10% chance in decreasing the profit range to (-1183.7404)-214.2496 million taka, 17% chance in remaining the same profit range, 29% possibility to raise in 1612.2396-3010.2296 million taka and only 43% chance to earn 4408.219 or more than 4408.219 million taka in the next year.

When the profit of NLIC starts with 1612.2396-3010.2296 million taka in the first year it decreases to (-581.73)-(-1183.74) millions in the next year with probability .008, there is 9% chance in decreasing the profit range to (-1183.7404)-214.2496 million taka, 14% possibility to decrease in 214.2496-1612.2396 million taka, 33% chance in remaining the same profit range, and only 43% chance to earn 4408.219 or more than 4408.219 million taka in the next year.

When the profit of NLIC starts with 4408.219 million taka or more than 4408.219 millions in the first year it decreases to (-581.73)-(-1183.74) millions in the next year with probability .010, there is 9% chance in decreasing the profit range to (1183.7404)-214.2496 million taka, 13% possibility to decrease in 214.2496-1612.2396 million taka, 28% likelihood for decreasing the profit to 1612.2396-3010.2296 million taka, and only 48% chances to remain in the same profit range in the next year.

$$p^3 = \begin{bmatrix} 0.016129032 & 0.10123542 & 0.1660947 & 0.2886067 & 0.4279341 \\ 0.009974539 & 0.09949116 & 0.1339595 & 0.2895816 & 0.4669932 \\ 0.009309894 & 0.09227328 & 0.136467 & 0.3042248 & 0.4572453 \\ 0.010543810 & 0.09335968 & 0.1372070 & 0.3027936 & 0.4560958 \\ 0.009083595 & 0.10038036 & 0.1345455 & 0.3014857 & 0.4545049 \end{bmatrix} \quad (19)$$

Similarly we can get p^4, p^5, p^6, p^7, p^8, p^9, p^{10}, p^{11}, p^{12}, p^{13} and finally in the matrix of order 14 and for higher order than 14 it will be convergence and the convergence matrix is as follows-

$$p^{14} = \begin{bmatrix} 0.009708738 & 0.09708738 & 0.1359223 & 0.3009709 & 0.4563107 \\ 0.009708738 & 0.09708738 & 0.1359223 & 0.3009709 & 0.4563107 \\ 0.009708738 & 0.09708738 & 0.1359223 & 0.3009709 & 0.4563107 \\ 0.009708738 & 0.09708738 & 0.1359223 & 0.3009709 & 0.4563107 \\ 0.009708738 & 0.09708738 & 0.1359223 & 0.3009709 & 0.4563107 \end{bmatrix} \quad (20)$$

3: LONG RUN (OR LIMITING) PROBABILITIES

From the convergence transition probability matrix in equation (5), we get the following long run probabilities

Table 3.1: Long run probabilities for different state of profit of GDLIC using MC during 2008-2011.

State	probabilities	Profit range
π_1	0.1794057	6-154
π_2	0.3182054	154-302
π_3	0.3188322	302-450
π_4	0.1448351	450-598
π_5	0.03872171	≥598

The highest long run probability 32% is associated with the profit range 154-302 and 302-450 million taka, followed by 18% with 6-154 million taka, 14% with 450-598 million taka and the minimum probability 4% associated with profit 598 million taka or more million taka. From the convergence transition probability matrix in equation (10), we get the following long run probabilities

Table 3.2: Long run probabilities for different state of profit of NLIC using MC during 2010-2011

State	probabilities	Profit range
π_1	0.1551616	(-156)-983.4
π_2	0.2028915	983.4-2122.8
π_3	0.331384	2122.8-3262.223
π_4	0.2146519	3262.223-4401.2
π_5	0.09591107	≥4401.2

The highest long run probability 33% is associated with the profit range 2122.8-3262.223 million taka, followed by 21% with 3262.223-4401.2 million taka, 20% with 983.4-2122.8 million taka, 16% with 983.4-2122.8 million taka and the minimum probability 9% associated with profit 4401.2 million taka or more million taka. From the convergence transition probability matrix in equation (15), we get the following long run probabilities

Table 3.3: Long run probabilities for different state of profit of GDLIC using MCMC during 2008-2011.

State	probabilities	Profit range
π_1	0.1446648	(2.54)-140.90
π_2	0.2413253	140.90-279.26
π_3	0.2900788	279.26-417.62
π_4	0.2510692	417.62-555.98
π_5	0.07286186	≥555.98

The highest long run probability 29% is associated with the profit range 279.26-417.62 million taka, followed by 25% with 417.62-555.98 million taka, 24% with 140.90-279.26, 14% with2.54-140.90 million taka and the minimum probability 7% associated with profit 555.98 million taka or more million taka. From the convergence transition probability matrix in equation (20), we get the following long run probabilities.

Table 3.4: Long run probabilities for different state of profit of NLIC using MCMC during 2010-2011.

State	probabilities	Profit range
π_1	0.009708738	(-581.73)-(-1183.74)
π_2	0.09708738	(-1183.7404)-214.2496
π_3	0.1359223	214.2496-1612.2396
π_4	0.3009709	1612.2396-3010.2296
π_5	0.4563107	≥4408.219

The highest long run probability 45% is associated with the profit range 4408.219 million taka or more than 4408.219 million taka, followed by 30% with 1612.2396-3010.2296 million taka, 13% with 214.2496-1612.2396 million taka, 9% with (-1183.7404)-214.2496 million taka and the minimum probability .009 associated with profit (-581.73)-(-1183.74) million taka.

4: GOODNESS-OF-FIT TEST FOR GENERATED OBSERVATIONS VIA METROPOLIS-HASTING ALGORITHM

To ensure about the generated (by M-H algorithm) profit's distribution, we use Kolmogorov-Smirnov test.

: Generated data follows normal distribution,

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: Generated data does not follow normal distribution.

At 5% level of significance we will test the above hypothesis

Table 4.1: Two-sample Kolmogorov-Smirnov test for generated profit of GDLIC and NLIC using M-H algorithm.

Company	Test Statistic	p-value	Decision
GDLIC	0.0769	0.5696	The generated data comes from normal distribution.
NLIC	0.2885	0.0003489	The generated data does not come from normal distribution

5. CONCLUSIONS:

From the transition probability matrix and long run probabilities, we have found that the highest long run probabilities in the separate profit range for the two different GDLIC and NLIC using MC simulation and MCMC simulation using M-H algorithm are different and finally the two sample Kolmogorov–Smirnov shows that for GDLIC, generated profit follows normal distribution but the generated profit of NLIC does not follows normal distribution. So on the basis of overall result, we can finally recommend that GDLIC is much more better for the policy makers in order to further more policy with the less risk compare to NLIC and company will be more benefited with their annual profit in case of Markov Chain Monte Carlo (MCMC) simulation using M-H algorithm than Markov Chain (MC) simulation.

6. REFFERENCE:

[1] Chakravart, Laha, and Roy (1967), A note on Kolmogorov-Smirnov Goodness-of-Fit Test. Journal of Statistics.

[2] C. Kulp and J. Hall, (1968) Casuality Insurance, Fourth Edition, , page 35.

[3] Peskun , P. (1973) Optimum Monte Carlo sampling using Markov Chains, Biometrica 60.

[4] Jagers, P. (1975), “Branching Process with Biological Applications” , Wiley.

[5] Daneil, Wayne W. (1978): Applied Non-parametric Statistics, Houghton Mifflin Company, Boston Dallas, Geneva.

[6] Bruss, F. Thomas (1984): "A Note on Extinction Criteria for Bisexual Galton– Watson Processes". Journal of Applied Probability 21: 915–919.

[7] Bhat, U.N.(1984), “Elements of Applied Stochastic Processes”, 2nd edition, John Wiley and Sons, NewYork.

[8] Medhi, j. (1994), “ Stochastic Processes”, 2nd edition, New Age International (P) Ltd. Publisher, New Delhi.

[9] Ross, S.M. (1995), Stochastic Processes. Wiley. ISBN 978-0471120629.

[10] Hamdy A. Taha (1999), “Operations Research An Introduction”, 6th ed. Prentice Hall of India (p) Ltd, New Delhi.

[11] Latouche, G. and V. Ramaswami (1999): Introduction to Matrix Analytic Methods in Modelling, 1st edition.

[12] Bhat, B.R. (2000), “Stochastic Models”, New Age International (p) Ltd. Publishers, New Delhi.

[13] Athanasios Papoulis, S. Unni krishna Pillai (2002), “Probability, Random Variables and Stochastic Processes”, 4th ed., Tata McGraw-Hill Publishing company Ltd, New Delhi.

[14] Gujarati, D.N. (2003), “Basic Econometrics”, 4th ed., McGraw Hill, New York.

[15] Hines, William W.; Douglas C. Montgomery; David M. Goldsman and Connine M.Borror (2003): Probability and Statistics in Engineering, John Wiley and Sons Inc.

[16] Ross, S. (2003), “ Probability Models”, 8th edition, Academic Press, USA.

[17] Hollis and Strauss, (2007) privacy, Driving Data and Automobile Insurance: An Economic Analysis (HTML).http://www.ifsl.org.uk/uploads/CBS_Insurance_2007.

[18] Gregory D. Squires (2003) Racial Profiling, Insurance Style: Insurance Redlining and the Uneven Development of Metropolitan Areas. Journal of Urban Affairs Volume 25 Issue 4, November 2003.

[19] M.K.Roy (2004), “Fundamentals of Probability & Probability Distributions”, 3rded.

[20] Suddhendu Biswas (2004), “Applied Stochastic Process”, 2nded. New central book agency (p) Ltd, New Delhi.

[21] William W.Hines, Douglas C. Montgomery, David M. Goldsman, Connie M.Borror (2004), “Probability and Statistics in Engineering”, 4th ed., John Wiley & Sons, Inc.

[22] Strauss and Hollis, (2007), Insurance Markets When Firms are Asymmetrically Informed: A Note (HTML)

[23] S.C. Gupta, V.K. Kapoor (2008), “Fundamentals of Mathematical Statistics”, 12th ed., Sultan Chand & Sons, New Delhi.

[24] Barbu, V.S, Limnios, N. (2008), Semi-Markov Chains and Hidden Semi-Markov Models toward Applications: Their Use in Reliability and DNA Analysis. ISBN 978-0387731711.

[25] Ghosh, S., Roy, M. K., & Biswas, S. C. (2016). Determination of the best fit probability distribution for monthly rainfall data in Bangladesh. *American Journal of Mathematics and Statistics*, *6*(4), 170-174.