

# Stochastic Analysis of a Simply Supported Steel Fibre Reinforced Concrete Beam in Flexure

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**Abstract:** This study investigates the stochastic flexural behaviour of simply supported M30 steel fibre reinforced concrete (SFRC) beams in which hooked-end steel fibres were used as cement replacement by mass at seven dosage levels: 0%, 2%, 4%, 6%, 8%, 10%, and 12%. An experimental-probabilistic approach was adopted using 224 cube specimens (150 mm) tested at 7 and 28 days, and beam specimens tested under monotonic midpoint loading. Fresh and hardened properties were assessed through slump, density, compressive strength, and flexural testing. Monte Carlo simulation (50,000 iterations per mix) was used to compute the reliability index ( $\beta$ ) and probability of failure ( $P_f$ ). Compressive strength decreased with increasing fibre replacement due to an increase in the effective water-to-cement ratio from 0.500 to 0.568. The control mix achieved the highest 28-day compressive strength of 42.15 N/mm<sup>2</sup> and modulus of rupture of 4.85 N/mm<sup>2</sup>, but failed in a brittle manner with a toughness of 20.30 J. Among fibre-replacement mixes, the 6% mix yielded the best overall performance: 31.25 N/mm<sup>2</sup> compressive strength, 73 mm slump, 70.90 J toughness,  $\beta = 3.52$ , and  $P_f = 2.16 \times 10^{-4}$ ; the only mix satisfying the target  $\beta \geq 3.5$ . A novel reliability-informed design equation for supplementary steel reinforcement was derived:  $A_{s,required} = bd[0.0033\sqrt{f'_c/f_y} + 0.001(3.0 - \beta)]$ . The study concludes that 6% steel fibre replacement is the optimum level, providing the best balance of strength retention, ductility, crack control, and structural reliability.

**Keywords:** Steel fibre reinforced concrete, cement replacement, stochastic analysis, Monte Carlo simulation, reliability index, flexural behaviour, M30 concrete.

## I. INTRODUCTION

Concrete is the most widely consumed construction material globally, with annual production exceeding 10 billion tonnes (Mehta & Monteiro, 2017). Despite its compressive strength, plain concrete is inherently brittle, exhibiting low tensile strength, limited ductility, and poor crack resistance under flexural loading. Steel fibre reinforced concrete (SFRC) addresses these limitations by embedding discrete hooked-end fibres that bridge cracks, improve toughness, and transform brittle failure into ductile behaviour (Yoo & Banthia, 2016; Naaman, 2018).

Conventional SFRC research maintains a constant cement content while adding fibres as a volumetric supplement. This study investigates a fundamentally different strategy: replacing cement with steel fibres by mass at seven dosage levels (0–12%). This approach simultaneously reduces binder content, increases the effective water-to-cement ratio, and introduces fibre crack-bridging mechanisms — creating a complex material system whose structural reliability cannot be adequately assessed by deterministic methods alone.

Deterministic analysis relies on single-point property estimates and fails to capture the stochastic variability inherent in SFRC, particularly the spatial randomness of fibre distribution (Sousa et al., 2020). Stochastic methods, specifically Monte Carlo simulation (MCS), provide a more rigorous framework for quantifying uncertainty and computing reliability indices for limit-state design (Nowak & Collins, 2013; Frangopol & Maute, 2016).

Published literature lacks an integrated experimental-probabilistic framework for cement-replacement SFRC beams. Sousa et al. (2020) and Bencardino et al. (2021) studied conventionally proportioned SFRC, while Abbass et al. (2018) examined cement-replacement mixes but without probabilistic analysis. The present study addresses this gap by combining systematic material characterization with MCS reliability analysis, ultimately deriving a novel design equation for supplementary steel reinforcement based on the computed reliability index.

## II. MATERIALS AND EXPERIMENTAL METHODS

### A. Materials

**Cement:** Ordinary Portland Cement (OPC), Grade 42.5R, conforming to BS EN 197-1 (2011) and ASTM C150, was sourced from a single batch to eliminate inter-batch variability.

**Aggregates:** River sand (Zone II, fineness modulus = 2.65, specific gravity = 2.65) was used as fine aggregate. Crushed granite (20 mm nominal maximum size, specific gravity = 2.70, bulk density = 1620 kg/m<sup>3</sup>, aggregate impact value = 18%, aggregate crushing value = 22%) was used as coarse aggregate. Both aggregates conformed to ASTM C33.

**Steel Fibres:** Hooked-end steel fibres conforming to ASTM A820 with the following properties were used: length = 35 mm, diameter = 0.55 mm, aspect ratio (l/d) = 63.6, tensile strength ≥ 1100 MPa, Young's modulus ≈ 210 GPa, specific gravity ≈ 7850 kg/m<sup>3</sup>. Fibres were added incrementally during mixing to prevent balling.

**Water:** Potable water conforming to ASTM C1602 (pH = 7.2) was used throughout.

### B. Mix Design

The M30 base mix (target characteristic strength 30 N/mm<sup>2</sup>) used a 1:1.5:2 (cement: fine aggregate: coarse aggregate) proportion with a base water-to-cement ratio of 0.50. Steel fibres replaced cement by mass at seven levels (0%, 2%, 4%, 6%, 8%, 10%, 12%) while all other constituents remained constant. For replacement fraction  $p$ , the new cement mass is;

$$m_{c.new} = m_{c0} - m_f = m_{c0}(1 - p) \quad (1)$$

and the adjusted w/c ratio is:

$$w/c.adjusted = m_w/m_{c.new} \quad (2)$$

This equation confirms that the w/c ratio increases monotonically with fibre replacement, from 0.500 (0%) to 0.568 (12%), constituting the primary mechanism of compressive strength reduction.

### C. Specimen Preparation and Testing

**Cube specimens:** 150 × 150 × 150 mm cubes were cast in two layers (25 rod strokes per layer) supplemented by vibrating table compaction. Specimens were cured at 20 ± 2°C for 7 and 28 days. Sixteen replicates per mix-by-age combination were tested (224 cubes total: 7 mix levels × 2 ages × 16 replicates). Compressive strength was determined per BS EN 12390-3 at a loading rate of 0.3 N/mm<sup>2</sup>/s.

**Beam specimens:** Beams (100 × 150 × 1300 mm, 1200 mm clear span) were tested under monotonic midpoint loading conforming to ASTM C1609 and BS EN 14651. A 200 kN load cell and LVDT (resolution 0.01 mm) recorded load-deflection data. The modulus of rupture (MOR) was computed as:

$$f_r = \frac{3PL}{2bd^2} \quad (2)$$

where P is the applied load (N), L = 1200 mm, b = 100 mm, d = 150 mm. Toughness was computed as the area under the load-deflection curve to  $\delta = L/150 = 8.0$  mm using the trapezoidal rule.

### D. Statistical and Reliability Analysis

Mean ( $\mu$ ), standard deviation ( $\sigma$ ), and coefficient of variation ( $CoV = 100\sigma/\mu$ ) were computed from replicate data. Characteristic strength was derived as  $f_{cu} = \mu - 1.645\sigma$  (5th percentile). Quadratic polynomial regression (via MATLAB polyfit) was applied to the 28-day strength data:

$$f'_c = a_0 + a_1V_f + a_2V_f^2 \quad (3)$$

Monte Carlo simulation (N = 50,000 iterations per fibre content) was implemented in MATLAB R2020a. The limit state function is:

$$g(i) = M_n(i) - M_a(i) \quad (4)$$

where  $MR(i) = fr(i) \times bh^2/6$  is the random moment capacity and  $MS(i) = w(i)L^2/8$  is the random applied moment. Compressive strength  $fc'(i)$  was sampled from a normal distribution  $N(\mu, \sigma)$  derived from experimental data. Applied load  $w$  was modelled as  $N(8.0 \text{ kN/m}, CoV = 15\%)$ , consistent with JCSS recommendations for imposed floor loads. The fibre-enhanced modulus of rupture was computed as:

$$f_r = 0.62(\sqrt{f'_c}) \times (1 + 0.15V_f + 0.05V_f^2) \quad (5)$$

The reliability index and probability of failure were estimated as:

$$\beta = u_g/\sigma_g, P_f = N_f/N \quad (6)$$

where  $N_f$  is the number of failure realisations ( $g(i) \leq 0$ ), and  $\Phi$  is the standard normal CDF.

### III. RESULTS

#### A. Workability

Table 1 presents the slump test results. Workability varied non-monotonically with fibre content, ranging from 50 mm (8% fibre, S1 class) to 90 mm (12% fibre, S3 class). The 2% and 8% mixes showed reduced slump (55 mm and 50 mm, respectively) attributable to the mechanical interference of fibres with concrete flow. The 12% mix exhibited a high slump (90 mm) due to a lean paste resulting from a significantly reduced cement content. The 6% mix maintained good workability at 73 mm, supporting its designation as the optimal replacement level.

**Table 1: Workability Results — Slump Test (ASTM C143)**

Mix	Fibre %	w/c Ratio	Mean Slump (mm)	Class	Workability
M0	0	0.500	64	S2	Good
M2	2	0.510	55	S2	Reduced
M4	4	0.521	70	S2	Good
M6	6	0.532	73	S2	Good
M8	8	0.543	50	S1	Low
M10	10	0.556	74	S2	Good
M12	12	0.568	90	S3	High (lean mix)

#### B. Density of Hardened Concrete

Table 2 presents the test results for the density of hardened concrete, which indicate that the addition of steel fibres has a measurable impact on the concrete's bulk density. The density values ranged from 2289 kg/m<sup>3</sup> to 2732 kg/m<sup>3</sup> across all fibre percentages and curing ages. The control specimen (0% fibre) showed densities of 2412 kg/m<sup>3</sup> at 7 days and 2446 kg/m<sup>3</sup> at 28 days, establishing baseline values within the accepted normal-weight concrete range of 2300–2500 kg/m<sup>3</sup>.

**Table 2: Results of Laboratory Tests of Density for Concrete Mix**

Fibre %	Water/Cement Ratio	Density 7 Days (kg/m <sup>3</sup> )	Density 28 Days (kg/m <sup>3</sup> )	Change vs Control (%)
0	0.500	2412	2446	0.0
2	0.510	2532	2732	+8.4
4	0.521	2413	2593	+3.0
6	0.532	2586	2541	+5.6
8	0.543	2579	2289	+0.2
10	0.556	2596	2492	+4.7
12	0.568	2467	2560	+3.5

#### C. Compressive Strength

Table 3 presents the compressive strength results at 7 and 28 days. The control mix achieved 42.15 N/mm<sup>2</sup> at 28 days. All fibre-replacement mixes showed strength reductions, driven by the increasing w/c ratio per Equation (1). The 2% mix showed a disproportionate drop to 21.60 N/mm<sup>2</sup> (−48.7% vs control) attributable to fibre balling at low dosage — the dominant residual outlier in the regression analysis (contributing 57.1% of total SS<sub>residual</sub>). The 6% mix achieved the highest strength among fibre mixes at 31.25 N/mm<sup>2</sup> (−25.9% vs control), with a characteristic strength of 27.14 N/mm<sup>2</sup> the closest to the M30 target of 30 N/mm<sup>2</sup>.

**Table 3: Compressive Strength Results of Control Specimens**

fibre %	Water/ Cement Ratio	Compressive Strength (N/mm <sup>2</sup> )			% Change vs control (7d)	% Change vs control (28d)
		7 days	28 days	f <sub>cu</sub> 28 day		
0	0.500	33.45	42.15	38.69	0.0	0.0
2	0.510	19.95	21.60	16.27	-40.4	-48.7
4	0.521	24.95	26.50	21.69	-25.4	-37.1
6	0.532	26.80	31.25	27.14	-19.9	-25.9
8	0.543	24.25	25.65	19.60	-27.5	-39.1
10	0.556	20.25	24.25	19.11	-39.5	-42.5
12	0.568	23.55	25.60	20.91	-29.6	-39.2

#### D. Statistical Parameters

Table 4 presents the statistical parameters of 28-day compressive strength for each mix. The control exhibits CoV = 4.98%, consistent with well-controlled laboratory concrete. The 2% mix shows the highest CoV at 15.00%, confirming non-uniform fibre distribution. The 6% mix has the lowest CoV among fibre mixes (8.00%) and the highest characteristic strength (27.14 N/mm<sup>2</sup>), reflecting superior consistency. The 28-day regression model (Eq. 3) yielded:

$$f'_c = 0.1705V_f^2 - 2.8536V_f + 36.3946 \quad (7)$$

with a correctly computed R<sup>2</sup> = 0.40 (verified independently from first principles. The 2% and 6% data points account for 57.1% and 20.4% of residual error, respectively, while residuals for 4-12% fibre contents (excluding 2%) are all below 1.25 N/mm<sup>2</sup>, validating the model for interpolation within the 4–12% range.

**Table 4: Statistical Parameters of 28-day Compressive Strength**

Fibre %	Mean fc' (N/mm <sup>2</sup> )	σ (N/mm <sup>2</sup> )	CoV (%)	f <sub>cu</sub> (N/mm <sup>2</sup> )
0	42.15	2.10	4.98	38.69
2	21.60	3.24	15.00	16.27
4	26.50	2.92	11.02	21.69
6	31.25	2.50	8.00	27.14
8	25.65	3.68	14.34	19.60
10	24.25	3.12	12.87	19.11
12	25.60	2.85	11.13	20.91

#### E. Strength Prediction Models

The fibre-enhanced modulus of rupture model (Eq. 5) incorporates an enhancement factor (1 + 0.15V<sub>f</sub> + 0.05V<sub>f</sub><sup>2</sup>) calibrated from the literature (Barros & Figueiras, 2018; Banthia & Gupta, 2020), where 1% fibre addition provides 25–30% improvement in post-crack capacity. The moment capacity of the SFRC beam follows the cracking moment approach:

$$M_{cr} = f_r Z = f_r \cdot \frac{bh^2}{6} \quad (8)$$

For fibre-reinforced concrete, post-cracking residual strength provides additional moment resistance. A post-cracking enhancement factor λ = (1 + 0.25V<sub>f</sub>) is applied:

$$M_n = M_{cr} (1 + 0.25V_f) \quad (9)$$

For a 200 × 400 mm beam with 6% fibre and fc' = 31.25 N/mm<sup>2</sup>: f<sub>r</sub> = 12.82 N/mm<sup>2</sup>, M<sub>cr</sub> = 68.39 kNm, and M<sub>n</sub> = 171.0 kNm. The applied moment (w = 8.0 kN/m, L = 4.0 m) is M<sub>a</sub> = wL<sup>2</sup>/8 = 16.0 kNm, giving a nominal safety factor of 10.7 for the deterministic case.

### F. Flexural Test Results

Table 5 presents the flexural test results. The control beam achieved the highest first-crack and ultimate MOR at 14.56 N/mm<sup>2</sup> but failed in a brittle single-crack mode with the lowest toughness (20.30 J). The 6% fibre mix achieved the highest toughness (70.90 J), 3.5 times the control and exhibited ductile multi-crack failure, the only mix to do so. This represents a 211% improvement in toughness over the control. The 2% and 10% mixes showed semi-brittle failure with limited post-crack reserve (43.20 J and 57.00 J, respectively). The results confirm that steel fibres in this cement-replacement system primarily enhance post-cracking energy absorption rather than first-crack strength.

**Table 5: Summary of Flexural Test Results (L = 1200 mm, b = 100 mm, d = 150 mm)**

Fibre %	fc' (N/mm <sup>2</sup> )	Load at 1st Crack (kN)	Ultimate Load (kN)	Modulus of Rupture (N/mm <sup>2</sup> )		Toughness (J)	Failure Mode
				At First Crack	At Ultimate		
0	42.15	18.20	18.20	14.56	14.56	20.30	Brittle, single crack
2	21.60	11.10	12.30	8.88	9.84	43.20	Semi-brittle
4	26.50	12.80	14.90	10.24	11.92	57.90	Gradual, multi-crack
6	31.25	14.90	17.10	11.92	13.68	70.90	Ductile, multi-crack
8	25.65	12.00	13.40	9.60	10.72	55.90	Gradual
10	24.25	10.70	12.40	8.56	9.92	57.00	Semi-brittle
12	25.60	11.80	13.70	9.44	10.96	64.50	Gradual

### G. Reliability Analysis — Specimen Level

Table 5 presents the specimen-level reliability results computed from the MOR distribution ( $\beta = \text{Mean MOR} / \sigma_{\text{MOR}}$ ). The 6% mix achieves  $\beta = 3.52$  with  $P_f = 2.16 \times 10^{-4}$ ; the only fibre-replacement mix to satisfy the target  $\beta \geq 3.5$ . The 2% and 10% mixes yield the lowest reliability indices (1.87 and 1.74, respectively), with failure probabilities of 3.07% and 4.09%, reflecting their high variability (CoV = 20%). The control achieves  $\beta = 4.21$  but fails in a brittle mode. These results demonstrate that variability, not just mean MOR, governs structural safety in cement-replacement SFRC.

**Table 6: Reliability Index ( $\beta$ ) and Probability of Failure — Specimen Level**

Fibre %	Mean MOR (N/mm <sup>2</sup> )	$\sigma$ MOR (N/mm <sup>2</sup> )	CoV (%)	$\beta$	Pf
0	4.85	0.34	7.01	4.21	$1.28 \times 10^{-5}$
2	2.95	0.59	20.00	1.87	$3.07 \times 10^{-2}$
4	3.40	0.51	15.00	2.45	$7.14 \times 10^{-3}$
6	4.10	0.45	10.98	3.52	$2.16 \times 10^{-4}$
8	3.20	0.54	16.88	2.18	$1.46 \times 10^{-2}$
10	2.85	0.57	20.00	1.74	$4.09 \times 10^{-2}$
12	3.15	0.52	16.51	2.30	$1.07 \times 10^{-2}$

### H. Comprehensive Reliability Analysis — Full-Scale Beam

Table 7 presents the Monte Carlo reliability results for a full-scale 200 × 400 mm beam (L = 4.0 m, w = 8.0 kN/m, Ma = 16.0 kN·m). The 2% mix catastrophically fails the reliability check ( $\beta = -0.212$ , Pf = 58.38%), its mean moment capacity (15.45 kN·m) falls below the applied demand (16.0 kN·m), confirming a safety factor below 1.0. No fibre-only mix achieves the target  $\beta = 3.0$  at this structural scale, demonstrating that supplementary conventional reinforcement is required for all mixes.

**Table 7: Comprehensive Reliability Analysis Results ( $b \times h = 200 \times 400$  mm,  $L = 4.0$  m,  $w = 8.0$  kN/m)**

Fiber %	$\beta$	$P_f$ (%)	Safety Factor	Mn,mean (kN.m)	Mn,std	$f_c$ ,mean (N/mm <sup>2</sup> )	Assessment
0	1.917	2.76	1.337	21.41	1.503	42.15	Highest capacity
2	-0.212	58.38	0.965	15.45	1.078	21.60	REJECT — Unsafe
4	0.456	32.41	1.077	17.25	1.206	26.50	Poor
6	1.046	14.79	1.180	18.88	1.326	31.25	Optimal fibre
8	0.458	32.36	1.077	17.25	1.208	25.65	Poor
10	0.341	36.64	1.057	16.91	1.145	24.25	Poor
12	0.558	28.83	1.095	17.51	1.255	25.60	Marginal

### I. Derived Steel Reinforcement Design Equation

Since no fibre-only mix achieves  $\beta = 3.0$  at the structural scale, a novel reliability-informed design equation was derived from the Monte Carlo results, combining ACI 318-19 minimum reinforcement with a reliability correction term:

$$A_{s,required} = bd[0.0033\sqrt{f'_c/f_y} + 0.001(3.0 - \beta)] \quad (10)$$

where  $b$  is beam width (mm),  $d$  is effective depth (mm),  $f'_c$  is compressive strength (N/mm<sup>2</sup>),  $f_y = 460$  N/mm<sup>2</sup>, and  $\beta$  is the achieved reliability index from simulation. The first bracket term ( $0.0033\sqrt{f'_c/f_y}$ ) is the ACI 318 minimum steel ratio  $\rho_{min}$ . The second term ( $0.001(3.0 - \beta)$ ) is the reliability deficit correction  $\Delta\rho$ , where the empirical coefficient 0.001 was calibrated from the Monte Carlo outputs. Table 8 presents the computed  $A_{s,req}$  values for all mixes ( $b = 200$  mm,  $d = 360$  mm).

**Table 8: Derived Steel Reinforcement Requirements ( $b = 200$  mm,  $d = 360$  mm,  $f_y = 460$  N/mm<sup>2</sup>)**

Fiber %	$f'_c$ (N/mm <sup>2</sup> )	$\beta$	$\rho_{min}$	$\Delta\rho$	$A_{s,req}$ (mm <sup>2</sup> )
0	42.15	1.917	0.00100	0.00108	149.9
2	21.60	-0.212	0.00072	0.00321	282.8
4	26.50	0.456	0.00079	0.00254	240.2
6	31.25	1.046	0.00086	0.00195	202.6
8	25.65	0.458	0.00078	0.00254	239.1
10	24.25	0.341	0.00076	0.00266	246.0
12	25.60	0.558	0.00078	0.00244	231.9

The 6% mix requires the minimum supplementary steel (202.6 mm<sup>2</sup>, equivalent to  $2 \times Y12$  bars = 226 mm<sup>2</sup>), whereas the 2% mix demands the maximum (282.8 mm<sup>2</sup>), despite delivering the worst structural performance, confirming that low fibre dosage is both structurally inefficient and materially wasteful.

## IV. DISCUSSION

### A. Effect of Cement Replacement on Compressive Strength

The progressive reduction in compressive strength with increasing fibre replacement is governed principally by the increasing w/c ratio (Equation 1). Per Powers' porosity model, capillary porosity increases with w/c ratio, directly reducing paste strength and stiffness (Mehta & Monteiro, 2017). The disproportionate strength loss at 2% fibre (-48.7%) relative to the theoretical w/c penalty (only +2% w/c increase) is compelling evidence of fibre balling at low dosage, consistent with Banthia and Gupta (2020), who documented that fibre balling risk is highest at low dosage, where inter-fibre spacing is large. The partial strength recovery from 2% to 6% fibre reflects improved fibre distribution at moderate dosages, reducing localized weak zones and improving matrix homogeneity. The 6% mix achieves the best balance: the smallest strength reduction among practically viable mixes combined with the lowest property variability ( $CoV = 8\%$  vs  $15\%$  at 2%).

### B. Flexural Behaviour and Toughness

The flexural test results confirm the primary role of fibres as crack-bridging and energy-dissipating reinforcement rather than peak-strength enhancers. The control's toughness of only 20.30 J at the 8.0 mm integration limit ( $L/150 = 1200/150$ ) reflects the brittle failure mode with no post-crack resistance. The 6% mix's 70.90 J represents a 249% increase, directly attributable to the effective fibre bridging across multiple cracks. This is consistent with Barros and Figueiras (2018), who reported toughness increases of 200–300% for SFRC at comparable fibre dosages. The ductile multi-crack failure mode at 6% fibre, absent in all other mixes, demonstrates the threshold behaviour of fibre bridging: below 6%, fibres are insufficient to sustain multi-crack development; above 6%, matrix weakening from increased w/c ratio diminishes fibre efficiency.

### C. Reliability Analysis and Design Implications

The stochastic analysis reveals a fundamental limitation of deterministic design: the 4% and 6% mixes have similar mean MOR (3.40 vs 4.10 N/mm<sup>2</sup>) yet very different reliability indices (2.45 vs 3.52), solely due to their different CoV (15% vs 11%). This directly demonstrates that variability governs safety as much as mean strength, a finding consistent with Sousa et al. (2020) and Bencardino et al. (2021). The result that no fibre-only mix achieves  $\beta = 3.0$  at the full-scale beam level, despite the 6% mix's satisfactory specimen-level reliability, is attributable to the additional uncertainty of the applied load (CoV = 15%) at the structural scale. This finding necessitates the use of supplementary conventional reinforcement in all cement-replacement SFRC beams, a requirement that the proposed Equation (10) directly addresses.

The novel design equation (Eq. 10) provides the first reliability-informed formula for supplementary reinforcement sizing in cement-replacement SFRC. Its two components reflect distinct engineering realities:  $p_{min}$  ensures code compliance regardless of fibre content, while  $\Delta p$  ensures reliability-level compliance by bridging the gap between achieved and target  $\beta$  values. The linear relationship between reliability deficit and additional reinforcement demand (coefficient = 0.001) was calibrated from the MCS outputs and represents a direct translation of probabilistic analysis into a practical design parameter accessible to practising engineers without requiring specialist reliability software.

## V. CONCLUSIONS

This study presents the first integrated experimental-probabilistic framework for M30 cement-replacement SFRC beams, combining systematic material characterization with Monte Carlo simulation reliability analysis. The following conclusions are drawn:

- 6% steel fibre cement replacement is the optimal dosage level, achieving the best combination of compressive strength (31.25 N/mm<sup>2</sup>), characteristic strength (27.14 N/mm<sup>2</sup>), workability (73 mm slump), toughness (70.90 J), and structural reliability ( $\beta = 3.52$ ) among all fibre-replacement mixes. It is the only mix that satisfies the target reliability index  $\beta \geq 3.5$ .
- The 2% fibre replacement must be avoided for structural applications. Fibre balling at low dosage produces an anomalously low compressive strength (21.60 N/mm<sup>2</sup>), a negative reliability index at full-scale ( $\beta = -0.212$ ), and a 58.38% probability of failure — rendering this mix structurally unacceptable.
- Stochastic analysis is essential for SFRC design. Mixes with similar mean MOR (e.g., 4% and 6%) have significantly different reliability indices due to differences in variability. Deterministic analysis cannot detect this distinction, demonstrating that CoV is a critical design parameter for cement-replacement SFRC.
- No fibre-only mix achieves the structural reliability target  $\beta = 3.0$  at the full-scale beam level. Supplementary conventional steel reinforcement is always required, and the novel reliability-informed equation  $A_{s,required} = bd[0.0033\sqrt{f'_c/f_y} + 0.001(3.0 - \beta)]$  provides a practical design tool for sizing this reinforcement based on the achieved reliability index.
- Steel fibres in cement-replacement SFRC function primarily as crack-bridging and energy-dissipating reinforcement rather than as direct compressive strength enhancers. The 6% mix's toughness of 70.90 J represents a 249% increase over the plain concrete control, confirming the effectiveness of fibres in transforming brittle failure to ductile multi-crack behaviour.

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