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Stimulated Raman Scattering in a Magnetized Electron-Positron-Ion Plasma

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Abstract **- Electron-positron plasmas that may be found in numerous environments, such as, active galactic nuclei, pulsar magnetosphere, solar flares, ultra-short laser-matter interaction and interplanetary space, have dielectric properties appreciably different from those of classical electron-ion plasmas since modes occurring on the ion timescale disappear due to a mass symmetry effect. These properties as well as excited instabilities have been studied earlier, whereas in the present work, we focus on stimulated Raman scattering instability (SRS) in a magnetized electron-positron plasma containing a fraction of ions (e-p-i plasma), where the background magnetic field is parallel to the pump electric field.**

1. INTRODUCTION

Electron-positron (e-p) plasmas can be found in the early universe, in astrophysical objects such as pulsars, active galactic nuclei, supernovae remnants, in γ -raybursts and at the center of the Milky Way galaxy [1,2]. These plasmas are created by collisions between particles that are accelerated by electromagnetic and electrostatic waves and/or by gravitational forces. High energy laser-plasma interactions and fusion devices can be used to produce e-p plasmas. Electron-positron plasmas are also created in large tokamaks[3] through collisions between Mev electrons and thermal particles. Due to the mass symmetry in e-p plasmas, there are fewer spatial and temporal scales on which collective effects (e.g, electrostatic and electromagnetic waves as well as their instabilities and coherent nonlinear structures, etc) can occur [1,2]. For instance, Iwamoto [4] has given an elegant description of the linear modes in a non relativistic pair plasma that is magnetized. In addition, Zank and Greaves [5] studied the linear properties of various electrostatic and electromagnetic waves in unmagnetized and magnetized pair plasmas. Recently, Annou and Bahamida[6] have studied the parametric coupling in electron-positron plasma and they obtained the nonlinear dispersion relation and the nonlinear growth rate of the parametrically generated modes in this kind of plasmas. On the other hand, Bornatici *et al.*[7], Liu and Rosenbluth[8]and Drake[9]studied also the parametric interaction of an intense coherent electromagnetic wave with collective modes in an electron-ion plasma. If the excited waves are both electrostatic, they may be absorbed in the plasma. In contrast, if one of the excited plasma waves is electromagnetic, it can be scattered. This process is called " stimulated Raman scattering" if the excited plasma wave is a Langmuir wave and it is called " stimulated Brillouin scattering" if the excited plasma wave is an ion-acoustic wave. Furthermore, while the unmagnetized plasma may reveal three wave modes, namely, Langmuir, ion-acoustic and electromagnetic modes, the magnetized plasma may support a great number of new wave modes [10]. The basic modes of magnetized plasmas such as those that are found in molecular clouds, cometary plasmas and stellar atmospheres are of a great interest [11], for instance, Alfvén waves are of importance to the understanding of many basic plasma phenomena [12,13]. It has been shown also that the magnetic field generated in laser-produced plasmas are strong enough to alter the spectrum of electrostatic modes in the plasma but not strong enough to modify the characteristics of propagation of the incident and scattered electromagnetic modes[14]. In the present paper, the stimulated Raman scattering is studied in a magnetized e-p plasma containing a fraction of ions (e-p-i plasma). As a matter of fact, astrophysical and laboratory plasmas contain in general a fraction of ions, e.g, electron-positron-ion plasmas are encountered in the magnetosphere of neutron stars, and also in the solar atmosphere[15-18]. Indeed, these plasmas received some attention from numerous authors[19-21]. The maximum growth rate of the stimulated Raman scattering process is calculated and compared to a previous study. The paper is organized as follows: in Sec.2, the problem is exposed and solved and we conclude in Sec.3

2. THEORY

We consider an (e-p-i) plasma embedded in a uniform background magnetic field $\vec{B}_0$. We consider a large amplitude electromagnetic pump wave with an electric field $\vec{E}_t$ parallel to $\vec{B}_0$, where,

$$\vec{E}_t = 2\vec{E}_{t0}\cos\left(\vec{k}_t.\vec{x}-\omega_t t\right), \tag{1}$$

The equilibrium state contains electrons, positrons and ions. The electrons and positrons oscillating respectively with the velocities $\vec{V}_{te^-}$ and $\vec{V}_{te^+}$ given by,

$$\vec{V}_{te^-} = \frac{2e}{m\omega_t}\vec{E}_{t0}\sin\left(\vec{k}_t.\vec{x}-\omega_t t\right), \tag{2}$$

$$\vec{V}_{te^+} = -\frac{2e}{m\omega_t}\vec{E}_{t0}\sin\left(\vec{k}_t.\vec{x}-\omega_t t\right) \tag{3}$$

where, m is the electron and positron mass. The ions are considered to be immobile because of their inertia. In the SRS process where electrons as well as positrons participate, the electromagnetic pump wave $(\omega_t, \vec{k}_t)$ decays into an electromagnetic sideband wave $(\omega_{t'}, \vec{k}_{t'})$ and an electrostatic modified Langmuir wave $(\omega_1, \vec{k}_1)$ with the following constraints,

$$\omega_t = \omega_{t'} + \omega_1 \tag{4}$$

and,

$$\vec{k}_t = \vec{k}_{t'} + \vec{k}_1 \tag{5}$$

Equations (4) and (5) when used with the dispersion relations of the modes involved prove to encompass both the forward and backward Raman processes as the characteristics of the parametrically generated mode are sensitive to the scattering angle. Let us perturb this equilibrium and study the time development of this perturbation using the linearized fluid equations and Maxwell's equations which are given by (c.f.Ref. [14]),

$$\frac{\partial n_{1e^-}}{\partial t} + N_{0e^-}\vec{\nabla}.\vec{V}_{1e^-} = 0 \tag{6}$$

$$\frac{\partial n_{1e^+}}{\partial t} + N_{0e^+}\vec{\nabla}.\vec{V}_{1e^+} = 0 \tag{7}$$

$$\frac{\partial \vec{V}_{1e^-}}{\partial t} + \frac{3KT_{e^-}}{mN_{0e^-}}\vec{\nabla}n_{1e^-} + \frac{e}{m}\left(\vec{E}_1 + c^{-1}\vec{V}_{1e^-}\times\vec{B}_0\right) = -\vec{\nabla}\left(\vec{V}_{te^-}\bullet\vec{V}_{t'e^-}\right) \tag{8}$$

$$\frac{\partial \vec{V}_{1e^+}}{\partial t} + \frac{3KT_{e^+}}{mN_{0e^+}}\vec{\nabla}n_{1e^+} - \frac{e}{m}\left(\vec{E}_1 + c^{-1}\vec{V}_{1e^+}\times\vec{B}_0\right) = -\vec{\nabla}\left(\vec{V}_{te^-}\bullet\vec{V}_{t'e^-}\right) \tag{9}$$

T_{e^-}, T_{e^+} being respectively the electron and positron temperatures, n_{1e^-}, n_{1e^+}, $\vec{V}_{1e^-}$, $\vec{V}_{1e^+}$ and $\vec{E}_1$ are respectively the electron density perturbation, the positron density perturbation, the electron velocity perturbation, the positron velocity perturbation and the electric field perturbation whereas, N_{0e^-} and N_{0e^+} are respectively, the equilibrium electron and positron densities. At the equilibrium state, we have $N_{0e^-} = N_{0i} + N_{0e^+}$, where N_{0i} is the equilibrium ions density. The velocities $\vec{V}_{t'e^-}$ and $\vec{V}_{t'e^+}$ which represent respectively the response of electrons and positrons to the sideband are given by,

$$\vec{V}_{t'e^-} = \frac{2e}{m\omega_{t'}}\vec{E}_{t'0}\sin\left(\vec{k}_{t'}.\vec{x}-\omega_{t'}t\right), \tag{10}$$

and,

$$\vec{V}_{t'e^+} = -\frac{2e}{m\omega_{t'}}\vec{E}_{t'0}\sin\left(\vec{k}_{t'}.\vec{x}-\omega_{t'}t\right), \tag{11}$$

where the field $\vec{E}_{t'} = 2\vec{E}_{t'0}\cos\left(\vec{k}_{t'}.\vec{x}-\omega_{t'}t\right)$ represents the sideband electromagnetic wave.

To obtain the equation (9) we have used the fact that

$$\vec{V}_{te^+} = -\vec{V}_{te^-}, \tag{12}$$

and,

$$\vec{V}_{t'e^+} = -\vec{V}_{t'e^-} \tag{13}$$

Taking the divergence of the equations (8) and (9), and using the following equation,

$$\vec{\nabla}.\vec{E}_1 = 4\pi e\left(n_{1e^+} - n_{1e^-}\right) \tag{14}$$

where, $n_{1e^-} = n^0_{1e^-}\cos\left(\vec{k}_1.\vec{x}-\omega_1 t\right)$ and $n_{1e^+} = n^0_{1e^+}\cos\left(\vec{k}_1.\vec{x}-\omega_1 t\right)$

We find the following set of equations,

$$\frac{\partial^2 n_{1e^-}}{\partial t^2} + \frac{3KT_{e^-}}{m}k_1^2 n_{1e^-} + \omega_{pe}^2\left(n_{1e^-} - n_{1e^+}\right) + \omega_{ce}^2 n_{1e^-} = N_{0e^-}\Delta\left(\vec{V}_{te^-}\bullet\vec{V}_{t'e^-}\right) \tag{15}$$

$$\frac{\partial^2 n_{1e^+}}{\partial t^2}+\frac{3KT_{e^+}}{m}k_1^2\,n_{1e^+}-\omega_{pp}^2\left(n_{1e^-}-n_{1e^+}\right)-\omega_{ce}^2\,n_{1e^+}$$
$$=N_{0e^+}\,\Delta\left(\vec{\boldsymbol{V}}_{\boldsymbol{te}^-}\bullet\vec{\boldsymbol{V}}_{\boldsymbol{t'e}^-}\right) \qquad (16)$$

where, $\omega_{pe}^2=\dfrac{4\pi N_{0e^-}e^2}{m}$, $\omega_{pp}^2=\dfrac{4\pi N_{0e^+}e^2}{m}$ and $\omega_{ce}^2=\left(\dfrac{eB_0}{mc}\right)^2$. The propagation has been considered perpendicular to $\vec{B}_0$. By using the following relationship,

$$n_{1e^+}=\alpha n_{1e^-} \qquad (17)$$

and,

$$N_{0e^+}=\delta N_{0e^-} \qquad (18)$$

With,

$$\alpha=-\left(\frac{N_{0e^+}}{N_{0e^-}}\right)=-\delta \qquad (19)$$

where, Eq.(18) may be imposed by experiment whereas Eq.(17) is derived by way of conservation equations (α and δ are constants), the subtraction of Eq.(16) from Eq. (15) leads to the following differential equation,

$$\left(\frac{\partial^2}{\partial t^2}+\omega_u^2\right)n_{1e^-}=\left(\frac{1-\delta}{1-\alpha}\right)N_{0e^-}\,\Delta\left(\vec{\boldsymbol{V}}_{\boldsymbol{te}^-}\bullet\vec{\boldsymbol{V}}_{\boldsymbol{t'e}^-}\right) \qquad (20)$$

To obtain Eq. (20), we have used the fact that,

$$T_{e^-}\ \square\ T_{e^+}\ \square\ T_e \qquad (21)$$

In Eq. (20), ω_u^2 is given by,

$$\omega_u^2=\left(\frac{1+\alpha}{1-\alpha}\right)\omega_{ce}^2+(1+\delta)\,\omega_{pe}^2+\frac{3KT_e}{m}k_1^2 \qquad (22)$$

$\omega_1^2=(1+\delta)\,\omega_{pe}^2+\dfrac{3KT_e}{m}k_1^2$, being the dispersion relation of the modified Langmuir wave.

On the other hand, the sideband $\vec{\boldsymbol{E}}_{\boldsymbol{t'}}=2\vec{\boldsymbol{E}}_{\boldsymbol{t'0}}\cos\left(\boldsymbol{k}_{\boldsymbol{t'}}.\vec{\boldsymbol{x}}-\omega_{\boldsymbol{t'}}\boldsymbol{t}\right)$ satisfies the following wave equation,

$$\Delta\vec{\boldsymbol{E}}_{\boldsymbol{t'}}-\frac{1}{c^2}\frac{\partial^2\vec{\boldsymbol{E}}_{\boldsymbol{t'}}}{\partial t^2}=\frac{4\pi}{c^2}\frac{\partial}{\partial t}\left[-e(n_{1e^-})\vec{\boldsymbol{V}}_{\boldsymbol{te}^-}+e(n_{1e^+})\vec{\boldsymbol{V}}_{\boldsymbol{te}^+}\right] \qquad (23)$$

By using Eq.(12), Eq.(23) may be cast as follows,

$$\Delta\vec{\boldsymbol{E}}_{\boldsymbol{t'}}-\frac{1}{c^2}\frac{\partial^2\vec{\boldsymbol{E}}_{\boldsymbol{t'}}}{\partial t^2}=-\frac{4\pi e}{c^2}\frac{\partial}{\partial t}\left[\left(n_{1e^-}+n_{1e^+}\right)\vec{\boldsymbol{V}}_{\boldsymbol{te}^-}\right] \qquad (24)$$

The scattered electromagnetic wave satisfies then the following equation,

$$\left(\frac{\partial^2}{\partial t^2}+\omega_{t'}^2\right)\vec{\boldsymbol{E}}_{\boldsymbol{t'}}=4\pi e(1+\alpha)\frac{\partial}{\partial t}\left(n_{1e^-}\vec{\boldsymbol{V}}_{\boldsymbol{te}^-}\right), \qquad (25)$$

where,

$$\omega_{t'}^2=k_{t'}^2c^2+(1+\delta)\,\omega_{pe}^2 \qquad (26)$$

In other hand, we have,

$$\Delta\left(\vec{\boldsymbol{V}}_{\boldsymbol{te}^-}\bullet\vec{\boldsymbol{V}}_{\boldsymbol{t'e}^-}\right)=-\frac{e^2k_1^2}{m^2\omega_t\omega_{t'}}\vec{\boldsymbol{E}}_{\boldsymbol{t}}.\vec{\boldsymbol{E}}_{\boldsymbol{t'}} \qquad (27)$$

and,

$$\frac{\partial}{\partial t}\left(n_{1e^-}\vec{\boldsymbol{V}}_{\boldsymbol{te}^-}\right)=-\frac{e\omega_{t'}}{m\omega_t}n_{1e^-}\vec{\boldsymbol{E}}_{\boldsymbol{t}} \qquad (28)$$

Replacing these results in Eqs. (20) and (25), we find,

$$\left(\frac{\partial^2}{\partial t^2}+\omega_u^2\right)n_{1e^-}=\lambda\,E_tE_{t'} \qquad (29)$$

$$\left(\frac{\partial^2}{\partial t^2}+\omega_{t'}^2\right)E_{t'}=\mu\,E_t\,n_{1e^-} \qquad (30)$$

where,

$$\lambda=-\left(\frac{1-\delta}{1-\alpha}\right)\frac{\omega_{pe}^2k_1^2}{\omega_t\omega_{t'}\,4\pi m}\vec{\boldsymbol{u}}_{\boldsymbol{t}}.\vec{\boldsymbol{u}}_{\boldsymbol{t'}}, \qquad (31)$$

and,

$$\mu=-(1+\alpha)\frac{\omega_{t'}}{\omega_t}\frac{\omega_{pe}^2}{N_{0e^-}}\,\vec{u}_t.\vec{u}_{t'}, \qquad (32)$$

since $\vec{E}_t$ and $\vec{E}_{t'}$ can be written as,

$$\vec{E}_t=E_t\,\vec{u}_t, \qquad (33)$$

and,

$$\vec{E}_{t'}=E_{t'}\,\vec{u}_{t'}, \qquad (34)$$

$\vec{u}_t$ and $\vec{u}_{t'}$ specify the direction of polarization of the fields $\vec{E}_t$ and $\vec{E}_{t'}$.

Using the Fourier transformation and after some algebra, one finds (c.f. Ref.[14]) the maximum growth rate $\gamma_{\max}$ of the stimulated Raman scattering process,

$$\gamma_{\max(e-p-i)}=\frac{\frac{(1-\delta)}{\sqrt{1+\delta}}\omega_{pe}^2\,k_1\left|\vec{u}_t.\vec{u}_{t'}\right|}{4\omega_t\left[\omega_{t'}\,\pi N_{0e^-}\,m\left(\left(\frac{1-\delta}{1+\delta}\right)\omega_{ce}^2+(1+\delta)\omega_{pe}^2+\frac{3KT_e}{m}k_1^2\right)^{1/2}\right]^{1/2}}E_{t0} \qquad (35)$$

The maximum growth rate of the instability is proportional to the amplitude of the electromagnetic pump wave E_{t0}.

On the other hand, the maximum growth rate of the SRS process calculated for an e-ion plasma in Ref.[14] is given by,

$$\gamma_{\max(e-ion)}=\frac{\omega_{pe}^2\,k_1\left|\vec{u}_t.\vec{u}_{t'}\right|E_{t0}}{4\omega_t\left[\omega_{t'sh}\,\pi N_0\,m\left(\omega_{ce}^2+\omega_{pe}^2+\frac{3KT_e}{m}k_1^2\right)^{1/2}\right]^{1/2}}, \qquad (36)$$

where,

$$\omega_{t'sh}^2=k_{t'}^2c^2+\omega_{pe}^2, \qquad (37)$$

By setting $\delta=0$ (no positron in the plasma) in the expression of $\gamma_{\max(e-p-i)}$ (c.f.Eq.(35))we get the expression of the maximum growth rate $\gamma_{\max(e-ion)}$ for an (e-i) plasma given by Eq.(36).

We plot on figures 1and 2, the variation of the growth rates ratio $\gamma_{\max(e-p-i)}/\gamma_{\max(e-ion)}$ in terms of $k_1\lambda_D^*$ and δ respectively in the case $k_{t'}^2c^2\langle\langle\omega_{pe}^2$ for an interstellar plasma, where $\gamma_{\max(e-p-i)}/\gamma_{\max(e-ion)}$ is given by,

$$\gamma_{\max(e-p-i)}/\gamma_{\max(e-ion)}=(1-\delta)(1+\delta)^{-3/4}\left[1+\frac{\delta-\left(\frac{2\delta}{1+\delta}\right)\frac{\omega_{ce}^2}{\omega_{pe}^2}}{\left(1+\frac{\omega_{ce}^2}{\omega_{pe}^2}+3\frac{T_e}{T^*}\left(k_1\lambda_D^*\right)^2\right)}\right]^{-1/4}, \qquad (38)$$

where λ_D^* is the Debye length corresponding to $T^*=0.01eV$. On figure 1, is plotted $\gamma_{\max(e-p-i)}/\gamma_{\max(e-ion)}$ versus $k_1\lambda_D^*$ for three different temperatures, where $\omega_{pe}=10^4\,s^{-1}$, $\omega_{ce}=10\,s^{-1}$, $\delta=0.3$, $T_e=10^{-1}eV$, $1eV$, *and* $10eV$. On figure 2, it is shown the variation of $\gamma_{\max(e-p-i)}/\gamma_{\max(e-ion)}$ in terms of δ where $\omega_{ce}=10\,s^{-1}$, $\omega_{pe}=10^4\,s^{-1}$, $T_e=1eV$ and $k_1\lambda_D^*=0.2$.

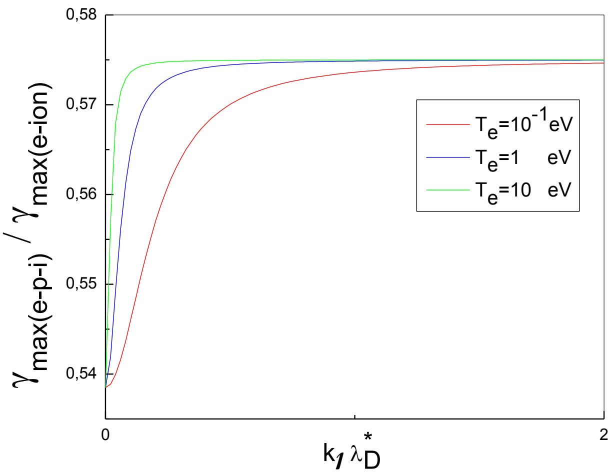

Fig.1 : Ratio $\gamma_{\max(e-p-i)}/\gamma_{\max(e-ion)}$ versus $k_1\lambda_D^*$ for different temperatures T_e.

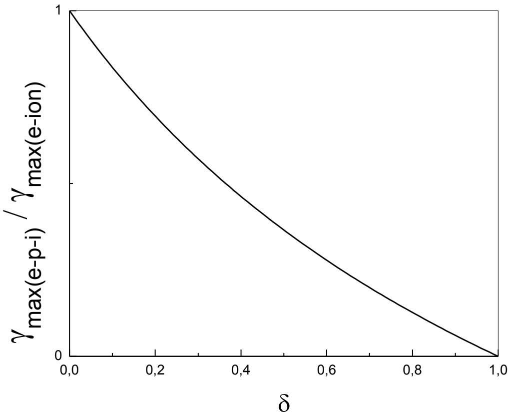

Fig.2 : Ratio $\gamma_{\max(e-p-i)} / \gamma_{\max(e-ion)}$ versus δ .

3. CONCLUSION

This paper is devoted to the study of the stimulated Raman scattering in a magnetized electron-positron-ion plasma. The maximum growth rate of this parametric process is calculated and compared to the maximum growth rate obtained in a previous study by Shivamoggi [14] in the case of a magnetized e-i plasma. It is found that the ratio $\gamma_{\max(e-p-i)} / \gamma_{\max(e-ion)}$ increases for increasing $k_1 \lambda_D^*$, whereas for a given wavelength, the ratio increases with temperature. We find also that the normalized maximum growth rate $\gamma_{\max(e-p-i)} / \gamma_{\max(e-ion)}$ decreases for increasing values of the parameter δ . This tendency has been revealed in an unmagnetized electron-positron-ion plasma (c.f.Ref.[2]).

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Figure caption

1/ Fig.1 : Ratio $\gamma_{\max(e-p-i)} / \gamma_{\max(e-ion)}$ versus $k_1 \lambda_D^*$ for different temperatures T_e .

2/Fig.2 : Ratio $\gamma_{\max(e-p-i)} / \gamma_{\max(e-ion)}$ versus δ .

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