# Static Spherically Symmetric Cosmological Space- Time in Bimetric Relativity

S. D. Deo
Head, Department of Mathematics
N. S. Science & Arts College, Bhadrawati, Dist.Chandrapur-442902(M.S) India

***Abstract*- Spherically symmetric static Kantowski - Sachs space – time is studied in the context of Rosen's (1940) bimetric relativity with the source of matter perfect fluid and scalar massive meson field respectively. It is observed that, Kantowski - Sachs cosmological model representing perfect fluid does not exist where as massive meson scalar field exists when the space time is Spherically symmetric static model and $M \neq 0$.Moreover ,when M=0 , the scalar meson field reduces to zero mass scalar field** .

***Key Words- Spherically symmetric, perfect fluid, scalar massive meson field, Bimetric relativity***

## 1. INTRODUCTION

In general theory relativity, the spherical symmetry has its own importance by virtue of its simplicity. Many noteworthy spherically symmetric space – times (for examples -Schwarzschild solutions (exterior and interior), the Robertson – Walker model of the expanding universe, etc.) are playing a vital role in general theory of relativity.

Space time symmetries are features of space time that can be described as exhibiting some form of symmetry .The role of symmetry in physics is important in simplifying solutions to many problems .Space time symmetries finding ample applications in the study of exact solutions of Einstein's field equations of general relativity.

Deo(1) studied Spherically symmetric Kantowski - Sachs space – time in the context of Rosen's (1973) bimetric relativity with the source of matter cosmic strings and domain walls respectively. Observed that, in this theory spherically symmetric Kantowski - Sachs space – time does not accommodate cosmic strings as well as domain walls. Hence, one can obtain the vacuum solutions. And it is interesting to note that resulting space – time represents the Robertson – Walker flat metric.

Here we have studied the static spherically symmetric space – times in the context of Rosen's (2-4) bimetric theory of relativity with the source of matter perfect fluid and scalar massive meson field respectively.

## 2. FIELD EQUATIONS

The field equations of bimetric relativity derived from variation principles are $K_i^j = N_i^j - \frac{1}{2} N g_i^j = -8\pi\kappa T_i^j$ (2.1)

Where $N_i^j = \frac{1}{2}\gamma^{\alpha\beta}\left(g^{hj}\, g_{hi}\big|_{\alpha}\right)\Big|_{\beta}$ (2.2)

$$N = N_{\alpha}^{\alpha}, \kappa = \left(\frac{g}{\gamma}\right)^{\frac{1}{2}} \quad (2.3)$$

$g$ = det.$g_{ij}$ ,$\gamma$ = det.$\gamma_{ij}$ (2.4)

Avertical bar ( | ) denotes a covariant differentiation with respect to $\gamma_{ij}$.

$T_i^j$ is the energy momentum tensor for the matter.

## 3. PERFECT FLUID

Here we consider the spherically symmetric Kantowski - Sachs space – time in the form

$$ds^2 = dt^2 - \lambda^2 dr^2 - k^2(d\theta^2 + \mathrm{Sin}^2\theta\, d\varphi^2) \quad (3.1)$$

Where $\lambda$ and k are functions of r only.

The background metric corresponding to the metric (3.1) is taken as

$$d\sigma^2 = dt^2 - dr^2 - d\theta^2 - \mathrm{Sin}^2\theta\, d\varphi^2 \quad (3.2)$$

For this metric the only non-vanishing $\gamma$ – Christoffel symbols are

$$\Gamma_{33}^{2} = \sin\theta\cos\theta\ ,\ \Gamma_{32}^{3} = \Gamma_{23}^{3} = \cot\theta \quad (3.3)$$

The energy momentum tensor for cosmic perfect fluid is given by

$$T_i^j = (\rho + p)v_i v^j - p g_i^j \quad (3.4)$$

Together with $v_4 v^4 = 1$

Here $v^i$is the four velocity vector of the fluid distribution having $p$ and $\rho$ as the proper pressure and energy density of the fluid, respectively.

Using co-moving coordinate system the field equations (2.1) for the metric (3.1) and (3.2) corresponding to the energy momentum tensor (3.4) in bimetric relativity can be written explicitly as

$$\left(\frac{\lambda'}{\lambda}\right)' - 2\left(\frac{k'}{k}\right)' = -16\pi kp \qquad (3.5)$$

$$\left(\frac{\lambda'}{\lambda}\right)' = 16\pi kp \qquad (3.6)$$

$$\left(\frac{\lambda'}{\lambda}\right)' + 2\left(\frac{k'}{k}\right)' = -16\pi k\rho \qquad (3.7)$$

Using the equations (3.5) and (3.6) we get –

$$\left(\frac{\lambda'}{\lambda}\right)' = \left(\frac{k'}{k}\right)' \qquad (3.8)$$

Using the equation (3.8) with (3.5) and (3.7) we get

$$\rho + 3p = 0 \qquad (3.9)$$

In view of reality conditions ,i.e. $\rho \ge 0, p \ge 0$ the equation (3.9) implies that

$$\rho = 0 = p \qquad (3.10)$$

Thus in bimetric relativity the spherically symmetric Kantowski-Sachs cosmological perfect fluid model does not exist and hence only vacuum model exists.

Using the equations (3.10) and (3.5)-(3.7) we have

$$\left(\frac{\lambda'}{\lambda}\right)' = 0 \qquad (3.11)$$

$$\left(\frac{k'}{k}\right)' = 0 \qquad (3.12)$$

Using the equations (3.11) and (3.12) we get $\lambda = n_1 e^{m_1 r}$ (3.13)

$$k = n_2 e^{m_2 r} \qquad (3.14)$$

Where $m_1$, $m_2$, $n_1$, $n_2$ are constants of integration.

Using the equations (3.13) and (3.14) in (3.1), then

$$ds^2 = dt^2 - e^{2m_1 r} dr^2 - e^{2m_2 r}\left(d\theta^2 + \sin^2\theta d\phi^2\right) \qquad (3.15)$$

For which $m_1=m_2=m$, then

$$ds^2 = dt^2 - e^{2mr}\left(dr^2 + d\theta^2 + \sin^2\theta d\phi^2\right) \qquad (3.16)$$

This vacuum model represents Robertson – Walker flat static metric.

## 4. MASSIVE SCALAR MESON FIELD

Here we consider the region of the space time with massive meson scalar field .The energy momentum tensor for massive scalar field is given by

$$T_{ij} = V_{,i}V_{,j} - \frac{1}{2}g_{ij}\left(V_{,m}V^{,m} - M^2V^2\right) \qquad (4.1)$$

Together with

$$g^{ij}V_{;ij} + M^2V = 0 \qquad (4.2)$$

Where M is the mass of the parameter of the scalar meson field V. And the suffix comma and the semicolon after a field variable represent ordinary and covariant differentiations with respect to rand $g_{ij}$, respectively.

Using the equations (3.1), (3.2) and (4.1) then (2.1) yield-

$$\left(\frac{\lambda'}{\lambda}\right)' - 2\left(\frac{k'}{k}\right)' = -8\pi k(V'^2 - M^2V^2) \qquad (4.3)$$

$$\left(\frac{\lambda'}{\lambda}\right)' = 8\pi k\left(V'^2 - M^2V^2\right) \qquad (4.4)$$

$$\left(\frac{\lambda'}{\lambda}\right)' + 2\left(\frac{k'}{k}\right)' = -8\pi k\left(V'^2 + M^2V^2\right) \qquad (4.5)$$

Using equations (4.3) and (4.4) we get $\left(\frac{\lambda'}{\lambda}\right)' = \left(\frac{k'}{k}\right)'$ (4.6)

Which give us

$$\lambda = kqh^r \qquad (4.7)$$

Where $h \ge 0$,q are constants of integration.

Using equations (4.6) in (4.3) and (4.5), we have

$$\left(\frac{k'}{k}\right)' = 8\pi\kappa\left(V'^2 - M^2V^2\right) \qquad (4.8)$$

$$3\left(\frac{k'}{k}\right)' = -8\pi\kappa\left(V'^2 - M^2V^2\right) \qquad (4.9)$$

This turns to be

$$2V'^2 - M^2V^2 = 0 \qquad (4.10)$$

Equation (4.10) gives two solutions for V as

$$V = \alpha \exp\left(\frac{M}{\sqrt{2}}r\right) \qquad (4.11)$$

$$V = \beta \exp\left(-\frac{M}{\sqrt{2}}r\right) \qquad (4.12)$$

Klein-Gordan equation (4.2) for the metric (3.1) is

$$V'' + \left(\frac{\lambda'}{\lambda} + 2\frac{k'}{k}\right)V' + M^2V = 0 \qquad (4.13)$$

Using the values of λ and *V* from the equations (4.7) and (4.11) and similarly from (4.7) and (4.12), we get the values of λ and *k*.

## CONCLUSION

It is shown that in Rosen's bimetric relativity, the static spherically symmetric cosmological model representing perfect fluid does not exist where as massive meson scalar field exists when the space time is (3.1)and $M \neq 0$ .Moreover ,when M=0 , the scalar meson field reduces to zero mass scalar field .

## ACKNOWLEDGEMENTS

The author wishes to acknowledge the UGC for sanctioning research project and financial support.

## REFERENCES

1. S. D. Deo, International Journal of Applied Computational Science and Mathematics
2. (1)2012, 23-28.
3. N. Rosen, Gen. Rela. Grav. 4 (1973) 435.
4. N. Rosen, Ann. Phys.84 (1974)455.
5. N. Rosen, Gen. Rela. Grav. 12 (1980) 493.