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Static and Vibration Analysis of Functionally Graded Metal-Ceramic Beam for Different Boundary Conditions

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***Abstract*—Static analysis of functionally graded metal-ceramic beam is shown in this paper for different boundary conditions. For the ceramic part Zirconium Carbide was chosen for its high toughness and melting point. Structural steel is used in the metal part. Properties of the beam varies in the thickness direction according to power law. This paper also shows the principal natural frequency under different boundary conditions of the functionally graded beam. The static loadings are applied at the metal surface of the beam and the analysis are carried out through finite element method using higher order shear deformation theory for different power law exponent of the FGM.**

***Keywords*—Functionally graded matrial; power law; static analysis; metal ceramic beam;**

I. INTRODUCTION

Functionally graded materials (FGM) are anisotropic materials normally consist of mixture of ceramic and metal. FGM is composed of various layers including pure ceramic at one end and pure metal at the other end. The layers in between the pure ceramic and metal are mixture of both ceramic and metal. The volume fraction of the fundamental materials is varying gradually through the thickness direction from one layer to another. By this way the composite can have the both specification of the both materials. As the layers have gradually varying properties along the thickness, the FGM can exhibit the ability to reduce the residual stress, thermal stress and stress concentration at the interface of the layer found in conventional composites. Ceramics usually have better thermal load capacity than the metal, but due to lack of their ductility they cannot withstand higher mechanical load. On the other hand, metals have better ability to withstand high mechanical load but as their co efficient of thermal expansion is higher than the ceramic, they can't withstand high thermal load. As mentioned above, FGM can exhibit the specification of both metals, they can withstand a very high thermal load and a mechanical load which is practically impossible compared to single materials. So, they are designed for high temperature applications such as, nuclear reactor, space shuttle, blade of a gas turbine armor protection for military vehicles, fusion energy devices, bio medical implants such as bone implants, dental implants etc.

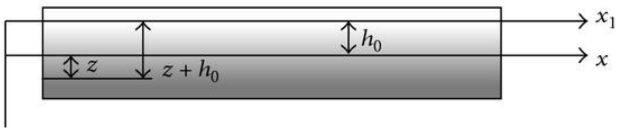

Fig. 1. A typical FGM

The material properties at the various layers of an FGM can be found by means of a power series [1]. An example of a typical FGM is shown in fig. 1.

M. Şimşek [2] solved the functionally graded simply supported beam subjected to a uniformly distributed load. The beam has been investigated by using Ritz method within the framework of higher order shear deformation theory and Timoshenko beam theory analytically. Aydogdu and Taskin [3] investigated the free vibration of an FGM using the Euler boundary bean theory, parabolic shear deformation theory and exponential shear deformation theory. H.J. Xiang and J. Yang [4] determined the free and forced vibrations of the FGM using the Timoshenko beam theory. Chakraborty and Gopalakrishnan [5] analyzed the wave propagation behavior of FG beam under high frequency impulse mechanical and thermal loading, by using a spectrally formulated finite element method. S. Alexraj, N. Vasiraja and P. Nagaraj [6] studied about the cantilever and simply supported FGM beam under mechanical and thermal load by means of finite element method. They [6] assumed two boundary conditions, cantilever and simply supported and also they used *Al* as the metal in the FGM and ZrO_2 as the ceramic. Yang J., Chen Y., Xiang Y. and Jia X.L. [7] studied about the free and forced vibration of cracked FGM subjected to an axial load and moving load. Benatta M.A., Mechab I., Tounsi A. and Adda Bedia E.A. [8] proposed an analytical solution of functionally graded short beams including warping effect.

In this paper, the static analysis of a ten layered ZrC-Structural steel FGM has done, and also investigated the natural frequency behavior of an FGM under no load for different boundary conditions were investigated. The analysis has been done by finite element method. It is assumed that the material property of the beam vary through its thickness direction according to the power law [1]. The boundary conditions are assumed as cantilever, simply supported and both ends fixed. In this study, various displacements, stresses and strains of the FGM beam for different material compositions are examined.

II. THEORY AND FORMUALTION

A functionally graded beam having length of L, width b, thickness h with three different boundary conditions is shown in figure 2. The beam is subjected to a uniformly distributed load q_0. In this paper, not only the behavior of FGM under bending loads but also the behavior of the FGM under compressive load is also presented. In the FGM the effective properties like

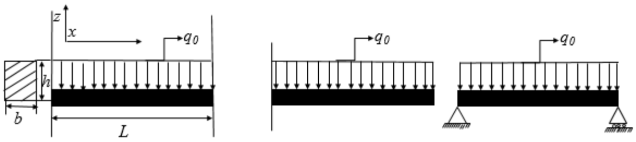

Fig. 2. (a) both ends fixed (b) cantilever (b) simply supported beam subjected to a uniformly distributed load.

Young's modulus E, Poisson's ratio v, coefficient of thermal expansion α and modulus of rigidity G vary continuously through the thickness direction according to the power law [1]. According to the power law form,

$$E_{fgm} = (E_m - E_c)V_c + E_c \qquad (1)$$

$$v_{fgm} = (v_m - v_c)V_c + v_c \qquad (2)$$

$$\alpha_{fgm} = (\alpha_m - \alpha_c)V_c + \alpha_c \qquad (3)$$

$$G_{fgm} = \frac{E_{fgm}}{2(1+v_{fgm})} \qquad (4)$$

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^N$$

Here, N is a non-negative variable parameter which describes the material property variation through the thickness direction. Constituents' c and m stands for ceramic and metal respectively. It is clear from equation (1-3) is that

$$E_{fgm} = E_c \quad v_{fgm} = v_c \quad \alpha_{fgm} = \alpha_c \;\; at \;\; z = -h/2$$

$$E_{fgm} = E_m \quad v_{fgm} = v_m \quad \alpha_{fgm} = \alpha_m \;\; at \;\; z = h/2$$

Variation of Young's modulus along the thickness direction of an FGM is shown in figure 3.

Based on the higher order shear deformation theory, the axial displacement, u_x, and the transverse displacement of any point of the beam, u_z, are given as [9]

$$u_x(x,z) = u_0(x) + z\phi(x) - kz^3(w_{0,x}(x) + \phi(x)) \qquad (5)$$

$$u_z(x,z) = w_0(x) \qquad (6)$$

Where u_0 and w_0 are the axial and the transverse displacement of any point on the neutral axis, ϕ is the rotation of the cross section, $k=4/3h^2$, and $( )_x$ denotes a derivatives with respect to x. Relationship between strain and displacement is given by

$$\varepsilon_{xx} = u_{x,x} = u_{0,x} + z\phi_{,x} - kz^3(\phi, x + w_{0,xx}) \qquad (7)$$

$$\gamma_{xy} = u_{x,y} + z_{y,x} = (1-3kz^2)(\phi + w_{0,x}) \qquad (8)$$

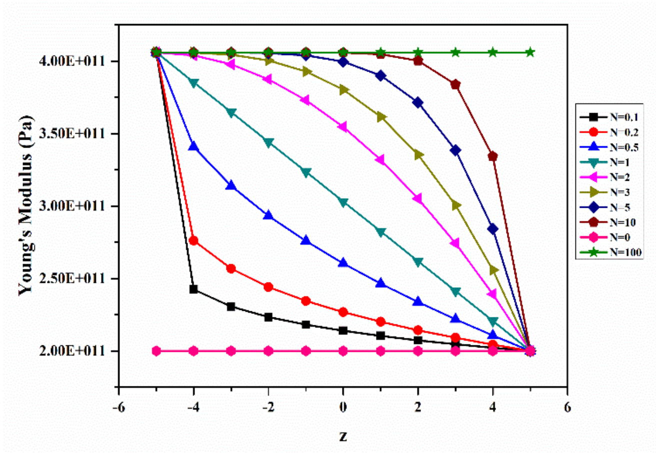

Fig. 3. Variation of Young's modulus along the thickness direction

Where ε_{xx} and γ_{xy} are the normal and the shear strain respectively. We are assuming that, the FGM obeys Hooke's law, the stresses in the beam become

$$\sigma_{xx} = E_{fgm}\varepsilon_{xx} \qquad (9)$$

$$\tau_{xy} = G_{fgm}\gamma_{xy} \qquad (10)$$

Where σ_{xx} and τ_{xy} are axial normal and the shear stresses. The strain energy of the beam is given by

$$U = \frac{1}{2}\int_0^L\int_A (\sigma_{xx}\varepsilon_{xx} + \tau_{xy}y_{xy})dAdx \qquad (11)$$

Where A is the cross-sectional area of the beam. The equivalent stress is given by

$$\sigma = \frac{\sigma_{xx}+\sigma_{yy}}{2} + \frac{\sigma_{xx}-\sigma_{yy}}{2}\cos 2\varphi + \tau_{xy}\sin 2\varphi \qquad (12)$$

$$\varepsilon = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} + \frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\cos 2\varphi + \gamma_{xy}\sin 2\varphi \qquad (13)$$

Where φ is an arbitrary angle between equivalent stress and the normal stress. Now, substituting equation (7),(8),(9),(10) into equation (11) gives

$$U = \frac{1}{2}\int_0^L \left\{ \begin{array}{l} A_{xx}(u_{0,x})^2 + 2(B_{xx} - kE_{xx}) \\ (u_{0,x})(\phi_{,x}) - 2kE_{xx}(u_{0,x}) \\ (w_{0,xx}) + (D_{xx} + k^2H_{xx} - \\ 2kF_{xx})(\phi_{,x})^2 + 2k(kH_{xx} \\ - F_{xx})(w_{0,xx})(\phi_{,x}) + k^2 \\ H_{xx}(w_{0,xx})^2 + (A_{xx} - \\ 6kD_{xy} + 9k^2F_{xy})\lfloor(\phi + w_{0,x})^2\rfloor \end{array} \right\} dx \qquad (14)$$

Where

$$(A_{xx}, B_{xx}, D_{xx}, E_{xx}, F_{xx}, H_{xx}) = \int_A E_{fgm}(1, z, z^2, z^3 z^4, z^6)dA$$

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$$(A_{xy}, D_{xy}, F_{xy}) = \int_A G_{fgm}(1, z^2, z^4)\, dA$$

Solving Equation (12) by variational method and formulating global stiffness matrices

$$\begin{bmatrix} [K_1]_{N\times N} & [K_2]_{N\times N} & [K_3]_{N\times N} \\ [K_4]_{N\times N} & [K_5]_{N\times N} & [K_6]_{N\times N} \\ [K_7]_{N\times N} & [K_8]_{N\times N} & [K_9]_{N\times N} \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \begin{bmatrix} f \\ 0 \\ 0 \end{bmatrix} \qquad (15)$$

Where $[K_1]$,…,$[K_9]$ are the stiffness matrices of the beam. f is the generalized load vector governed by the uniformly distributed loads that is applied on the beam.

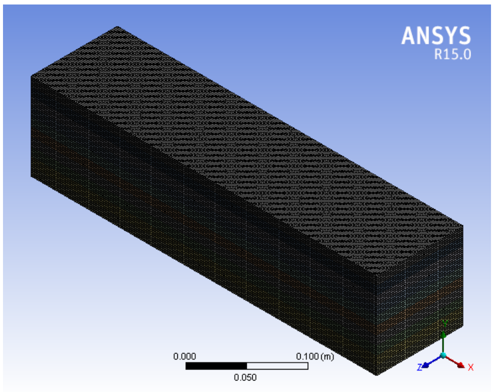

Fig. 4. Modeling of FGM in ANSYS

III. METHODOLOGY

The finite element analysis of the beam is carried out in this paper to investigate the static behavior of an FGM under mechanical load and natural frequency under no load. There are few software like ANSYS, ABAQUS that can analyze finite element method. In this paper, ANSYS is used to carry out the calculation of finite element method.

The functionally graded beam is composed of zirconium carbide (ZrC; $E_c=406GPa$, $v_c=0.19$, $\alpha_c=1.41\times10^{-7}K^{-1}$) and structural steel ($E_m=200GPa$, $v_m=0.3$, $\alpha_m=1.2\times10^{-5}K^{-1}$). The properties of the transitional layers between metal and ceramic changes through the thickness of the beam according to the power law. The bottom surface of the beam is pure ceramic and the upper surface of the beam is pure metal. The length, width and the thickness of the beam is kept as $L=0.4m$, $b=0.1m$, $h=0.1m$. Uniformly distributed load of *1000N, 1500N, 2000N, 2500N, 3000N, 5000N, 10000N, 20000N, 50000N* and *100000N* are applied on the beam respectively and the performance is studied. The mechanical load was applied on the metal surface. The FGM beam was modeled in ANSYS. The mesh size of 1mm is taken in this paper. Total number of nodes is 2342610 and the total number of elements is 500000. Figure 4 shows the modeling of an FGM in ANSYS.

IV. RESULTS

The responses under static loading is shown in this section. Table 1. represents the response total deformation and directional deformation under 100kN force. The total deformation under other applied forces are shown in fig. 8-10. From the figures, deformation is maximum for the minimum value possible for power law exponent. That means the deformation is maximum for metal constituents. After that, with the increment of power law exponent, the deformation decreases.

Table 1 Maximum Deformation Under 100kN Load.

N	Cantilever (mm)		Both ends fixed (mm)		Simply supported (mm)	
	Total	*Directional (w_0)*	*Total*	*Directional (w_0)*	*Total*	*Directional (w_0)*
0	0.5103	0.078861	0.017646	0.0040622	0.086457	0.0796
0.1	0.40887	0.059327	0.0147	0.0035339	0.065761	0.061451
0.5	0.37634	0.058783	0.012961	0.0032437	0.057165	0.053582
1	0.35699	0.05822	0.011939	0.0030674	0.057061	0.041172
3	0.3281	0.057259	0.010598	0.0027924	0.05336	0.049347
5	0.31577	0.054277	0.010163	0.002679	0.052005	0.04797
10	0.30171	0.050854	0.0097539	0.0025517	0.047644	0.0404
100	0.25271	0.039671	0.0085072	0.0020308	0.042738	0.025332

The stress and strain under 100kN force is tabulated in table 2 and table 3 respectively. And responses under other loads are shown in fig. 5-7 for different boundary conditions. Both the stress and strain are maximum for $N=0.1$ for all boundary conditions except shear stress for both ends fixed beam. For both ends fixed beam, shear stress is maximum for metal constituents only. There is a dissimilarity in this stress and strain section compared to the previous deformation section. For cantilever FG beam, both the equivalent and shear stress is maximum for $N=0.1$. After that point, stress is reducing gradually with the decrease in power law exponent. For both ends fixed beam, equivalent stress is maximum for $N=0.1$ but shear stress is maximum for metal constituent. After that point the power law exponent was taken as 0.5, for this exponent, both equivalent and shear stress decreased, but after that point, both the stresses increased and then decreased with the increment of power law exponent. Same phenomenon happened for simply supported FG beam too.

The same response is also true for strains too. It can clearly be seen from table 3 and fig. 5-7.

Table 2. Maximum Stress under 100kN Load

N	Cantilever (MPa)		Both ends fixed (MPa)		Simply supported (MPa)	
	Equivalent (σ)	*Shear (σ_{xy})*	*Equivalent (σ)*	*Shear (σ_{xy})*	*Equivalent (σ)*	*Shear (σ_{xy})*
0	231.99	63.876	65.976	20.48	513.89	125.31
0.1	376.58	79.442	74.893	18.747	1506.4	390.78
0.5	329.53	74.655	48.158	12.452	507.56	129.42
1	306.82	70.889	61.412	16.348	1462.9	380.22
3	287.29	9.648	55.89	14.95	1453.2	377.79
5	282.62	9.1192	54.315	14.407	1452	377.47
10	277.57	8.5166	52.874	13.823	497.22	126.89
100	201.88	5.9141	55.89	13.567	497.06	126.81

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Table 3. Maximum Strain under 100kN Load

N	Cantilever		Both ends fixed		Simply supported	
	Equivalent *(ℰ)*	*Shear* *($ℰ_{xy}$)*	*Equivalent* *(ℰ)*	*Shear* *($ℰ_{xy}$)*	*Equivalent* *(ℰ)*	*Shear* *($ℰ_{xy}$)*
0	0.00116	0.00083	0.00033	0.000266	0.00257	0.001629
0.1	0.001476	0.00103	0.000298	0.000244	0.00371	0.002291
0.5	0.001382	0.000971	0.000179	0.000162	0.00125	0.000759
1	0.00131	0.000922	0.000255	0.000213	0.003603	0.002229
3	0.001167	0.000125	0.000229	0.000194	0.003579	0.002215
5	0.001097	0.000119	0.000219	0.000187	0.003576	0.002213
10	0.001018	0.000111	0.000208	0.00018	0.001225	0.000744
100	0.000497	0.0000347	0.000142	0.0000795	0.001224	0.000743

Fig. 11-13 show the variation of directional deformation (Y-Axis) along the length of the FG beam for different boundary conditions. It is very clear from the figures is that, the transverse displacement is maximum for metal rich FG beam and minimum for ceramic rich FG beam as we know that deformation is inversely proportional to the Young's modulus.

Metals have lower Young's modulus compared to the ceramic. The functionally graded beam gradually becomes ceramic rich from the metal portions. As a result, the deformation is maximum for metal rich FG beam, but with the increment of power law exponent, the beam is intending to become more ceramic rich material. So, with the increment of power law exponent, the deformation is decreasing.

The variation of transverse deformation curves are showing general trend regarding their boundary conditions when subjected to a uniformly distributed load. That confirmation depicts the validity of this work that is shown in this paper.

The variation of shear stress (xy plane) along the length of the FG beam for different boundary conditions are shown in fig. 14-16.

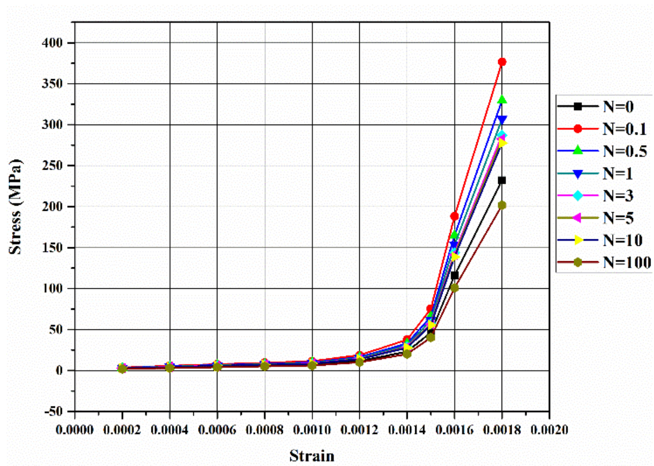

Fig. 5. Equivalent stress vs equivalent strain for the cantilever beam

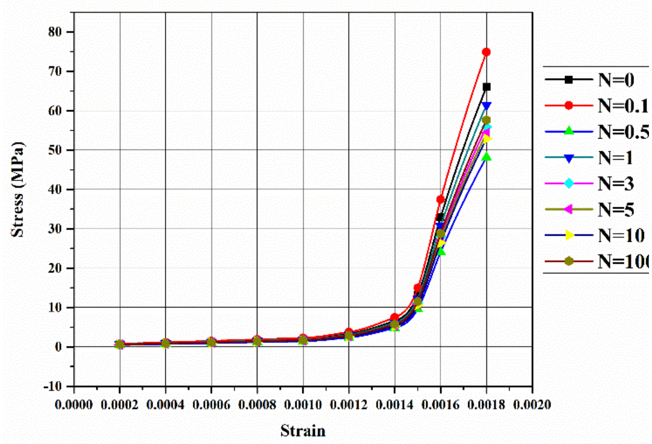

Fig. 6. Equivalent stress vs equivalent strain for both ends fixed beam.

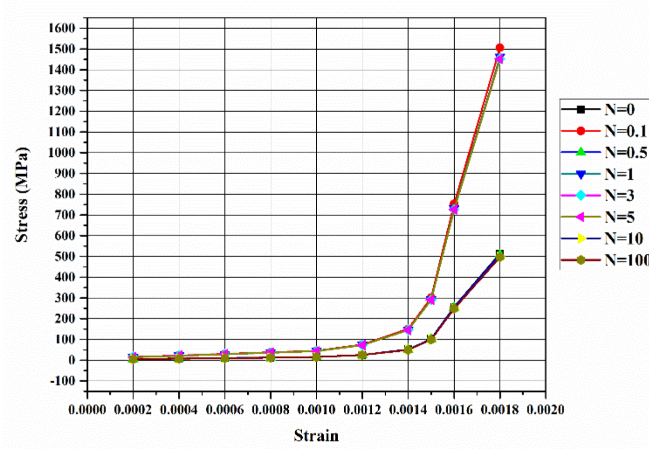

Fig. 7. Equivalent Stress vs strain for simply supported beam

This phenomenon also obeys the stress relation with power law exponent that were mentioned earlier in this paper.

All the variations along the length of FG beams are taken from the upper side of the beam. Similarly, along the thickness direction, the results can be examined. In this paper, variation of responses along the length from the upper layer of the beam is presented only.

One question may arise, why are the responses showing pretty much same curve. Well the answer is : the responses under static loading represents similar curves because the applied loads were following pretty much same trend as the total deformation or stress vs strain curve.

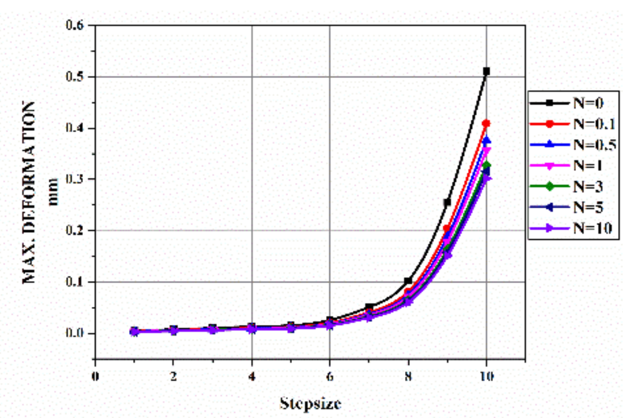

Fig. 8. Total deformation for cantilever beam

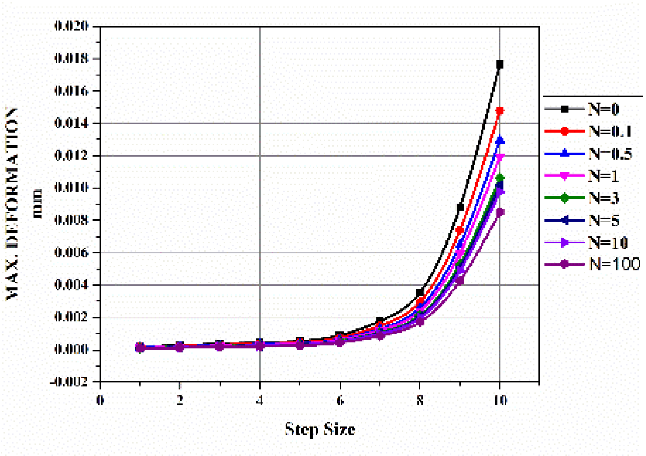

Fig. 9. Total deformation for both ends fixed beam

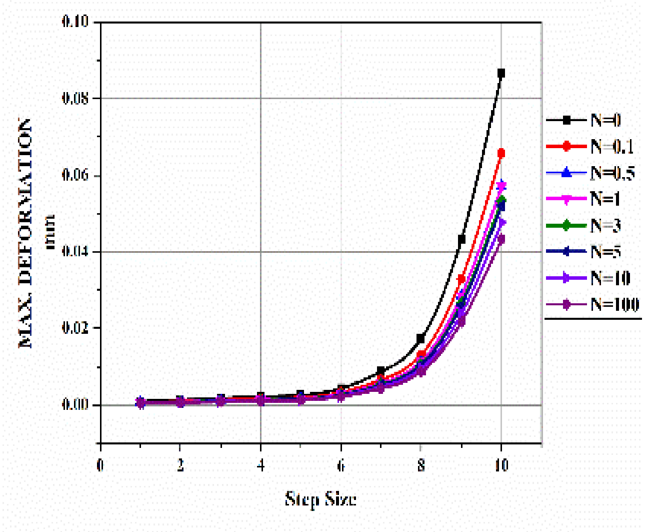

Fig. 10. Total deformation for simply supported beam

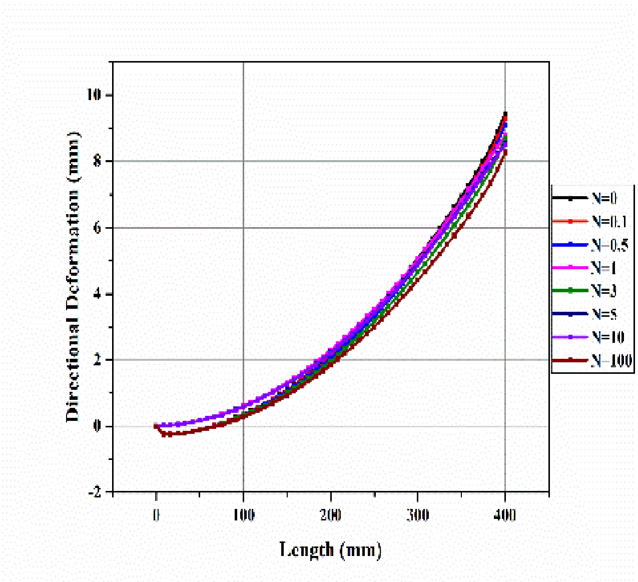

Fig. 11. Variation of directional deformation along the length of the FG cantilever beam

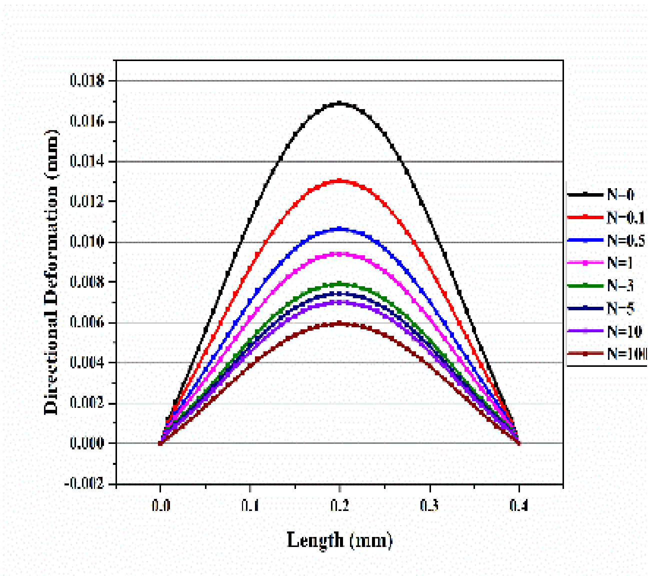

Fig. 12. Variation of directional deformation along the length of the FG both ends fixed beam

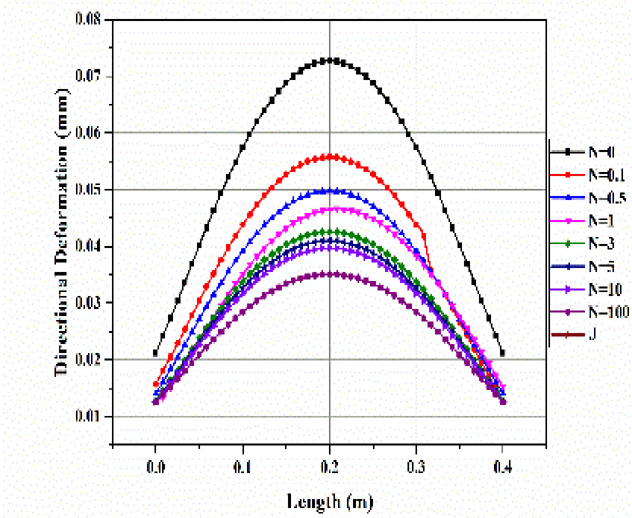

Fig. 13. Variation of directional deformation along the length of the FG simply supported beam

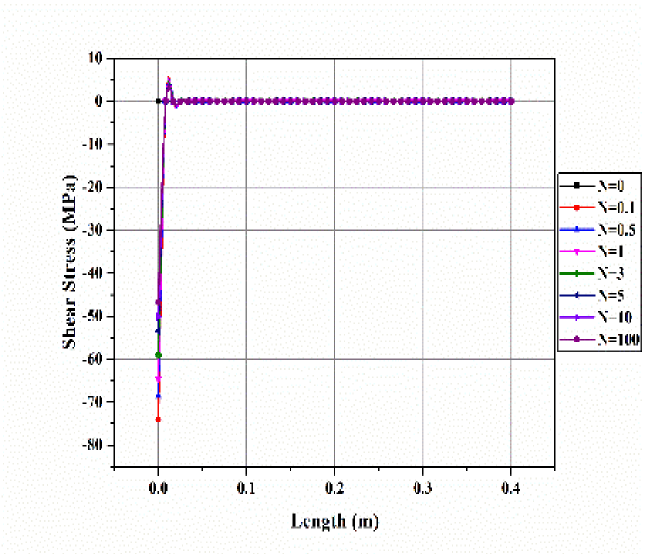

Fig. 14. Variation of shear stress along the length of the FG cantilever beam

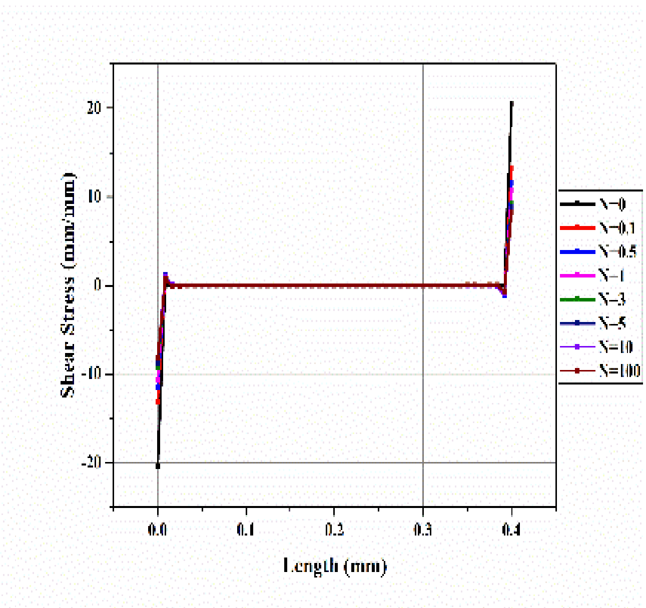

Fig. 15. Variation of shear stress along the length of the FG both ends fixed beam

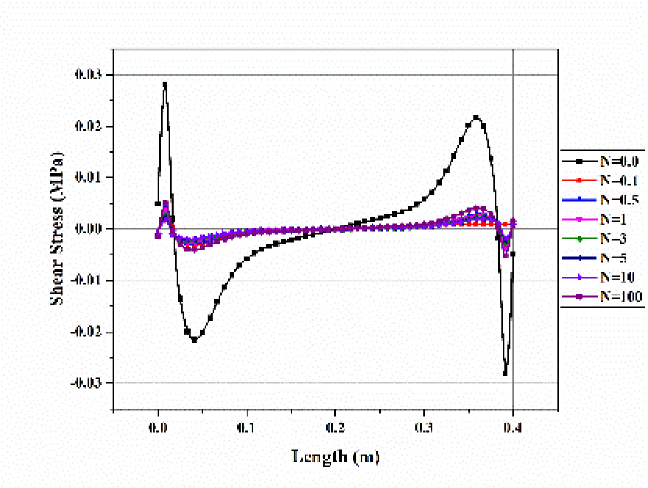

Fig. 16. Variation of shear stress along the length of the FG simply supported beam

Free vibration means the rate of vibration in a system when no load is applied on the system despite of its weight. Natural frequency depends on the material property of the structure. To be specific, the natural frequency depends on the density and Young's modulus of any structure. Natural frequency is proportional to the square root of the modulus of elasticity and inversely proportional to the square root of the density [10]. Natural frequency also depends on different boundary condition as it affects the vibration modes. So, for specific application boundary condition should be specified properly before calculating the natural frequency.

As beams have infinite numbers of degree of freedom the natural frequency is also infinite. Because, each degrees of freedom have their own natural frequency. In this study, the principal mode of natural frequency is determined for three different boundary conditions.

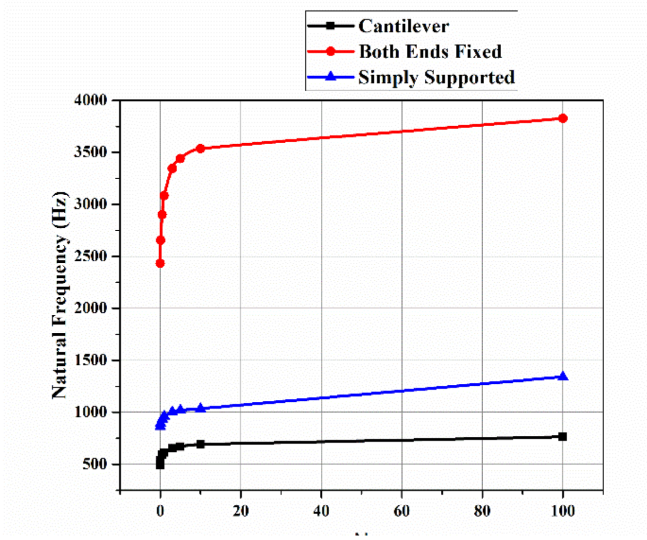

Fig. 17. Natural frequency of the FG beam under different boundary conditions

Table 4. Natural frequency of *ZrC/Steel* based FGM

N	*Cantilever* Hz	*Fixed* Hz	*Simply Supported* Hz
0	491.51	2431.2	862.14
0.1	538.95	2654.5	897.79
0.5	591.45	2901.7	934.79
1	614.95	3084.6	961.56
3	653.67	3344.9	1003.4
5	670.61	3440.1	1019.1
10	690.18	3535.7	1035.3
100	763.34	3825.6	1341

Table 4. shows the exact values of natural frequency for different power law exponent and for different boundary conditions. Fig. 17. shows a graphical representation of these values in terms of different boundary conditions and in terms of different values of power law exponent. It is clear from fig. 17 that natural frequency is maximum for both ends fixed beam. Also, whenever the FG beam intended to become ceramic rich, natural frequency increases.

V. CONCLUSION

Static and free vibration analysis of ZrC/Steel based FGM was carried out in this paper by finite element method. The results indicated that the response and stress propagation in the FG beam for different boundary conditions were much more different than those found in the isotropic beams. The free vibration analysis indicated that whenever the beam is intended to become ceramic rich, natural frequency increased. During the analysis, the material property at the different layers of the FG beam were strictly following the power law curve that has been shown in figure 3.

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