Static And Modal Analysis Of Composite Drive Shaft And Development Of Regression Equations.

Miss Priya Dongare.¹

Prof. Dr. Suhas Deshmukh.²

1. Student, Sinhgadh College of Engg. Kondhawa (Bk), Pune.

2. Professor, Sinhgad Academy of Engineering, Kondhawa (Bk), Pune.
Abstract

In the present work an attempt is made to evaluate the suitability of composite material such as Carbon/Epoxy for drive shaft in automotive transmission. ANSYS FEM solver is used for evaluation of shaft properties (such as maximum stress, maximum deformation) with variation in fiber orientation in each layer. Further it is studied the effect of fiber orientation on structural properties of composite shaft. Effect of fiber orientation is further analysed to determine variation of modal frequencies of composite shaft under simply supported condition. A FEM result shows that composite shaft has maximum torque transmission capacity at 45° fiber orientation. Variation of maximum stress and deformation is further correlated with fiber orientation using regression analysis and it is observed that results of regression model and FEM results are having good matching. Similar sort of regression analysis is carried out for modal analysis also and it is observed that results of FEM and regression model are quite matching with each other.

1. Introduction

Drive shafts as power transmission tubing are used in many applications, including cooling towers, pumping sets, aerospace, trucks and automobiles. In metallic shaft design, knowing the torque and the allowable shear stress for the material, the size of the shaft’s cross section can be determined. Polymer matrix composites such as carbon/epoxy or glass/epoxy offer better fatigue characteristics because micro cracks in the resin do not freely propagate as in metals, but terminate at the fibres. Generally, composites are less susceptible to the effects of stress concentration, such as are caused by notches and holes, compared with metals. [2]

An efficient design of composite drive shaft could be achieved by selecting the proper variables, which are specified to minimize the chance of failure and to meet the performance requirements. In the optimal design of the drive shaft, these variables are constrained by the lateral natural frequency, torsional vibration, torsional strength and torsional buckling of the shaft.

Present work aims at study the effect of fiber orientation on maximum deformation and stress in composite shaft and development of regression model of using FEM results. Further modal analysis is carried out to estimate natural frequencies of composite shaft and regression model is developed for modal frequencies. Ultimate aim of present work is to compare composite shaft and solid shaft based on its maximum deformation, stress, modal frequencies and weight of shaft.

Paper is outlined as follows, section 2 explains about static analysis, section 3 explains about FEM modelling procedure adopted using ANSYS APDL, section 4 explains about modal analysis and section 5 discusses about regression modelling and comparison with FEM results. Finally section 6 gives concluding remarks.

2. Static analysis

Static analysis deals with the conditions of equilibrium of the bodies acted upon by forces. A static analysis can be either linear or non-linear. All types of non-linearity are allowed such as large deformations, plasticity, creep, stress stiffening, contact elements etc. A static analysis calculates the effects of steady loading conditions on a structure, while ignoring inertia and damping effects, such as those carried by time varying loads. A static analysis can however include steady inertia loads such as gravity, spinning and time varying loads. In static analysis loading and response conditions are assumed, that is the loads and the structure responses are assumed to vary slowly with respect to time. The kinds of loading that can be applied in static analysis includes, Externally applied forces, moments and pressures, Steady state inertial forces such as gravity and spinning, imposed non-zero displacements. [4]

A static analysis result of structural displacements, stresses and strains and forces in structures for components caused by loads will give a clear idea about whether the structure or components will withstand for the applied maximum forces. If the stress values obtained in this analysis crosses the allowable values it will result in the failure of the structure in the static condition itself. To avoid such a failure, this analysis is necessary. In present work following model of composite shaft is taken into account for analysis. Figure 1 shows a composite shaft with its boundary conditions.

Torque at mid span

Simply supported B.C.

\[ \text{Figure 1 Drive Shaft and its boundary conditions} \]

Geometric and material properties of composite drive shaft under analysis is listed in table below.
3. Finite element method

The finite element method is the most powerful numerical technique ever devised for solving solid and structural mechanics problems in geometrically complicated regions. The finite element analysis of a problem is so systematic that it can be divided into a set of logical steps that can be implemented on a digital computer and can be used to solve a wide range of problems by merely changing the data input defining the domain, its physical properties and initial and boundary conditions. It is this feature that gave the finite element method such remarkable success in the modelling and simulation of practical engineering problems.

The finite element technique is endowed with two distinct features that no other method shares:

1. The domain of the problem is viewed as a collection of nonintersecting simple sub domains, called finite element. The word domain is used to denote the physical system, or material region over which the governing equations are to be solved. The subdivision of a domain into elements is termed finite element discretization. The collection of the elements is called the finite element mesh of the domain. In general, the finite element mesh is an approximation to the domain.
2. Over each finite element, the solution of the governing equations is approximated by a linear combination of undetermined parameters and preselected approximation functions, almost always polynomials.[3]

Finite element models of the drive shaft were generated and analyzed using ANSYS version 13 commercial software. 3D model of hollow composite shaft is generated using APDL language available in ANSYS Multiphysics. Hollow shaft is meshed using layered element SHELL281. Thickness, material properties and fiber orientations of each layer is provided for element via command

```
KEYOPT,1,8,1
SECTYPE,1,SHELL,
secdata,T1,1,-90
secdata,T2,1,0
secdata,T3,2,0
secdata,T4,1,0
```

Four numbers of layers are taken into account for analysis. Appropriate simply supported boundary conditions are applied at both ends and torque is applied at mid-span of drive shaft. Application of torque is critical issue in FEA analysis, this is simplified by providing tangential force at exterior nodes of shaft and local co-ordinate system is rotated to provide a tangential directional force. This is possible by command

```
csys,1,nrotat,all
```

Static analysis is carried out to estimate maximum deformation and stress occurred in shaft. Similar fashion modal analysis is carried out to estimate five fundamental natural frequencies of shaft. Next section explains about results and its interpretation.

4. FEM Results

The composite shaft is considered as the simply supported beam with torque applied at the centre of the shaft as shown in the Figure 2.
Figure 3 Change in the deformation along the length of the composite shaft.

Figure 3 shows the variation of the deformation in mm along the length of shaft. The deformation is maximum at the centre of the shaft. Figure 4 compares the variation in the layer for the fibre orientation angle.

Figure 4 Comparison of deformation in all layers of composite shaft

Similarly the stress variation is observed along the length of the composite shaft. Figure 6 compares the variation stress in the layers at fibre orientation angle from -90 to +90.

Figure 6 Variation of the stress in composite shaft

Regression analysis

Regression analysis is a statistical tool for the investigation of relationships between variables.

In general, there are two types of regression analyses:
- Linear regression analysis-A linear regression analysis assumes that the regression model is a linear function with respect to the parameters of the regression model, i.e., the regression parameters are the coefficients of the regression terms.
- Nonlinear regression analysis-For a nonlinear regression analysis, the regression model is a nonlinear function with respect to the parameters of the regression model.

Multiple regression is a statistical technique that allows us to predict someone’s score on one variable on the basis of their scores on several other variables. E.g. The composite shaft has four layers. The regression analysis is done to get the variation of the orientation of the fiber with respect to the stress and deformation. It will have one equation for each layer which will give the stress and deformation in terms of fibre angle orientation.

5. Regression Result-

II. Static and Regression Analysis for the deformation in the composite shaft :-

The results obtained by static analysis for the deformation by varying in fiber angle orientation of the first layer of composite shaft and the layer 2,3,4 keeping 0° (constant). The regression equation for first layer is as below.

Deformation for θ1,
\[ \Delta \theta_1 = 0.9911 + 0.0002 \times \sin(\theta_1) + 1.0281 \times \cos(\theta_1) + 0.0856 \times \sin(\theta_1^2) - 0.0001 \times \sin(\theta_1^2) + 0.3953 \times (\theta_1^2) - 0.0001 \times (\theta_1) \]
Figure 7 Comparison of ANSYS result and regression result of first layer of composite shaft

Same as above the second layer fibre angle is varied keeping layer 1, 3, 4 constant (0°). The equation of the second layer is as below:

Deformation for \( \theta_2 \):
\[
\Delta \theta_2 = -1.0032 + 1.0402 \times \cos \theta_2 + 1.0281 \times \cos \theta_1 + 0.0866 \times \sin(\theta_2^2) + 0.3999 \times (\theta_2^2)
\]

Figure 8 Comparison of ANSYS result for deformation and regression result of second layer of composite shaft

Similarly the equations are obtained for the third and forth layer as below.

Deformation for \( \theta_3 \):
\[
\Delta \theta_3 = -1.5493 + 1.5842 \times \cos \theta_3 + 0.1346 \times \sin(\theta_3^2) + 0.6089 \times (\theta_3^2)
\]

Deformation for \( \theta_4 \):
\[
\Delta \theta_4 = 1.5842 \times \sin \theta_4 + 0.1346 \times \cos \theta_4 + 0.6089 \times \sin \theta_4^2
\]

II. Static and Regression Analysis for the stress in the composite shaft :-

Same as deformation we can plot the results of variation in the stress values of the each layer.

Stress for \( \theta_1 \):
\[
\sigma_{\theta_1} = 2.1927 - 2.1783 \times \cos \theta_1 - 0.1927 \times \sin \theta_1^2 - 0.8369 \times \theta_1^2
\]

Figure 10 The effect of fiber orientation angle on stress of First layer of composite shaft

Stress for \( \theta_2 \):
\[
\sigma_{\theta_2} = 1.4391 - 1.4235 \times \cos \theta_2 - 0.1244 \times \sin \theta_2^2 - 0.5496 \times \theta_2^2
\]

Figure 11 The effect of fiber orientation angle on deformation of second layer of composite shaft

Stress for \( \theta_3 \):
\[
\sigma_{\theta_3} = 3.1961 - 3.1590 \times \cos \theta_3 - 0.2810 \times \sin(\theta_3^2) - 1.2214 \times (\theta_3^2)
\]

Stress for \( \theta_4 \):
\[
\sigma_{\theta_4} = 3.1961 - 3.1590 \times \cos \theta_4 - 0.2810 \times \sin(\theta_4^2) - 1.2214 \times (\theta_4^2)
\]

III. Modal Analysis :-

The natural frequencies for each layer are obtained using ANSYS 13 by varying the orientation of fiber in layer 2 by 5° and keeping the orientation of other layers constant i.e. 0°.

Figure 15 Effect of the change in fiber orientation angle on natural frequency in second layer.
Similarly natural frequencies of the second, third and forth layer is obtained by keeping respective other layers constant

Figure 16 Effect of the change in fiber orientation angle on natural frequency in third layer.

Figure 17 Effect of the change in fiber orientation angle on natural frequency in forth layer.

6. CONCLUSION

Finite element analysis provides an insight on effect of variation of fiber orientation in composite drive shaft on statics and dynamics properties of drive shaft. In present study four-layer-composite drive shaft is studied. A model of hybridized layers was generated incorporating both carbon-epoxy and glass-epoxy as material. Fiber orientation in one layer is varied and other layer orientations are kept constant and variation of the orientation is studied in more detail and following conclusion can be easily drawn,

1. Shear stress in that layer increases up to 45 degree orientation, then decreases till 90 degree and then again starts increasing; it shows sine nature of graph.
2. While shear stress in other layers decreases till 45 degree, then increases till 90 degree and again starts decreasing; it shows cosine nature of graph.
3. Deflection in each layer remains same for any angle of orientation; it increases for 90 degree and 0 degrees.
4. 2nd, 4th and 5th natural frequencies increases with angle of orientation and then remains constant.

3. While 6th natural frequencies decreases with angle of orientation and then remains constant.
4. by doing regression analysis we have obtained relations between stress and fiber angle orientation, deflection and fiber angle orientation.

Further it is necessary to study the effect of dependencies of each layer on properties of composite drive shaft. One can further analyze and develop a correlation to study buckling behaviour.

7. References