State Space Modelling of High Gain DC-DC Boost Converter with Coupling Inductor

Preti Tyagi¹, V. C. Kotak², B. Mathew ³, V. P. Sunder Singh⁴

Associate Professor, Dept of Electronics Engineering, Shah &Anchor Kutchhi Engineering College, Mumbai, India²
Assistant Professor, Dept of Electronics Engineering, Shah &Anchor Kutchhi Engineering College, Mumbai, India³
Rtd. Professor, Dept. of Electrical Engineering, Indian Institute of Technology (IIT- Powai), Mumbai, India⁴

Abstract: Power electronic converters are periodic time variant systems because of their switching operation. The high gain boost DC-DC converters is especially useful in boosting low solar panel voltage (12V) to high voltage. The generalized state space method discussed in this paper is a way to model converters as time independent system, defined by a unified set of differential equation. This paper demonstrates the state space modeling of a proposed dc-dc boost converter topology using coupling inductor.

Keywords: state space model of DC-DC converter, coupling inductor, High gain boost converter.

I. INTRODUCTION

High gain DC-DC boost converter operating at high voltage regulation is mainly used in much industrial application. Theoretically, a dc–dc boost converter can achieve a high step-up voltage gain with an extremely high duty ratio near to 100%. However, in practice, the step up voltage gain is limited due to the effect of power switches, rectifier diode, the equivalent series resistance (ESR) of inductors and capacitors. The coupled inductor technique used in Proposed High gain DC-DC Boost converter provides solution to achieve a high voltage gain, low voltage stress on the active switch, and high efficiency while maintaining high duty ratio. For modeling of converters mainly two techniques are commonly used, first one is circuit averaging based and another one is state space averaging based technique. In this paper we are deriving state space model of high gain dc-dc boost converter having coupling inductor topology.

II. TOPOLOGY SELECTION

There are several topologies of DC-DC converters based on their components count and efficiency. Considering the cost of the electricity produced from PV module, an efficient converter is required. The non-isolated is more efficient than the isolated ones [2].

A new proposed topology in which, The boost input inductor was replaced by a coupled-inductor switching cell formed by a transistor. The voltage stress on the transistor was clamped by the output voltage. The rectifier diode withstands the output voltage. In order to even further reduce the current stresses on the switches, a new approach is considered, where a current limiting inductor connecting in the series with the secondary coil of the coupled inductor. Two diodes and a capacitor are added. The new converter features a high conversion ratio, high gain combined with a small voltage stress on the switches. It keeps the soft-switching operation and the alleviation of the recovery problem of the diodes. Its operation in a switching steady-state cycle is analyzed. The state space modeling techniques is used to model the High gain DC-DC boost converter. The new purposed design of boost converter is as follow:

Fig.(1) Proposed dc-dc converter circuit

The proposed High gain DC-DC boost converter is a having coupling inductor provides 360Vdc from the small input voltage of 12Vdc with good efficiency.
III. OPERATION OF CIRCUIT

The circuit is working in just continuous mode because the inductor current just reaches to zero not exact zero. So it is possible to ensure that the inductor fully discharges its stored energy. The operation of the circuit is explained in the following diagrams. The operation of the proposed converter circuit operated into two modes: mode 1 (when switch is on), and mode 2 (when switch is off).

Mode 1 when switch is on: The figure (2) Shows the inductor L1 draws current from the battery. In this stage the current will flow through inductor, switching device and back to the source. In this case the inductor current rises and energy is store in the inductor. The voltage across the inductor is equal to the input dc voltage (Vin) with the polarity as shown in the diagram. The diode is not conduct it remains in reverse bias condition. But due to the coupled inductor L1 with inductor L2, current also will flow through L2 inductor, L3, diode D2, and capacitor C2 and capacitor C2 gets charged. The inductor L3 (current limiting inductor) is use to control current flowing through diode d2 when the switch is ON. So the current flow in two loops when switch is in ON condition.

Analysis (when switch is ON): The equivalent circuit of mode 1 is as follow.

Current across inductor L1:

\[ V_s = L_1 \frac{dI_{L1}}{dt} \]

\[ I_{L1} = \frac{V_s}{L_1} \times \frac{1}{s} \] (Valid for t=0 to Ton)  ............. (1)

Voltage across the capacitor C2:

\[ V_1 = V_s \times \frac{N_2}{N_1} \]

At t=0, Vc2=0, IL3=0,

\[ V_1 = L_3 \frac{dI_{L3}}{dt} + V_{C2} \]

\[ I_{L3} = C_2 \frac{dV_{C2}}{dt} \]

Put the value of \( I_{L2} \), we get

\[ V_1 = L_3 C_2 \frac{dV_{C2}}{dt} + V_{C2} \] (Valid for \( I_{L3} \geq 0 \))

\[ V_{C2} = \frac{V_1}{1+sL_3C_2} \] ......................... (2)

At the end of \( T_{ON} \), \( V_{C2} \) and \( I_{L1} \) are known i.e. \( V_{C2}(T_{ON}) \) and \( I_{L1}(T_{ON}) \).

Mode 2 when switch is off: In figure. (5) shows the next stage i.e. mode 2 when switch is in OFF condition, in this case diode D1 becomes forward biased and it conducts. The current will flow through inductor L1, diode D1, capacitor C1 and back to the supply. In order to maintain the flow of current in
same direction, there will be an induced voltage with the polarities as shown in fig.(3). The magnitude of voltage is \( L \frac{di}{dt} \). This voltage comes in series with input voltage \( V_{in} \). So the total voltage across the switch \( S_1 \) is \( V_{in} + L \frac{di}{dt} \). The diode \( D_1 \) is forward biased and stored energy in the inductor transferred to the load. The inductor \( L_1 \) current reduces as shown in the waveform. In this case capacitor \( C_2 \) act as a voltage source, it makes the diode \( D_3 \) forward biased along with the induced voltage in \( L_2 \) with polarities shown in fig.(3) and helps to maintain the output voltage. The capacitor \( C_1 \) and capacitor \( C_3 \) continue provide the output value of 360Vdc power supply.

Fig.(5) MODE 2: When switch is OFF

Analysis (when switch is OFF): The equivalent circuit of mode 2 is as follow.

![Equivalent Circuit](image)

Calculate the inductor current and output voltage. Current across the inductor \( L_1 \):

\[
V_{C4} = V_{C3} \frac{N_1}{N_2} \\
V_{C2} = V_{C2} \left( T_{on} \right) \left( \frac{N_1}{N_2} \right)^2 \\
C4 = C2 \left( \frac{N_2}{N_1} \right)^2
\]

At \( t=0 \), \( V_{C1} = 0 \), \( V_{C4} = 0 \)

\[
I_{L1} = C1 \frac{dV_{C1}}{dt} + C4 \frac{dV_{C4}}{dt} \\
V_{C4} = V_{C1} + V_{C2} \left( T_{on} \right) \frac{N_2}{N_1}.
\]

Current across the inductor \( L_1 \):

\[
I_{L1} = (C1 + C4) \frac{dV_{C2}}{dt}, \quad \text{............... (3)}
\]

Substitute the value of \( I_{L1} \) in this equation and we get value of voltage across the capacitor.

\[
V_2 = L1 \frac{dL_1}{dt} + V_{C1} \\
V_2 = L1(C1 + C4) \left( \frac{d^2 C_{L1}}{dt} \right) + V_{C1}
\]

Voltage across the capacitor \( C_1 \):

\[
V_{C1} = \frac{V_2}{1 + L1(C1+C4)S^2} \quad \text{............... (4)}
\]

\( V_{C1} \) nothing but the output voltage.

IV. STATE SPACE APPROACH

The state space approach offer a more convenient way to analyze such system with the basic knowledge of matrix algebra. In state space approach, the analysis is carried out in time domain by representing the system in form of first order differential equations and is further arranged in the matrix form.

In single input or single output (SISO) system, the output consists of linear combination of state variables and \( D \) is the null matrix. Hence the state equation of SISO system can be given by

\[
\dot{x} = Ax + Bu \\
y = C x
\]

The representation of the equivalent circuit of switching mode, DC-DC high gain boost converter shown in state space form. The circuit equation can be written by applying KVL and KCL to describe the dynamic behavior of the circuit as follow[6]:

STATE SPACE REPRESENTATION OF MODE 1:

Applying KVL in loop 1fig (3):

![State Space Diagram](image)
Applying KVL in loop 2 fig (4):
\[ L_2 \frac{dI_{L_2}}{dt} + V_{C_3} = V_1 \]

Applying KCL in loop 2 fig (4):
\[ C_3 \frac{dV_{C_3}}{dt} - I_{L_1} = 0 \]

These equations can be rearranged as:
\[ \frac{dI_{L_1}}{dt} = \frac{V_s}{L_1} \] (5)
\[ \frac{dI_{L_2}}{dt} = \frac{1}{L_2} (V_1 - V_{C_3}) \] (6)
\[ \frac{dV_{C_3}}{dt} = \frac{1}{C_3} I_{L_1} \] (7)

Equation (5) (6) (7) can be expressed in state space form as follows:
\[
\begin{bmatrix}
\frac{dI_{L_1}}{dt} \\
\frac{dI_{L_2}}{dt} \\
\frac{dV_{C_3}}{dt}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 \\
0 & \frac{1}{L_2} & 0 \\
0 & 0 & \frac{1}{C_3}
\end{bmatrix}
\begin{bmatrix}
I_{L_1} \\
I_{L_2} \\
V_{C_3}
\end{bmatrix}
+ \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
V_s \\
0 \\
0
\end{bmatrix}
\] (8)

The output equation is written as:
\[
Y = C_2 x
\]
\[ Y_1 = I_{L_1}, Y_2 = V_{C_3} \]
\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
I_{L_1} \\
I_{L_2} \\
V_{C_3}
\end{bmatrix}
\]

Equation (8) can be written in standard state space form as:
\[
\frac{dx(t)}{dt} = A_1 x(t) + B_1 u(t)
\]

Where \( x(t) \) is the (3x1) state variable vector, \( u(t) \) is the (2x1) input, \( A_1 \) and \( B_1 \) are matrices with constant elements given by
\[
A_1 =
\begin{bmatrix}
0 & 0 & 0 \\
0 & -\frac{1}{L_2} & 0 \\
0 & \frac{1}{C_3} & 0
\end{bmatrix}
\]
\[
B_1 =
\begin{bmatrix}
\frac{1}{L_1} & 0 \\
0 & \frac{1}{L_2} \\
0 & 0
\end{bmatrix}
\]

STATE SPACE REPRESENTATION OF MODE 2:

Applying KVL in the loop fig (6):
\[ L_1 \frac{dI_{L_1}}{dt} = V_s - V_{C_1} \]

Applying KCL at the node 1:
\[ (C_1 + C_4) \frac{dV_{C_1}}{dt} - I_{L_1} = 0 \]

These equations can be rearranged as:
\[ \frac{dI_{L_1}}{dt} = \frac{1}{L_1} V_s - \frac{1}{L_1} V_{C_1} \] (9)
\[ \frac{dV_{C_1}}{dt} = \frac{1}{C_1+4} I_{L_1} \] (10)

Equation (9) and (10) can be expressed in state space form as follows:
\[
\begin{bmatrix}
\frac{dI_{L_1}}{dt} \\
\frac{dV_{C_1}}{dt}
\end{bmatrix} =
\begin{bmatrix}
0 & -\frac{1}{L_1} \\
\frac{1}{L_1} & \frac{1}{C_1+4}
\end{bmatrix}
\begin{bmatrix}
I_{L_1} \\
V_{C_1}
\end{bmatrix}
+ \begin{bmatrix}
1 \\
0
\end{bmatrix}
\begin{bmatrix}
V_s \\
0
\end{bmatrix}
\] (11)

The output equation is written as:
\[
Y = C_2 x
\]
\[ Y_1 = I_{L_1}, Y_2 = V_{C_1} \]
\[
\begin{bmatrix}
Y_1 \\
Y_2
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
I_{L_1} \\
V_{C_1}
\end{bmatrix}
\]

Equation (11) can be written in standard state space form as:
\[
\frac{dx(t)}{dt} = A_2 x(t) + B_2 u(t)
\]

Where \( x(t) \) is the (2x1) state variable vector, \( u(t) \) is the single input, \( A_2 \) and \( B_2 \) are matrices with constant elements given by
\[
A_2 =
\begin{bmatrix}
0 & -\frac{1}{L_1} \\
\frac{1}{L_1} & 0
\end{bmatrix}
\]
\[
B_2 =
\begin{bmatrix}
\frac{1}{L_1} \\
0
\end{bmatrix}
\]
\[
C_2 =
\begin{bmatrix}
1 & 0
\end{bmatrix}
\]
V. CONCLUSION

The State space technique used for modeling the High gain DC-DC boost converter having coupling inductor has a number of advantages over circuit averaging technique, like more compact representation of equations, ability to obtain more transfer functions. This state space averaged model equation can be used for simulation of converter by using MATLAB instead of the model prepared with switches that takes long simulation time. This State space equation can be use for analyzing the system performance and development of controller for stability studies, thus derivation of state space models of high gain DC-DC boost converter topology helps in analyzing circuits with ease.

VI. REFERENCES

2. Qun Zhao “Performance Improvement of Power Conversion by Utilizing Coupled Inductors” Doctor of Philosophy In Electrical Engineering February 03, 2003 Blacksburg, Virginia.
4. Theodore Soong, Peter Lehn “A Transformerless High Boost DC-DC Converter for use in Medium / High Voltage Applications” University of Toronto, 10 King’s College Rd., Toronto, 2012 IEEE.
6. Dr. Shailendra Jain, “Modelling and Simulation using MATLAB Simulink” Wiley India, 2011