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Starting two steps-four off steps method accurately for the solution of second order initial value problems

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Abstract

We derive an order five hybrid method through collocation of the differential-system and interpolation of the approximate solution which is implemented in predictor- corrector mode. Continuous block method was used to generate the independent solution which served as predictor. The efficiency of our method was tested on some second order initial value problem and was found to give better approximation than the existing methods.

Keywords : hybrid method, collocation, differential system, interpolation, approximate solution, continuous block method, independent solution, predictor-corrector mode

A.M.S Subject Classification: 65L05, 65L06, 65D30

1 Introduction

This paper considers the approximate solution to second order initial value problem of the form

$$y'' = f(x, y, y'), \qquad y^k(x_0) = y_n^k, \quad k = 0, 1 \tag{1}$$

where f is continuous and differentiable within the interval of integration.

Convectionally, higher order ordinary differential equation are solved by method of reduction to syatem of first order ordinary differential equation. This method is extensively discussed by Awoyemi and Kayode[5], Adesanya, Anake and Udoh [4], Kayode and Awoyemi [14], Jator [11], Awoyemi and Idowu [7] to mention few. These authors suggested that the direct method for solving higher order ordinary differential equation are more efficient since the method of reduction increased the dimension of the resulting system fo first order ordinary differential equation; hence it waste alot of computer and human effort.

Many scholars have proposed method implemented in predictor corrector mode for the direct solution of (1), among them are Awoyemi [6], Kayode [13], Olabode [16], Adesanya, Anake and Oghoyon [3], Kayode and Adeyeye [12]. These authors proposed an implicit multistep method in which seperate predictors are needed to implement the corrector. The major setback of this method is that the predictors are reducing order of accuracy, therefore it has an great effect on the accuracy of the method.

Scholars later proposed block method to cater for some setbacks of the predictor-corrector method. Block method has the properties for Runge kutta method of being self starting and does not require developing seperate predictors and evaluate fewer function per step. Among scholars that proposed block method are: Jator [10], Jator and Li [9], Awoyemi et al. [8], Adesanya et al. [1], Majid,Azmi and Suleiman [15], Adesanya et al. [2], Omar et al. [17],Siamak [19], Omar and Suleiman [18]. It was observwd that in block method, the number of interpolation points must be equal to the order of the differential equation, hence this method does not exhaust all possible interpolation points therefore a method of lower order is developed.

In this paper, we developed a method which is implemented in predictor-corrector method but the predictors are constant order predictors hence cater for the setbacks of the convectional predictor- corrector method. The predictors are developed adopting block method hence these methods combine the properties of both predictor-corrector method and block method, therefore address some of the setbacks of the method.

2 Methodology

2.1 Development of the corrector

We consider a power series approximate solution of the form

$$y(x) = \sum_{j=0}^{r+s-1} a_j x^j \tag{2}$$

The second derivatives of (2) gives

$$y'' = \sum_{j=2}^{r+s-1} j(j-1) a_j x^{j-1} \tag{3}$$

Substituting (3) into (1) gives

$$f(x, y, y') = \sum_{j=2}^{r+s-1} j(j-1) a_j x^{j-1} \tag{4}$$

where r and s are the number of interpolation and collocation points respectively. Interpolating (2) at $x_{n+r}, r = 0 \left(\frac{2}{3}\right) \frac{4}{3}$ and collocating (4) at $x_{n+s}, s = 0 \left(\frac{2}{3}\right) 2$, gives a non linear system of equation

$$AX = U \tag{5}$$

where

$$X = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 \\ 1 & x_{n+\frac{2}{3}} & x_{n+\frac{2}{3}}^2 & x_{n+\frac{2}{3}}^3 & x_{n+\frac{2}{3}}^4 & x_{n+\frac{2}{3}}^5 & x_{n+\frac{2}{3}}^6 \\ 1 & x_{n+\frac{4}{3}} & x_{n+\frac{4}{3}}^2 & x_{n+\frac{4}{3}}^3 & x_{n+\frac{4}{3}}^4 & x_{n+\frac{4}{3}}^5 & x_{n+\frac{4}{3}}^6 \\ 0 & 0 & 2 & 6x_n & 12x_n^2 & 20x_n^3 & 30x_n^4 \\ 0 & 0 & 2 & 6x_{n+\frac{2}{3}} & 12x_{n+\frac{2}{3}}^2 & 20x_{n+\frac{2}{3}}^3 & 30x_{n+\frac{2}{3}}^4 \\ 0 & 0 & 2 & 6x_{n+\frac{4}{3}} & 12x_{n+\frac{4}{3}}^2 & 20x_{n+\frac{4}{3}}^3 & 30x_{n+\frac{4}{3}}^4 \\ 0 & 0 & 2 & 6x_{n+2} & 12x_{n+2}^2 & 20x_{n+2}^3 & 30x_{n+2}^4 \end{bmatrix}$$

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \end{bmatrix}^T$$

$$U = \begin{bmatrix} y_n & y_{n+\frac{2}{3}} & y_{n+\frac{4}{3}} & f_n & f_{n+\frac{2}{3}} & f_{n+\frac{4}{3}} & f_{n+2} \end{bmatrix}^T$$

solving (5) for $a_j's$ and substituting into (2) gives a continuous hybrid linear multistep method in the form

Where

$$y(x) = \alpha_0 + \alpha_{\frac{2}{3}} y_{n+\frac{2}{3}} + \alpha_{\frac{4}{3}} y_{n+\frac{4}{3}} + h^2 \left(\sum_{j=0}^{2} \beta_{2j} f_{n+2j} + \beta_{\frac{2}{3}} f_{n+\frac{2}{3}} + \beta_{\frac{4}{3}} f_{n+\frac{4}{3}} \right) \tag{6}$$

$$\alpha_0 = \frac{-1}{64} \left(243t^6 - 1458t^5 + 2970t^4 - 2160t^3 + 432t - 64 \right)$$

4

$$\alpha_{\frac{2}{3}} = \frac{1}{32}\left(243t^6 - 1458t^5 + 2970t^4 - 2160t^3 + 384t\right)$$

$$\alpha_{\frac{4}{3}} = \frac{1}{64}\left(243t^6 - 1458t^5 + 2970t^4 - 2160t^3 + 336t\right)$$

$$\beta_0 = \frac{1}{8640}\left(1215t^6 - 7533t^5 + 16470t^4 - 14760t^3 + 4320t^2 + 128t\right)$$

$$\beta_{\frac{2}{3}} = \frac{1}{960}\left(1350t^6 - 8019t^5 + 16050t^4 - 11280t^3 + 1664t\right)$$

$$\beta_{\frac{4}{3}} = \frac{1}{960}\left(135t^6 - 891t^5 + 2010t^4 - 1560t^3 + 256t\right)$$

$$\beta_2 = \frac{1}{8640}\left(243t^5 - 810t^4 + 720t^3 - 128``t\right)$$

$t = \frac{x-x_n}{h}$, $y_{n+j} = y(x_n + jh)$, $f_{n+j} = f\left(x_n, y\left(x_n + jh\right), y'\left(x_n + jh\right)\right)$

evaluating (6) at $t = 2$ gives a discrete scheme

$$y_{n+2} = 3y_{n+\frac{4}{3}} - 3y_{n+\frac{2}{3}} + y_n + \frac{h^2}{27}\left(f_{n+2} + 9f_{n+\frac{2}{3}} - 9f_{n+\frac{2}{3}} - f_n\right) \qquad (7)$$

equation (7) is our corrector

2.2 Development of the predictor

Interpolating (2) at $x_{n+r}, r = 0, 1$ and collocating (4) at $x_{n+s}, s = 0\left(\frac{1}{3}\right)2$ gives equation (5) where

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \end{bmatrix}^T$$

$$U = \begin{bmatrix} y_n & y_{n+1} & f_n & f_{n+\frac{1}{3}} & f_{n+\frac{2}{3}} & f_{n+1} & f_{n+\frac{4}{3}} & f_{n+\frac{5}{3}} & f_{n+2} \end{bmatrix}^T$$

$$X=\begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 & x_{n+1}^8 \\ 0 & 0 & 2 & 6x_n & 12x_n^2 & 20x_n^3 & 30x_n^4 & 42x_n^5 & 56x_n^6 \\ 0 & 0 & 2 & 6x_{n+\frac{1}{3}} & 12x_{n+\frac{1}{3}}^2 & 20x_{n+\frac{1}{3}}^3 & 30x_{n+\frac{1}{3}}^4 & 42x_{n+\frac{1}{3}}^5 & 56x_{n+\frac{1}{3}}^6 \\ 0 & 0 & 2 & 6x_{n+\frac{2}{3}} & 12x_{n+\frac{2}{3}}^2 & 20x_{n+\frac{2}{3}}^3 & 30x_{n+\frac{2}{3}}^4 & 42x_{n+\frac{2}{3}}^5 & 56x_{n+\frac{2}{3}}^6 \\ 0 & 0 & 2 & 6x_{n+1} & 12x_{n++1}^2 & 20x_{n++1}^3 & 30x_{n++1}^4 & 42x_{n+1}^5 & 56x_{n++1}^6 \\ 0 & 0 & 2 & 6x_{n+\frac{4}{3}} & 12x_{n+\frac{4}{3}}^2 & 20x_{n+\frac{4}{3}}^3 & 30x_{n+\frac{4}{3}}^4 & 42x_{n+\frac{4}{3}}^5 & 56x_{n+\frac{4}{3}}^6 \\ 0 & 0 & 2 & 6x_{n+\frac{5}{3}} & 12x_{n+\frac{5}{3}}^2 & 20x_{n+\frac{5}{3}}^3 & 30x_{n+\frac{5}{3}}^4 & 42x_{n+\frac{5}{3}}^5 & 56x_{n+\frac{5}{3}}^6 \\ 0 & 0 & 2 & 6x_{n+2} & 12x_{n+2}^2 & 20x_{n+2}^3 & 30x_{n+2}^4 & 42x_{n+2}^5 & 56x_{n+2}^6 \end{bmatrix}$$

solving for a$'_j s$ using Guassian elimination method and substituting into (2) givesa continuous hybrid linear multistep method in the form

$$y(x)=\sum_{j=0}^{1}\alpha_j y_{n+j}+h^2\left(\sum_{j=0}^{2}\beta_j f_{n+j}+\beta_{\frac{1}{3}}f_{n+\frac{1}{3}}+\beta_{\frac{2}{3}}f_{n+\frac{2}{3}}+\beta_{\frac{4}{3}}f_{n+\frac{4}{3}}+\beta_{\frac{5}{3}}f_{n+\frac{5}{3}}\right) \quad (8)$$

where $t=\frac{x-x_n}{h}$, $y_{n+j}=y(x_n+jh)$, $f_{n+j}=f\left(x_n,y\left(x_n+jh\right),y'\left(x_n+jh\right)\right)$

$$\alpha_0=1-t \qquad \alpha_1=t$$

$$\beta_0=\frac{1}{13440}\left(\begin{array}{c}243t^8-2268t^7+8820t^6-18522t^5+22736t^4-16464t^3\\ +6720t^2-1265t\end{array}\right)$$

$$\beta_{\frac{1}{3}}=\frac{-1}{2240}\left(243t^8-2160t^7+7872t^6+14616t^5+14616t^4-6720t^3+267t\right)$$

$$\beta_{\frac{2}{3}}=\frac{1}{4480}\left(1215t^8-10260t^7+34524t^6-58086t^5+49140t^4-16800t^3+267t\right)$$

$$\beta_1=\frac{-1}{3360}\left(1215t^8-9720t^7+30492t^6-46872t^5+35560t^4-11200t^3+525t\right)$$

6

$$\beta_{\frac{4}{3}} = \frac{1}{4480}\left(1215t^8 - 9180t^7 + 26964t^6 - 38682t^5 + 27320t^4 - 8400t^3 + 363t\right)$$

$$\beta_{\frac{5}{3}} = \frac{-1}{2240}\left(243t^8 - 1728t^7 - 4788t^6 - 6552t^5 + 2536t^4 - 1344t^3 + 57t\right)$$

$$\beta_2 = \frac{1}{13440}\left(243t^8 - 1620t^7 + 4284t^6 - 5670t^5 + 3836t^4 - 1120t^3 + 47t\right)$$

Solving for the independent solution y_{n+s}, $s = 1\left(\frac{1}{3}\right)2$, gives a continuous hybrid block formula

$$y(x) = \sum_{j=0}^{1} \frac{(jh)^m}{m!} y_n^{(m)} + h^2 \left(\sum_{j=0}^{2} \Psi_j f_{n+j} + \Psi_{\frac{1}{3}} f_{n+\frac{1}{3}} + \Psi_{\frac{2}{3}} f_{n+\frac{2}{3}} + \Psi_{\frac{4}{3}} f_{n+\frac{4}{3}} + \Psi_{\frac{5}{3}} f_{n+\frac{5}{3}} \right) \tag{9}$$

where

$$\Psi_0 = \frac{1}{13440}\left(243t^8 - 2268t^7 + 8820t^6 - 18522t^5 + 22736t^4 - 16464t^3 + 6720t^2\right)$$

$$\Psi_{\frac{1}{3}} = \frac{-1}{2240}\left(243t^8 - 2160t^7 + 7872t^6 + 14616t^5 + 14616t^4 - 6720t^3\right)$$

$$\Psi_{\frac{2}{3}} = \frac{1}{4480}\left(1215t^8 - 10260t^7 + 34524t^6 - 58086t^5 + 49140t^4 - 16800t^3\right)$$

$$\Psi_1 = \frac{-1}{3360}\left(1215t^8 - 9720t^7 + 30492t^6 - 46872t^5 + 35560t^4 - 11200t^3\right)$$

$$\Psi_{\frac{4}{3}} = \frac{1}{4480}\left(1215t^8 - 9180t^7 + 26964t^6 - 38682t^5 + 27320t^4 - 8400t^3\right)$$

$$\Psi_{\frac{5}{3}} = \frac{-1}{2240}\left(243t^8 - 1728t^7 - 4788t^6 - 6552t^5 + 2536t^4 - 1344t^3\right)$$

$$\Psi_2 = \frac{1}{13440}\left(243t^8 - 1620t^7 + 4284t^6 - 5670t^5 + 3836t^4 - 1120t^3\right)$$

7

evaluating (9) at $t = 0\left(\frac{1}{3}\right)2$, gives a discrete block formula in the form

$$A^{(0)}\mathbf{Y}_m = \mathbf{e}y_n + h^2\mathbf{d}f\left(y_n\right) + h^2\mathbf{b}\mathbf{F}(\mathbf{Y}_m) \tag{10}$$

where $A^{(0)} = 6 \times 6$ identity matrix

$$\mathbf{Y}_m = \begin{bmatrix} y_{n+\frac{1}{3}} & y_{n+\frac{2}{3}} & y_{n+1} & y_{n+\frac{4}{3}} & y_{n+\frac{5}{3}} & y_{n+2} \end{bmatrix}^T$$

$$f\left(y_n\right) = \begin{bmatrix} y_{n-1} & y_{n-2} & y_{n-3} & y_{n-4} & y_{n-5} & y_n \end{bmatrix}^T$$

$$\mathbf{F}(\mathbf{Y}_m) = \begin{bmatrix} f_{n+\frac{1}{3}} & f_{n+\frac{2}{3}} & f_{n+1} & f_{n+\frac{4}{3}} & f_{n+\frac{5}{3}} & f_{n+2} \end{bmatrix}^T$$

$$\mathbf{e} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{d} = \begin{bmatrix} \frac{28549}{1088640} & \frac{1027}{17010} & \frac{253}{2688} & \frac{1088}{8505} & \frac{35225}{217728} & \frac{41}{210} \end{bmatrix}^T$$

8

$$\mathbf{b} = \begin{bmatrix} \frac{275}{5184} & \frac{194}{945} & \frac{165}{448} & \frac{1504}{2835} & \frac{8375}{12096} & \frac{6}{7} \\ \frac{-5717}{120960} & \frac{-8}{81} & \frac{-267}{4480} & \frac{-8}{945} & \frac{3125}{72576} & \frac{3}{35} \\ \frac{10621}{272160} & \frac{788}{8505} & \frac{5}{32} & \frac{2624}{8505} & \frac{3125}{72576} & \frac{68}{105} \\ \frac{-7703}{362880} & \frac{-97}{1890} & \frac{-363}{4480} & \frac{-8}{81} & \frac{-625}{24192} & \frac{3}{70} \\ \frac{403}{60480} & \frac{46}{2835} & \frac{57}{2240} & \frac{32}{945} & \frac{275}{5184} & \frac{6}{35} \\ \frac{-199}{217728} & \frac{-19}{8505} & \frac{-47}{13440} & \frac{-8}{1701} & \frac{-1375}{217728} & 0 \end{bmatrix}$$

Substituting (10) into the first derivative of (9) at $t = \frac{1}{3}\left(\frac{1}{3}\right)2$ gives

$$y'_{n+\frac{1}{3}} = y'_n + h\left[\begin{array}{c} \left(\frac{19087}{181440}\right) f_n + \left(\frac{2713}{7560}\right) f_{n+\frac{1}{3}} - \left(\frac{15487}{60480}\right) f_{n+\frac{2}{3}} + \left(\frac{586}{2835}\right) f_{n+1} \\ - \left(\frac{6737}{60480}\right) f_{n+\frac{4}{3}} + \left(\frac{263}{7560}\right) f_{n+\frac{5}{3}} - \left(\frac{863}{181440}\right) f_{n+2} \end{array}\right]$$

$$y'_{n+\frac{2}{3}} = y'_n + h\left[\begin{array}{c} \left(\frac{1139}{11340}\right) f_n + \left(\frac{94}{189}\right) f_{n+\frac{1}{3}} + \left(\frac{11}{3780}\right) f_{n+\frac{2}{3}} + \left(\frac{332}{2835}\right) f_{n+1} \\ - \left(\frac{269}{3780}\right) f_{n+\frac{4}{3}} + \left(\frac{22}{945}\right) f_{n+\frac{5}{3}} - \left(\frac{37}{11340}\right) f_{n+2} \end{array}\right]$$

$$y'_{n+1} = y'_n + h\left[\begin{array}{c} \left(\frac{137}{1344}\right) f_n + \left(\frac{27}{56}\right) f_{n+\frac{1}{3}} + \left(\frac{387}{2240}\right) f_{n+\frac{2}{3}} + \left(\frac{34}{105}\right) f_{n+1} \\ - \left(\frac{243}{2240}\right) f_{n+\frac{4}{3}} + \left(\frac{9}{280}\right) f_{n+\frac{5}{3}} - \left(\frac{29}{6720}\right) f_{n+2} \end{array}\right]$$

$$y'_{n+\frac{4}{3}} = y'_n + h\left[\begin{array}{c} \left(\frac{286}{2835}\right) f_n + \left(\frac{464}{945}\right) f_{n+\frac{1}{3}} + \left(\frac{128}{945}\right) f_{n+\frac{2}{3}} + \left(\frac{1504}{2835}\right) f_{n+1} \\ + \left(\frac{58}{945}\right) f_{n+\frac{4}{3}} + \left(\frac{16}{945}\right) f_{n+\frac{5}{3}} - \left(\frac{8}{2835}\right) f_{n+2} \end{array}\right]$$

$$y'_{n+\frac{5}{3}} = y'_n + h\left[\begin{array}{c} \left(\frac{3715}{36288}\right) f_n + \left(\frac{725}{1512}\right) f_{n+\frac{1}{3}} + \left(\frac{2125}{12096}\right) f_{n+\frac{2}{3}} + \left(\frac{250}{567}\right) f_{n+1} \\ + \left(\frac{3875}{12096}\right) f_{n+\frac{4}{3}} + \left(\frac{235}{1512}\right) f_{n+\frac{5}{3}} - \left(\frac{275}{36285}\right) f_{n+2} \end{array}\right]$$

$$y'_{n+2} = y'_n + h\left[\begin{array}{c} \left(\frac{41}{420}\right) f_n + \left(\frac{18}{35}\right) f_{n+\frac{1}{3}} + \left(\frac{9}{140}\right) f_{n+\frac{2}{3}} + \left(\frac{68}{105}\right) f_{n+1} \\ + \left(\frac{9}{140}\right) f_{n+\frac{4}{3}} + \left(\frac{18}{35}\right) f_{n+\frac{5}{3}} + \left(\frac{41}{420}\right) f_{n+2} \end{array}\right]$$

3 Analysis of the basic properties of the block

3.1 Order of the method

We defined a linear operator on (6) to give

$$\mathcal{L}\{y(x):h\} = y(x) - \left[\alpha_0 + \alpha_{\frac{2}{3}} y_{n+\frac{2}{3}} + \alpha_{\frac{4}{3}} y_{n+\frac{4}{3}} + h^2 \left(\sum_{j=0}^{2} \beta_{2j} f_{n+2j} + \beta_{\frac{2}{3}} f_{n+\frac{2}{3}} + \beta_{\frac{4}{3}} f_{n+\frac{4}{3}}\right)\right] \tag{11}$$

Expanding y_{n+j} and f_{n+j} in Taylor series and comparing the coefficient of h gives

$$\begin{aligned} \mathcal{L}\{y(x) \ : \ h\} &= C_0 y(x) + C_1 h y'(x) + ... + C_p h^p y^p(x) + C_{p+1} h^{p+1} y^{p+1}(x) \\ &+ C_{p+2} h^{p+2} y^{p+2}(x) + ... \end{aligned} \tag{12}$$

Definition 1 *Order*

The difference operator $\mathcal{L}$ and the associated continuous linear multistep method (6) are said to be of order p if $\mathrm{C}_0 = C_1 = ... = C_p = C_{p+1} = 0$ and C_{p+2} is called the error constant and implies that the local truncation error is given by $t_{n+k} = C_{p+2} h^{(p+2)} y^{(p+2)}(x) + O\left(h^{p+3}\right)$

The order of our discrete scheme is 5, with error constant $C_{p+2} = \frac{-8}{32805}$

3.2 Consistency

A linear multistep method (6) is said to be consistent if it has order $p \geq 1$ and if $\rho(1) = \rho'(1) = 0$ and $\rho''(1) = 2!\sigma(1)$ where $\rho(r)$ is the first characteristic polynomial and $\sigma(r)$ is the second characteristic polynomial.

For our method,

$\rho(r) = r^2 + 3r^{\frac{4}{3}} - 3r^{\frac{2}{3}} + 1$

and $\sigma(r) = \frac{1}{27}\left(r^2 + 9r^{\frac{4}{3}} - 9r^{\frac{2}{3}} - 1\right).$

Clearly $\rho(1) = \rho'(1) = 0$ and $\rho''(1) = 2!\sigma(1)$.

Hence our method is consistent

3.3 Zero Stability

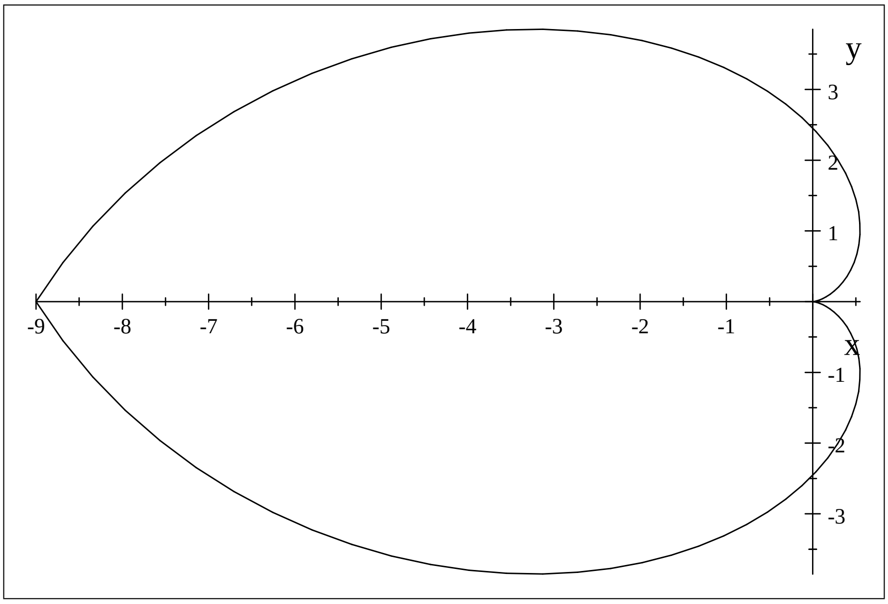
A linear multistep method is said to be zero stable, if the zeros of the first characteristic polynomial $\rho(r)$ satisfies $\mid r \mid \leq 1$ and for $\mid r \mid = 1$ is simple

Hence our method is not zero stable

3.4 Stability region

The method (13) is said to be absolute stable if for a given h, all roots z_s of the characteristic polynomial $\pi\left(z, h\right) = \rho\left(z\right) + h^2\sigma\left(z\right) = 0$, satisfies $\mid z_s \mid < 1, s = 1, 2, ..., n$. where $h = -\lambda^2 h^2$ and $\lambda = \frac{\partial f}{\partial y.}$.

The boundary locus method is adopted to determine the region of absolute stability. Substituting the test equation $y'' = -\ \lambda^2 h^2$ into (6) and writing $r = \cos\theta + i\sin\theta$ gives the stability region as shown in fig (1)



fig(1)

4 Numerical Experiments

4.1 Test Problems

We test our method with second order initial value problems

Problem 1: Consider the non-linear initial value problem (I.V.P)

$y'' - x(y')^2 = 0, y(0) = 1, y'(0) = \frac{1}{2}, h = 0.05$

Exact solution: $y(x) = 1 + \frac{1}{2} \ln \left(\frac{2+x}{2-x}\right)$

This problem was solved by Awoyemi [5] where a method of order 8 is proposed and it is implemented in predictor-corrector mode with $h = 1/320$. Jator [10] also solved this problem in block method where a block of order 6 and step-length of 5 is proposed with $h = 0.05$.We compared our result with these two results as shown in table 1. Though the result of Awoyemi [5] was not shown in this paper but

Jator [10] has shown that there method is better.

Problem 2: We consider the non-linear initial value problem (I.V.P)

$y'' = \frac{(y')^2}{2y} - 2y, y(\frac{\pi}{6}) = \frac{1}{4}, y'(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$.

Exact solution:$(\sin x)^2$

This problem was solved by Awoyemi [5] where a method of order 8 is proposed and it is implemented in predictor-corrector mode with $h = 1/320$. Jator [10] also solved this problem in block method where a block of order 6 and step-length of 5 is proposed with $h = 0.049213$. We compared our result with these two results as shown in table 2.Though the result of Awoyemi [5] was not shown in this paper but Jator [10] has shown that there method is better.

Error=|Exact result-Computed result|

Table 1 for problem 1

x	Exact result	Computed result	Error	Error in [10]
0.1	1.050041729278914	1.0500417292784907	6.6613(-16)	7.1629(-12)
0.2	1.1003353477310756	1.1003353477310676	7.9936(-15)	1.5091(-11)
0.3	1.1511404359364668	1.1511404359364292	3.7525(-14)	4.5286(-11)
0.4	1.2027325540540821	1.2027325540539671	1.1501(-13)	1.0808(-10)
0.5	1.2554128118829952	1.2554128118826986	2.9665(-13)	1.7818(-10)
0.6	1.3095196042031119	1.3095196042024151	6.9677(-13)	4.4434(-10)
0.7	1.3654437542713964	1.3654437542698326	1.5638(-12)	7.4446(-10)
0.8	1.4236489301936022	1.4236489301901405	3.4616(-12)	1.5009(-09)
0.9	1.4847002785940522	1.4847002785863199	7.7322(-12)	3.7579(-09)
1.0	1.549306144340554	1.5493061443162750	1.7780(-11)	4.7410(-09)

13

Table 2 for problem 2

x	Exact result	Computed result	Error	Error in [10]
1.1069	0.7998266847638	0.799826684754	9.7840(-12)	2.8047(-10)
1.2069	0.8733436578646	0.873343657851	1.3090(-11)	2.7950(-10)
1.3069	0.9319765974804	0.931976597464	1.5780(-11)	2.1490(-10)
1.4069	0.9733879933357	0.972338799331	1.7581(-11)	5.4975(-11)
1.5069	0.9959269037589	0.995926903740	1.8360(-11)	1.1545(-10)
1.6069	0.9986947735170	0.998694773498	1.8103(-11)	4.4825(-10)
1.7069	0.9815812563774	0.981581256055	1.6880(-11)	7.7969(-10)
1.8069	0.9452681426358	0.945268614248	1.4862(-11)	1.1840(-09)
1.9069	0.8912045176254	0.891204517613	1.2232(-11)	1.6318(-09)
2.0069	0.8215443313867	0.821544331377	9.2473(-12)	2.0567(-09)

5 Conclusion

We have proposed a two-steps four off steps method of order five in this paper. Continuous block method which has the properties of evaluation at all points within the interval of integration is adopted to give the independent solution at non overlapping intervals as the predictor. This new method forms a bridge between the predictor-corrector method and block method. Hence it shares the properties of both method. The new method performed better than the existing method i.e. block method and the predictor corrector method as shown in the numerical examples.

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