Stability of Flexibly supported Finite Oil Journal Bearings including Fluid Inertia and surface Roughness Effect: A Non-linear Transient Analysis

A. K. Bandyopadhyay[1],
Assistant Professor
Department of Mechanical Engineering,
DR. B.C Roy Engineering College,
Durgapur, West Bengal, India

S. K. Mazumder[2],
Professor,
Department of Mechanical Engineering,
DR. B.C Roy Engineering College,
Durgapur, West Bengal, India

M. C. Majumdar[3]
Professor,
Department of Mechanical Engineering,
NIT,Durgapur,
West Bengal,India

Abstract: **The aim of this study is to analyse the Non-linear transient stability of finite oil journal bearing including the effect of fluid inertia and bearing surface roughness. The inertia effect is usually ignored in view of its negligible contribution compared to viscous force.However,fluid inertia effect is to be taken in the analysis when modified Reynolds number is around one.This investigation deals with the stability of flexibly supported finite rough oil journal bearing with fluid film inertia effect using finite difference method.An attempt has been made to evaluate the critical mass parameter. A non-linear time transient method is used to simulate the journal centre trajectory to estimate the stability parameter,which is a function of speed.**

In the present work,a modified form of Reynolds equation is developed to include the combined influence of fluid inertia and surface roughness for the analysis of finite oil journal bearing.The modified average Reynolds equation considering inertia effect with flow simulation model of rough surfaces (Patir and Cheng [1, 2] is solved by a finite difference method with a successive over-relaxation scheme (Gauss-Siedel),while the equation of motion of both the journal and bearing are solved by the fourth-order Runge-Kutta method. The stability increases with the increase of eccentricity ratio and modified Reynold's numbers.

Keyword- Modified Reynolds number, stability, critical massparameter, Surface Roughness parameter,Fluid film inertia.

I INTRODUCTION

Ever-increasing demand for the hydrodynamic journal bearing systems to operate under high speed and high eccentricity makes it imperative to design this class of bearings accurately.In such cases,the familiar assumptions of smooth surface can no longer be employed to accurately predict the performance of journal bearings systems as no machining surfaces are perfectly smooth. Therefore,it is imperative to include the influence of surface roughness and fluid inertia effects in the design and analysis of journal bearings

In classical lubrication theory,the Reynolds equation has been used to provide an explanation for the process of hydrodynamic lubrication.TheNavier-Stokes equation is reduced to the Reynolds equation under the assumptions that the inertia forces of the lubricant is negligible and the flow is laminar.The validity of this conventional thin film theory is justified for small values of Reynolds number.Recently,owing to some practicalapplications,the need to include inertia effects has arisen because of the increasing number of lubrication problems which involve moderately large Reynold's numbers.Such application include large size bearing operating with non-conventional lubricants,bearings and seals operating with non-conventional lubricants such as liquid metals and water,the use of high speed bearings etc.In these cases,it is adequate to extend the Reynolds equation to include the inertial effects.

The effect of fluid inertia has been studied by many researchers for turbulent flow using long and short bearing approximations.However,there are few publications which deal only with the intermediate regime for finite oil journal bearings.Reinhardt and Lund [19] studied the dynamic characteristics based on first-order perturbation solution starting from the Navier-stokes equation.Banerjee et al.[5] introduced an extended form of Reynolds equation to include the effect of fluid inertia adopting an iteration scheme.

Kakoty and Majumdar [7- 9] carried out a first order perturbation technique in modified Reynolds number as was done by Reinhardt and Lund [19], to study the stability of an oil journal bearing.
The hydrodynamic lubrication theory of rough surfaces has been subject of growing interest as the bearing surfaces, in

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practice are all rough."Stochastic concept" introduced by Tzeng and Saibel [15] has fascinated many researchers and simulated a fair amount of work in this field.A theoretical analysis of the effect of surface roughness in a finite width bearing was done by Christensen et.al [16] based upon stochastic theory of hydrodynamic lubrication. A modified Reynolds equation considering combined effect of turbulence and surface roughness was derived by Hashimoto and Wada [18] to a high speed journal bearing .Majumdar and Ghosh[13] studied the stability of rigid rotors supported on finite rough oil journal bearings using perturbationmethod.Non-linear transient stability analysis has been performed by R.Turaga et.al[2,6]to study the sub-synchronous whirl stability of a rigid rotor supported on two symmetric hydrodynamic bearings with rough surfaces subjected to unidirectional constant load. Theoretical analysis to study the effect of support stiffness and damping on the transient response of flexibly supported rotor bearing systems,considering surface roughness effect was done by Ramesh,. J. et.al.[14].

An attempt is being made here to study the effect of fluid inertia and surface roughness effect on the stability of oil film journal bearings under unidirectional constant load.The governing equations are deduced starting from the Navier-Stokes equation and flow continuity equations.These equations are identical (except for time dependent terms) to the ones developed by Constatinescu and Galetuse[3] which also include turbulent flow regime.In the present study the authors are particularly concerned with the laminar flow regime.Since closed-form solution is not possible,an attempt is made to solve the system of nonlinear partial differential equation using Gauss-Siedel iteration method in a finite difference scheme.

A nonlinear time transient method is used to simulate the journal centre trajectory and thereby to estimate the stability parameters,which is a function of speed.

II BASIC THEORY

A. Considering fluid Inertia Effect only

The modified average Reynolds equation for fully lubricated surfaces is derived starting from the Navier-Stokes equations and the continuity equation with few assumptions. The non-dimensional form of the momentum equations and the continuity equation for a journal bearing may be written as (Figure.1)

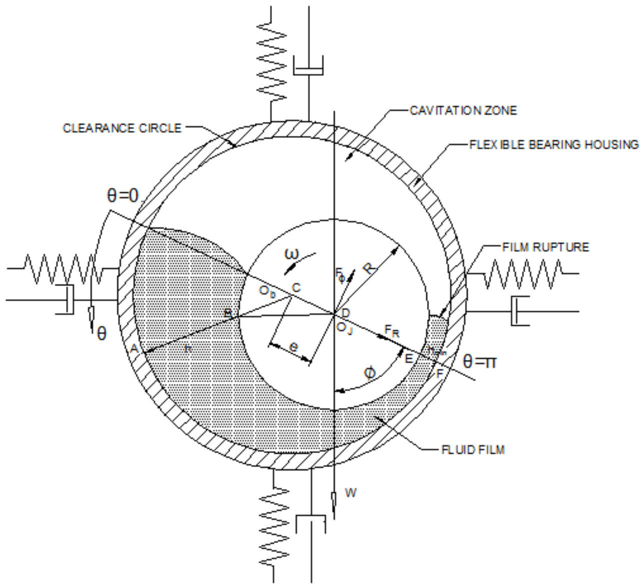

Fig.1 The schematic diagram of flexibly supported oil Journal Bearing

$$R^*_e\left[\Omega\frac{\partial\bar u}{\partial\tau}+\bar u\frac{\partial\bar u}{\partial\theta}+\bar v\frac{\partial\bar u}{\partial\bar y}+\bar w\left(\frac{D}{L}\right)\frac{\partial\bar u}{\partial\bar z}\right]=-\frac{\partial\bar p}{\partial\theta}+\frac{\partial^2\bar u}{\partial\bar y^2} \quad (1)$$

$$\frac{\partial\bar p}{\partial\bar y}=0 \quad (2)$$

$$R^*_e\left[\Omega\frac{\partial\bar w}{\partial\tau}+\bar u\frac{\partial\bar w}{\partial\theta}+\bar v\frac{\partial\bar w}{\partial\bar y}+\bar w\left(\frac{D}{L}\right)\frac{\partial\bar w}{\partial\bar z}\right]=-\left(\frac{D}{L}\right)\frac{\partial\bar p}{\partial\bar z}+\frac{\partial^2\bar w}{\partial\bar y^2} \quad (3)$$

$$\frac{\partial\bar u}{\partial\theta}+\frac{\partial\bar v}{\partial\bar y}+\left(\frac{D}{L}\right)\frac{\partial\bar w}{\partial\bar z}=0 \quad (4)$$

Where, $\bar z=\frac{z}{L/2}$, $\bar y=\frac{y}{c},\theta=\frac{x}{R}$, $\tau=\omega_p.t$, $\Omega=\frac{\omega_p}{\omega}$,

$\bar u=\frac{u}{\omega R},\bar v=\frac{v}{c\omega}$ $\bar w=\frac{w}{\omega R}$ $\bar p=\frac{pc^2}{\eta\omega R^2}$ and

$R_e^*=R_e.\frac{c}{R}=\frac{\rho\omega c^2}{\eta}$

The variation in the density with time is considered to be negligible. Since there is no variation in pressure across fluid film the second momentum equation is not used.

The fluid film thickness can be given as

$$h=c+e\cos\theta \quad (5)$$

$$\bar h=1+\varepsilon\cos\theta \quad (6)$$

where, $\bar h=\frac{h}{c},\varepsilon=\frac{e}{c}$,

After Constantinescu and Galetuse[3] the velocity components are approximated by the parabolic profiles. The velocity components may be expressed in non-dimensional form as follows:

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$$\bar{u} = \left[\frac{\bar{y}}{\bar{h}} + Q_\theta \left(\frac{\bar{y}^2}{\bar{h}^2} - \frac{\bar{y}}{\bar{h}} \right) \right] \tag{7}$$

$$\bar{w} = \left[Q_z \left(\frac{\bar{y}^2}{\bar{h}^2} - \frac{\bar{y}}{\bar{h}} \right) \right] \tag{8}$$

Q_θ and Q_z are dimensionless flow parameter in θ and $\bar{z}$ direction respectively.
Substituting these two into momentum equations and integrating give

$$Q_\theta = \frac{\bar{h}^2}{2} \left(\frac{\partial \bar{p}}{\partial \theta} \right) + R_e^* \times I_X \tag{9}$$

$$Q_z = \frac{\bar{h}^2}{2} \left(\frac{D}{L} \right) \left(\frac{\partial \bar{p}}{\partial \bar{z}} \right) + R_e^* \times I_Z \tag{10}$$

Where,

$$I_X = \frac{\bar{h}}{2} \left[\begin{array}{l} \frac{1}{2}\Omega \left(1 - \frac{1}{3} Q_\theta \right) \frac{\partial \bar{h}}{\partial \tau} - \frac{1}{6} \Omega \bar{h} \frac{\partial Q_\theta}{\partial \tau} - \frac{1}{3} \left(1 - \frac{1}{2} Q_\theta + \frac{1}{10} Q_\theta^2 \right) \frac{\partial \bar{h}}{\partial \theta} + \\ \frac{1}{3} \bar{h} \left(\frac{1}{5} Q_\theta - \frac{1}{2} \right) \frac{\partial Q_\theta}{\partial \theta} + \frac{1}{30} \left(\frac{D}{L} \right) \bar{h}\, Q_z \frac{\partial Q_\theta}{\partial \bar{z}} + \frac{1}{6} \left(\frac{D}{L} \right) \bar{h} \left(\frac{1}{5} Q_\theta - \frac{1}{2} \right) \frac{\partial Q_z}{\partial \bar{z}} \end{array} \right] \tag{11}$$

$$I_Z = \frac{\bar{h}}{2} \left[\begin{array}{l} -\frac{1}{6} \Omega\, Q_z \frac{\partial \bar{h}}{\partial \tau} - \frac{1}{6} \Omega \bar{h} \frac{\partial Q_z}{\partial \tau} - \frac{1}{6} Q_z \left(\frac{1}{5} Q_\theta - \frac{1}{2} \right) \frac{\partial \bar{h}}{\partial \theta} - \\ \frac{1}{6} \bar{h} \left(\frac{1}{5} Q_\theta - \frac{1}{2} \right) \frac{\partial Q_z}{\partial \theta} + \frac{1}{30} \bar{h} Q_z \frac{\partial Q_\theta}{\partial \theta} + \frac{1}{15} \left(\frac{D}{L} \right) \bar{h}\, Q_z \frac{\partial Q_z}{\partial \bar{z}} \end{array} \right] \tag{12}$$

From continuity equation one can obtain the following form of modified Reynold's equation in rotating coordinate system considering fluid inertia effect.

$$\frac{\partial}{\partial \theta} \left(\bar{h}^3 \frac{\partial \bar{p}}{\partial \theta} \right) + \left(\frac{D}{L} \right)^2 \frac{\partial}{\partial \bar{z}} \left(\bar{h}^3 \frac{\partial \bar{p}}{\partial \bar{z}} \right) = 12\ \Omega \frac{\partial \bar{h}}{\partial \tau} + \tag{13}$$

$$6 \left(1.0 - 2.0\ \Omega \frac{\partial \phi}{\partial \tau} \right) \frac{\partial \bar{h}}{\partial \theta} - 2 \times R_e^* \times \left[\frac{\partial}{\partial \theta} \left(\bar{h} \times I_X \right) + \left(\frac{D}{L} \right) \frac{\partial}{\partial \bar{z}} \left(\bar{h} \times I_z \right) \right]$$

B. Considering surface roughness effect

It has been reported by many researcher that the surface roughness patterns significantly influence the steady state and dynamic characterises of hydrodynamic bearings. Consider two real surfaces with normal film gap h in the sliding motion. Local film thickness h_T is defined to be of the form

$$h_T = h + \delta_1 + \delta_2 \tag{14}$$

Where h is the normal film thickness (compliance) defined as the distance between levels of the two surfaces. δ_1 and δ_2 are the random roughness amplitudes of the two surfaces measured from their mean levels.

We assume δ_1 and δ_2 have a Gaussian distribution of heights with zero mean and standard deviations σ_1 and σ_2 respectively.

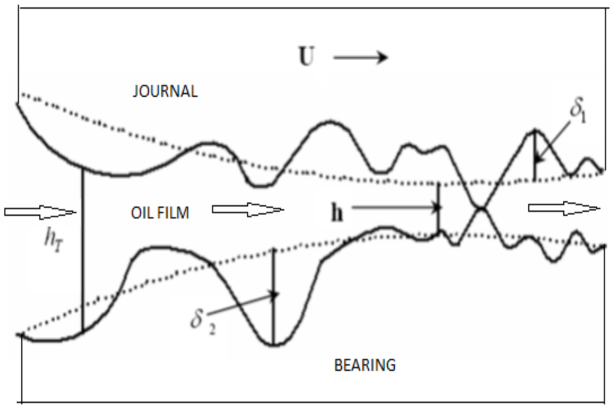

Fig. 2. Two rough surfaces in relative motion

The combined roughness $\delta = \delta_1 + \delta_2$ (15)

and has a variance $\sigma^2 = \sigma_1^2 + \sigma_2^2$ (16)

The ratio of h/σ is an important parameter showing the effects of surface roughness.
To study surfaces with directional properties the surface characteristic γ can be used. The parameter γ can be viewed as the length to width ratio of a representative asperity. There are mainly three sets of asperity patterns are identified purely

1. Transverse roughness pattern γ <1
2. Isotropic roughness pattern γ =1
3. Longitudinal roughness pattern γ >1

Considering the bearing and journal surface are rough surface having random roughness amplitudes of the two surfaces h_T can be written as

$$h_T = \int_{-h}^{\infty} (h + \delta) f(\delta) d\delta \tag{17}$$

Where $f(\delta)$ is the probability density function of composite roughness. Where δ_1 *and* δ_2 are the random roughness amplitudes of the two surfaces measured from their mean levels. σ_1 *and* σ_2 are the standard deviations.
For a Gaussian distribution, the normal probability function of δ is

$$f(\delta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\delta^2}{2\sigma^2}} \tag{18}$$

From equation (17) and (18) we have

$$h_T = \frac{1}{\sigma\sqrt{2\pi}} \int_{-h}^{\infty} (h + \delta) e^{-\frac{\delta^2}{2\sigma^2}} d\delta \tag{19}$$

After integration we have

$$\bar{h}_T = \frac{\bar{h}}{2} \left[1 + erf \left(\frac{\Lambda\, \bar{h}}{\sqrt{2}} \right) \right] + \frac{\bar{\sigma}}{\sqrt{2\pi}} e^{-\bar{h}^2 / 2\bar{\sigma}^2} \tag{20}$$

Where, $\bar{\sigma} = \frac{\sigma}{c}$ or $\Lambda = \frac{c}{\sigma} = \frac{1}{\bar{\sigma}}$.

Where, Λ is called surface roughness parameter.

$$\bar{h}_T = \frac{\bar{h}}{2}\left[1+erf\left(\frac{\Lambda\,\bar{h}}{\sqrt{2}}\right)\right] + \frac{1}{\Lambda\sqrt{2\pi}}\, e^{-0.5*\left(\Lambda\,\bar{h}\right)^2} \qquad (21)$$

Differentiating $\bar{h}_T$ with respect to $x, \bar{z}\, and\ \tau$, we get

$$\frac{\partial \bar{h}_T}{\partial \theta} = \frac{1}{2}\left[1+erf\left(\frac{\Lambda\,\bar{h}}{\sqrt{2}}\right)\right]\frac{\partial \bar{h}}{\partial \theta} = \frac{1}{2}\left[1+erf\left(\frac{\Lambda\,\bar{h}}{\sqrt{2}}\right)\right]\times\left(-\varepsilon\sin\theta\right) \qquad (22)$$

$$\frac{\partial \bar{h}_T}{\partial \tau} = \frac{1}{2}\left[1+erf\left(\frac{\Lambda\,\bar{h}}{\sqrt{2}}\right)\right]\frac{\partial \bar{h}}{\partial \tau} = \frac{1}{2}\left[1+erf\left(\frac{\Lambda\,\bar{h}}{\sqrt{2}}\right)\right]\times\frac{\partial \varepsilon}{\partial \tau}\cos\theta \qquad (23)$$

$$\frac{\partial \bar{h}_T}{\partial \bar{z}} = \frac{1}{2}\left[1+erf\left(\frac{\Lambda\,\bar{h}}{\sqrt{2}}\right)\right]\frac{\partial \bar{h}}{\partial \bar{z}} = 0 \qquad (24)$$

As $\dfrac{\partial \bar{h}}{\partial \bar{z}} = 0$

C. Pressure Flow Factors

Patir and Cheng [1] and [2] introduced pressure flow factors ϕ_x and ϕ_z in circumferential and axial direction are obtained through numerical simulation. The pressure flow simulation factors are given by the empirical relation of the form:

$$\phi_x = 1 - Ce^{-rH} \quad for\ \gamma \le 1 \qquad (25)$$

$$\phi_x = 1 + CH^{-r} \quad for\ \gamma > 1 \qquad (26)$$

Where $H = \dfrac{h}{\sigma}$.The constants C and r are given as A functions of γ in Table.1

ϕ_z is equal to ϕ_x value corresponding to the directional properties of the z profile. In functional form it is given as:

$$\phi_z\left(\frac{h}{\sigma},\gamma\right) = \phi_x\left(\frac{h}{\sigma},\frac{1}{\gamma}\right) \qquad (27)$$

Table 1. Coefficients of equations (25), (26) for ϕ_x

γ	C	r	Range
1/9	1.48	0.42	H > 1
1/6	1.38	0.42	H > 1
1/3	1.18	0.42	H > 0.75
1	0.90	0.56	H > 0.5
3	0.225	1.5	H > 0.5
6	0.520	1.5	H > 0.5
9	0.870	1.5	H > 0.5

D. Shear Flow Factors

Similar to the pressure flow factors, the shear flow factor is a function of the film thickness and roughness parameters only. However, unlike ϕ_x which only depends on the statistics of the combined roughness δ, and the shear flow factors depends on the statistical parameter of $\delta_1\ and\ \delta_2$ separately .Therefore, ϕ_s is a function of h/σ, the standard deviations $\sigma_1\ and\ \sigma_2$ and the surface pattern parameters γ_1 and γ_2 of the two opposing surfaces. Through numerical experimentation, ϕ_s is found to depend on these parameters through the functional form:

$$\phi_s = V_{r1}\Phi_s\left(\frac{h}{\sigma},\gamma_1\right) - V_{r2}\Phi_s\left(\frac{h}{\sigma},\gamma_2\right) \qquad (28)$$

Where V_{r1} and V_{r2} are the variance ratios given by:

$$V_{r1} = \left(\frac{\sigma_1}{\sigma}\right)^2,\ V_{r2} = \left(\frac{\sigma_2}{\sigma}\right)^2 = 1 - V_{r1} \qquad (29)$$

Φ_s is a positive function of h/σ and the surface pattern parameter of the given surface.

The shear flow factor Φ_s is plotted as a function of h/σ and γ in [1, 2].starting with zero for purely longitudinal roughness $(\gamma = \infty)$, the shear flow factor increases with decreasing γ ,and retains highest value for purely transverse roughness $(\gamma = 0)$ .Through numerical simulation and using nonlinear least square program they are of the form:

$$\Phi_s = A_1 H^{\alpha_1} e^{-\alpha_2 H + \alpha_3 H^2} \quad H \le 5 \qquad (30)$$

Where H=h/σ.For extrapolation beyond H=5 the following relation should be used:

$$\Phi_s = A_2 e^{-0.25H} \quad H > 5 \qquad (31)$$

The coefficients $A_1, A_2, \alpha_1, \alpha_2, \alpha_3$ are listed as functions of γ in Table 2.

Table 2: Coefficients of equations (26),(27) for Φ_s (range $H > 0.5$)

γ	A_1	α_1	α_2	α_3	A_2
1/9	2.046	1.12	0.78	0.03	1.856
1/6	1.962	1.08	0.77	0.03	1.754
1/3	1.858	1.01	0.76	0.03	1. 561
1	1.899	0.98	0.92	0.05	1.126
3	1.560	0.85	1.13	0.08	0.556
6	1.290	0.62	1.09	0.08	0.388
9	1.011	0.54	1.07	0.08	0.295

Now introducing pressure flow factors $\phi_x\ and\ \phi_z$ with shear flow factors ϕ_s we get modified Reynolds's equations considering combined effect of fluid inertia and surface roughness in dimensionless form as:

$$\frac{\partial}{\partial\theta}\left(\phi_X\,\bar{h}^3\frac{\partial \bar{p}}{\partial\theta}\right) + \left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(\phi_z\,\bar{h}^3\frac{\partial \bar{p}}{\partial\bar{z}}\right) = 12\,\Omega\,\frac{\partial \bar{h}}{\partial\tau} + \qquad (32)$$

$$6\left(1.0 - 2.0\,\Omega\frac{\partial\phi}{\partial\tau}\right)\frac{\partial \bar{h}}{\partial\theta} + 6\,\bar{\sigma}\frac{\partial\phi_s}{\partial\theta} - 2\times R_e^{*}\times\left[\frac{\partial}{\partial\theta}\left(\bar{h}_T\times I_X\right) + \left(\frac{D}{L}\right)\frac{\partial}{\partial\bar{z}}\left(\bar{h}_T\times I_z\right)\right]$$

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Or,

$$\frac{\partial}{\partial\theta}\left(\phi_x \bar{h}^3 \frac{\partial \bar{p}}{\partial\theta}\right)+\left(\frac{D}{L}\right)^2 \frac{\partial}{\partial \bar{z}}\left(\phi_z \bar{h}^3 \frac{\partial \bar{p}}{\partial \bar{z}}\right)=12\,\Omega\times\frac{1}{2}\left[1+erf\left(\frac{\Lambda \bar{h}}{\sqrt{2}}\right)\right]\times\frac{\partial\varepsilon}{\partial\tau}\cos\theta$$

$$+\,6\left(1.0-2.0\,\Omega\frac{\partial\phi}{\partial\tau}\right)\times\frac{1}{2}\left[1+erf\left(\frac{\Lambda \bar{h}}{\sqrt{2}}\right)\right]\times(-\varepsilon\sin\theta)\,+$$

$$6\,\bar{\sigma}\frac{\partial\phi_s}{\partial\theta}-2\times R_e^*\times\left[\frac{\partial}{\partial\theta}\left(\bar{h}_T\times I_x\right)+\left(\frac{D}{L}\right)\frac{\partial}{\partial\bar{z}}\left(\bar{h}_T\times I_z\right)\right] \quad (33)$$

Where, I_x and I_z are same as equation (11) and (12) above,

Boundary conditions for equation (33) are as follows

1. The pressure at the ends of the bearing is assumed to be zero (ambient):

$$\bar{p}\,(\theta,\pm 1)=0$$

2. The pressure distribution is symmetrical about the mid-plane of the bearing:

$$\frac{\partial\bar{p}}{\partial\bar{z}}(\theta,0)=0$$

3. Cavitation boundary condition is given by:

$$\frac{\partial\bar{p}}{\partial\theta}\left(\theta_2,\bar{z}\right)=0 \;\; and \;\; \bar{p}\left(\theta,\bar{z}\right)=0 \;\; for \;\; \theta_1\geq\theta\geq\theta_2$$

The equations (9), (10), (11), (12) and (33) are first expressed in finite difference form and solved simultaneously using Gauss-Siedel method in a finite difference scheme.

III METHOD OF SOLUTION

To find out steady-state pressure all the time derivatives are set equal to zero in Equations. (9), (10), (11), (12) and (33). For $(\varepsilon_0 \leq 0.2)$ the pressure distribution and flow parameters Q_θ and Q_z are evaluated from inertia less ($\mathrm{Re}^*=0$) solution, i.e., solving classical Reynold's equation. These values are then used as initial value of flow parameters to solve Eqs.(9) and (10) simultaneously for Q_θ and Q_z Using Guss-Siedel method in a finite difference scheme. Then update I_x & I_z and then calculate Q_θ and Q_z for use to solve Eq.(33) with particular surface roughness pattern (γ) and surface roughness parameter (Λ) for new pressure p with inertia effect by using a successive over relaxation scheme. The latest values of Q_θ, Q_z and p are used iteratively to solve the set of equations until all variables converges. The convergence criterion adopted for pressure is $\left|\left(1-\left(\sum\bar{p}_{new}/\sum\bar{p}_{old}\right)\right)\right|\leq 10^{-5}$ and also same criterion for Q_θ and Q_z. For higher eccentricity ratios $(\varepsilon_0>0.2)$ the initial values for the variables are taken from the results corresponding to the previous eccentricity ratios. Very small increment in ε is to be provided as Re^* increases. The procedure converges up to a value of $\mathrm{Re}^*=1.5$ which should be good enough for the present study. Since the bearing is symmetrical about its central plane ($\bar{z}$ =0),only one half of the bearing needs to be considered for the analysis.

A. *Fluid film forces*

The non-dimensional fluid film forces along line of centers and perpendicular to the line of centers are given by

$$\bar{F}_r\left(=\frac{F_r C^2}{\eta\omega R^3 L}\right)=\int_0^1\int_{\theta_1}^{\theta_2}\bar{p}\cos\theta\,d\theta\,d\bar{z} \quad (34)$$

$$\bar{F}_\phi\left(=\frac{F_\phi C^2}{\eta\omega R^3 L}\right)=\int_0^1\int_{\theta_1}^{\theta_2}\bar{p}\sin d\theta\,d\bar{z} \quad (35)$$

where θ_1 and θ_2 are angular coordinates at which the fluid film commences and cavitates respectively.

B. *Steady state load*

The steady state non-dimensional load and attitude angle are given by

$$\bar{W}_0=\sqrt{\left(\bar{F}_{r_0}{}^2+\bar{F}_{\phi_0}{}^2\right)} \quad (36)$$

$$\phi_o=\tan^{-1}\left(\frac{-\bar{F}_{\phi_0}}{\bar{F}_{r_0}}\right) \quad (37)$$

Since the steady state film pressure distribution has been obtained at all the mesh points, integration of equations (34) and (35) can be easily performed numerically by using Simpson's 1/ 3 rd. rule to get $\bar{F}_r$ and $\bar{F}_\phi$. The steady state load $\bar{W}_0$ and the attitude angle (ϕ_0) are calculated using equations (36) and (37).

C. *Equation of Motion*

The equation of motion for a rigid rotor supported on four identical flexibly supported bearings are given by,

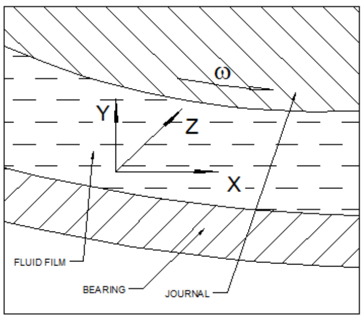

Fig. 3: Coordinate system in oil journal bearing arrangement

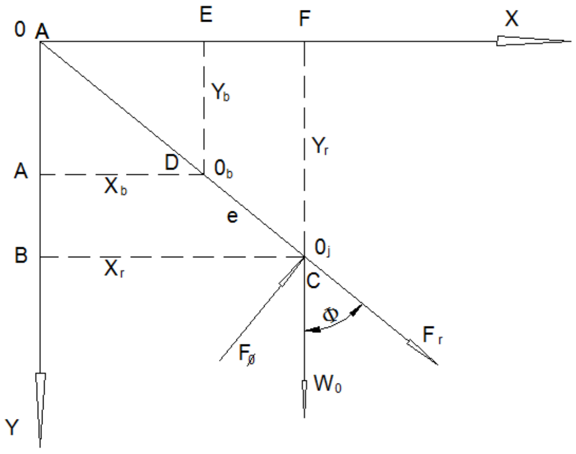

Fig. 4: Hydrodynamic fluid film forces in circumferential & radial direction

$$M_r.\frac{d^2X_r}{dt^2} = F_r\,Sin\phi + F_\phi\,Cos\phi \quad (38)$$

$$M_r.\frac{d^2Y_r}{dt^2} = F_r\,Cos\phi - F_\phi Sin\phi + W_0 \quad (39)$$

$$M_b.\frac{d^2X_b}{dt^2} = -F_\phi\,Cos\phi - F_r Sin\phi - B.\frac{dX_b}{dt} - KX_b \quad (40)$$

$$M_b.\frac{d^2Y_b}{dt^2} = F_\phi\,Sin\phi - F_r Cos\phi - B.\frac{dY_b}{dt} - KY_b \quad (41)$$

The relation between rotor & bearing motion are given by,

$$X_r = X_b + e\,Sin\phi \quad (42)$$

$$Y_r = Y_b + e\,Cos\phi \quad (43)$$

The above two equations are substituted in equations of motion. Finally the equations of motion are expressed in non-dimensional form as follows,

$$\dot{\bar{X}}_b = \frac{d\bar{X}_b}{d\tau} \quad (44)$$

$$\dot{\bar{Y}}_b = \frac{d\bar{Y}_b}{d\tau} \quad (45)$$

$$\ddot{\bar{X}}_b = \frac{1}{m.\bar{M}.\bar{W}_0.\Omega^2}\begin{bmatrix} -\bar{F}_\phi Cos\phi - \bar{F}_r Sin\phi \\ -\Omega.\bar{W}_0.\bar{B}.\dot{\bar{X}}_b - \bar{W}_0.\bar{K}.\bar{X}_b \end{bmatrix} \quad (46)$$

$$\ddot{\bar{Y}}_b = \frac{1}{m.\bar{M}.\bar{W}_0.\Omega^2}\begin{bmatrix} \bar{F}_\phi Sin\phi - \bar{F}_r Cos\phi \\ -\Omega.\bar{W}_0.\bar{B}.\dot{\bar{Y}}_b - \bar{W}_0.\bar{K}.\bar{Y}_b \end{bmatrix} \quad (47)$$

$$\dot{\varepsilon} = \frac{d\varepsilon}{d\tau} \quad (48)$$

$$\dot{\phi} = \frac{d\phi}{d\tau} \quad (49)$$

$$\ddot{\varepsilon} = \frac{A_3.F - A_4.E}{A_2.A_3 - A_1.A_4} \quad (50)$$

$$\ddot{\phi} = \frac{A_2.E - A_1.F}{A_2.A_3 - A_1.A_4} \quad (51)$$

where,

$$C = -2\dot{\varepsilon}.\dot{\phi}Cos\phi + \varepsilon.Sin\phi.\left(\dot{\phi}\right)^2 + \frac{1}{\bar{M}.\bar{W}_0.\Omega^2}\left[\bar{F}_r Sin\phi + \bar{F}_\phi Cos\phi\right]$$

$$D = 2\dot{\varepsilon}.\dot{\phi}.Sin\phi + \varepsilon.Cos\phi.\left(\dot{\phi}\right)^2 + \frac{1}{\bar{M}.\bar{W}_0.\Omega^2}\left[\bar{F}_r Cos\phi - \bar{F}_\phi Sin\phi + \bar{W}_0\right]$$

$$A_1 = Sin\phi,\ A_2 = Cos\phi,\ A_3 = \varepsilon.Cos\phi,\ A_4 = -\varepsilon.Sin\phi$$

$$G = \ddot{\bar{X}}_b$$

$$H = \ddot{\bar{Y}}_b$$

$$E = C - G$$

$$F = D - H$$

D. *Solution Scheme*:

For stability analysis, a non-linear time transient analysis is carried out using the equations of motion to compute a new set of $\varepsilon, \phi, X_b, Y_b$ & their derivatives for the next time step for a given set of. $Re^*, \gamma, L/D, \varepsilon_0, \bar{M}$ (Mass parameter) for a particular roughness parameter, Λ. The forth order Runge-Kutta method is used for solving the equations of motion. The hydrodynamic forces are computed for every time step by solving the partial differential equation for pressure satisfying the boundary conditions.

E. *Stability Analysis*

To study the combined effect of fluid inertia and surface roughness on journal centre trajectory of flexibly supported bearings a set of trajectories of journal centre and bearing has been studied and it is possible to construct the trajectories for numbers of complete revolution of the journal the plots shows the stability of the journal when the trajectory of journal and bearing centre ends in a limit cycle. Critical mass parameter for a particular eccentricity ratio, slenderness ratio, modified Reynold's number, surface roughness parameter and roughness pattern is found when the trajectories ends with limit cycle (Fig. 9& Fig.10) or it changes its trend from stable to unstable.

IV RESULTS AND DISCUSSIONS

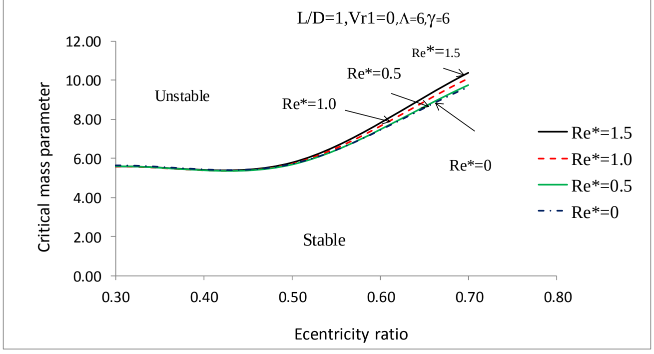

Figure 5.Variation of Critical mass parameter with ecentricity ratio including fluid inertia and surface roughness effect and different modified Reynolds number

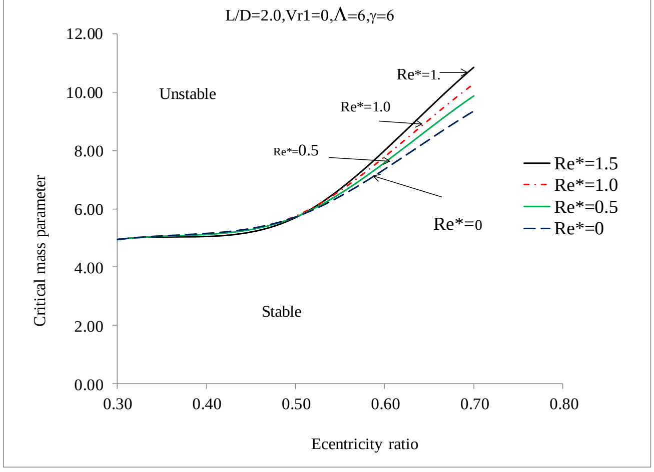

Figure 6.Variation of Critical mass parameter with ecentricity ratio including fluid inertia and surface roughness effect for difeeernt modified Reyolds number

A *Effect of Modified Reynold's Number (Re*)with eccentricity ratio*

Figure 5 and 6 shows the variations of stability at different values of Re*(0.5,1.0 and 1.5) and L/D (1.0 and 2.0).From the figure smaller L/D ratio gives better stability in the case of inertialess solution for all eccentricity ratios.

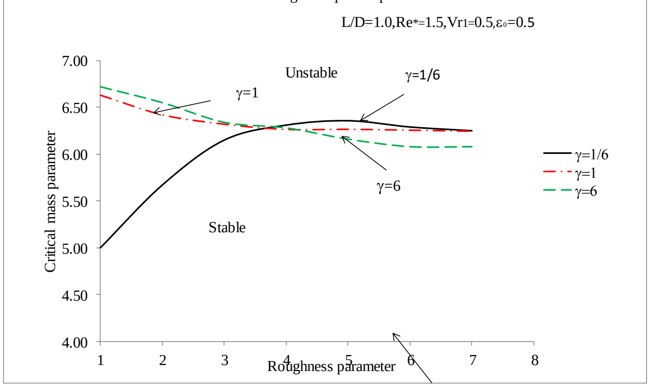

Figure 7.Variation of Critical mass parameter with surface roughness parameter for different roughness pattern parameter

B *Effect of Roughness Pattern (γ) with Surface roughness Parameters*

Figure 7 shows the variation of mass parameter with surface roughness parameter for various roughness pattern parameters.It is seen that stability is better for transversely oriented roughness pattern.

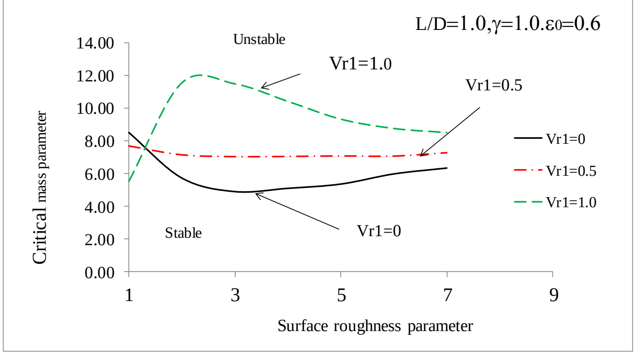

Figure 8.Variation of critical mass parameter with surface roughness parameter for different variance ratio.

C *Effect of variance ratio (Vr) with Surface roughness Parameter*

Figure 8 shows the variation of mass parameter with surface roughness parameter for different variance ratio.when the journal surface is rough and the bearing surface is smooth $(Vr_1 = 1.0)$,the stability is seen to decrease sharply for small values of Λ $(\Lambda < 3)$.On the other hand, when the journal surface is smooth and the bearing surface is rough (i.e., $Vr_1 = 0$),the bearing is highly stable for small roughness parameter $(\Lambda < 3)$.A bearing having identical roughness structure (i.e.,$Vr_1 = 0.5$) gives intermediate values of stability.

Figure 9.Journal centre trajectory of flexibly supported oil journal bearing considering fluid film inertia and surface roughness effect.

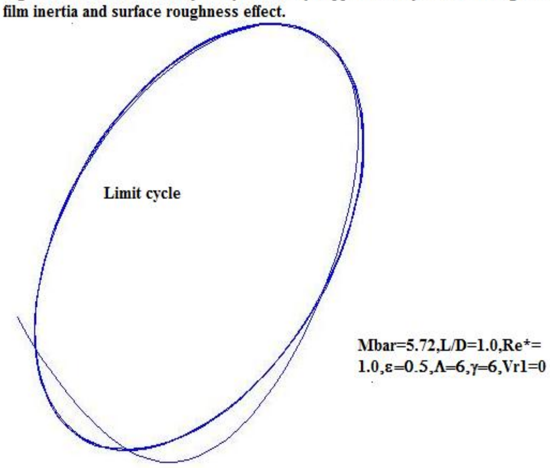

Fig. 9. Journal centre Trajectory of flexibly supported finite oil journal bearing with fluid inertia and surface roughness effect.

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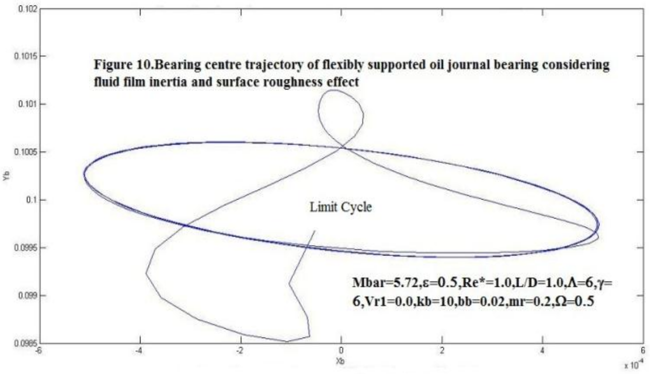

Fig. 10.Bearing centre Trajectory of flexibly supported finite oil journal bearing with fluid inertia and surface roughness effect

V CONCLUSIONS

1.The effect of inertia on the stability is affected considerably at higher L/D ratios and eccentricity ratios (Figure 5 and 6).The probable reason may be that higher L/D ratios and eccentricity ratios,the circumferential component of flow will be overtaking the axial flow.The inertia effect of the circumferential flow will possibly add more stiffness in the film,thereby improving the stability.It is also noted that higher Re* means higher surface speed of the shaft (when other parameter remain constant).This will further increase couette flow which is a part of circumferential flow.One can see this particular effect for L/D=2.0 and for $\varepsilon \geq 0.5$(Figure 6).

2. When the journal surface is very rough $(\Lambda < 3)$ and the bearing is smooth,the stability decreases drastically.

3. When the bearing surface is very rough $(\Lambda < 3)$ and the journal is smooth,the stability improves significantly.

4.The stability can be improved by employing higher L/D ratio.

NOMENCLATURE

c = Radial clearance (m)

D = Diameter of Journal (m)

e = eccentricity (m)

F_ε, F_ϕ = Hydrodynamic forces (N) ,

$\bar{F}_r$, $\bar{F}_\phi$ = Dimensionless Hydrodynamic film forces

$\bar{F}_\varepsilon\left(=\dfrac{F_r C^2}{\eta\omega R^3 L}\right)$, $\bar{F}_\phi\left(=\dfrac{F_\phi C^2}{\eta\omega R^3 L}\right)$

h = Film thickness, (m)

$\bar{h}$ = Dimensionless film thickness, h/c

L = Length of the bearing in m

p = Film pressure in Pa

$\bar{p}$ = dimensionless film pressure $\left(\dfrac{pc^2}{\eta\omega R^2}\right)$

R = radius of journal in m

R_e = Reynolds number, $\dfrac{\rho c R\omega}{\eta}$

R^*_e = Modified Reynolds number, $\left(\dfrac{c}{R}\right)R_e$

t = time in s

$u, v\ w$ = velocity components in x, y, z directions in m/s

$\bar{u},\bar{v},\bar{w}$ = dimensionless velocity components

$x, y\ z$ = coordinates

$\theta,\bar{Y},\bar{Z}$ = Dimensionless coordinates, $\dfrac{x}{R},\dfrac{y}{c},\dfrac{z}{L/2}$

$\bar{V}$ = $\dfrac{\partial h}{\partial\theta}+\Omega\left(\dfrac{d\varepsilon}{d\tau}\cos\theta+\varepsilon\dfrac{d\phi}{d\tau}\sin\theta\right)$

W_0 = steady-state load bearing capacity in N

$\bar{W}_0$ = Dimensionless steady-state load $\dfrac{W_0 c^2}{\eta\omega R^3 L}$

$\varepsilon,\varepsilon_0$ = Eccentricity ratio e/c (dimensionless), steady-state eccentricity ratio

ρ = Density of the lubricant (kg m^{-3})

ω = Angular velocity of journal(rad s^{-1})

ω_p = Angular velocity of whirl (rad s^{-1})

Ω = Whirl ratio, $\dfrac{\omega_p}{\omega}$

η = Absolute viscosity of lubricating film (N s m^{-1})

ϕ = Attitude angle

Q_θ = Dimensionless flow parameter in θ direction

Q_z = Dimensionless flow parameter in $\bar{z}$ direction

$\bar{Q}$ = Dimensionless side leakage

θ_1, θ_2 = Angular coordinates at which film commences and cavitates.

γ = Surface pattern parameter

Λ = Roughness Parameter, $\Lambda = c/\sigma$

H = h/σ

ϕ_x , ϕ_s = Pressure flow factors

Φ_s = Shear flow factor

σ = Composite r.m.s roughness,σ= $\sqrt{\sigma_1{}^2+\sigma_2{}^2}$

σ_1 , σ_2 = Standard deviation of δ_1 and δ_2

δ_1 , δ_2 = Random roughness amplitudes (heights) of surfaces.

δ = Combined roughness [m], $\delta = \delta_1 + \delta_2$

Vr = Variance ratio, $Vr1=\left(\sigma_1/\sigma\right)^2, Vr2=\left(\sigma_2/\sigma\right)^2$

X_b = Coordinate of bearing centre in x-direction

Y_b = Coordinate of bearing centre in y-direction

X_r = Coordinate of rotor centre in x-direction

Y_r = Coordinate of rotor centre in y-direction

M_r = Mass of rotor or journal

M_b = Mass of bearing

$m=\dfrac{M_b}{M_r}$ Mass ratio

$\bar{M}=\dfrac{M_r.c.\omega^2}{W_O}$ Critical Mass Parameter

$\bar{K}=kb$ Bearing support stiffness coefficient

$\bar{B}=bb$ Bearing support damping coefficient

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