Stability of Equilibrium Points in the Generalised Photogravitational Restricted Three Body Problem When It Is Coplanar

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Abstract: The Restricted Three Body Problem Is Generalised To Include The Effects Of An Inverse Square Distance Radiation Pressure Force On The Infinitesimal Mass Due To The Primaries, Which Are Both Radiating. In This Paper We Investigate The Stability Of Coplanar Equilibrium Points, Based On Equations In Variations. We Have Found The Characteristic Equation For The Complex Normal Frequencies Which Is A Sixth Order Polynomial .Thus We Conclude That Coplanar Equilibrium Points Are Unstable Due To Positive Real Part In Complex Roots.

Keywords: Stability; coplanar points; generalised; photogravitational; RTBP

I. INTRODUCTION

Radzievskii (1950) showed that in the restricted photogravitational three body problem, allowing for the gravitational attraction and light pressure of primaries, coplanar equilibrium points (L₆, L₇) exist in addition to three collinear and two triangular ones. Chernikov (1970) described the photogravitational restricted three body problem. Perezhogin (1976) discussed the stability of coplanar equilibrium points in the absence of a repulsive force from the smaller of the primaries. An investigation of the stability of collinear and triangular solutions in this problem was made by Kunitsyn and Tureshbaev (1983), (1985). A.T. Tureshbaev (1986) investigated the stability of the relative equilibrium positions (coplanar libration points) for a particle in a gas-dust cloud subject to the gravitational field and radiation pressure of a binary star. Luk’yanov (1987) obtained regions of stability for libration points L₆ and L₇ for any values of three parameters (μ, q₁, q₂). Perezhogin and Tureshbaev (1989) showed that stability for the majority of initial conditions and formal stability occur almost everywhere in the domain of first order stability of coplanar libration points.


Hence, we thought to examine the stability of equilibrium points L₆, L₇ in the generalised photogravitational coplanar restricted three body problem. Both the primaries are radiating and smaller primary is supposed to be an oblate spheroid.

We linearise the equations of motion. We have found the characteristic equation. The partial derivatives are evaluated at the equilibrium point L₆. We have found the roots of characteristic equation. We conclude that due to positive real part in complex roots, the out-of-plane equilibrium points are unstable.

II. STABILITY OF EQUILIBRIUM POINTS

The equations of motion (1) of the infinitesimal mass are given by Douskos and Markellos (2006).

\[ \dot{x} = 2\Omega y + \frac{\dot{\Omega}}{\Omega} \]
\[ \dot{y} = 2\Omega x + \frac{\dot{\Omega}}{\Omega} \]
\[ \dot{z} = \frac{\Omega}{iz} \]

where

\[ \Omega = \frac{1}{2} n^2 (x^2 + y^2) + \left( \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \right) \left( 1 + \frac{A_2}{r_2^2} = \frac{3z^2}{2r_2^2} \right) \]

\[ d = n^2 = 1 + \frac{3}{2} A_2 \]

\[ r_1^2 = (x + \mu)^2 + y^2 + z^2 \]
\[ r_2^2 = (x + \mu - 1)^2 + y^2 + z^2 \]
\[ x_0 = (1 - \mu) + \frac{6\sqrt{3}(1 - \mu)(1 - q_1)}{\mu q_2} A^2 / 2 \]
\[ z_0 = \sqrt{3} \sqrt{A_2} - \frac{9\sqrt{3}q_1 A^2}{\mu q_2} \]

\( A_2 \) is the oblateness co-efficient of the smaller primary
\( n \) is the mean motion
\( q_1 \) is the radiation co-efficient of the bigger primary
\( q_2 \) is the radiation co-efficient of the smaller primary
\( \mu = \frac{m_2}{m_1 + m_2} \)

The equilibrium point \( L_6 \) is given by Ishwar et al. (2010) (using Mathematica) and we transfer the origin to equilibrium point \((x_0, z_0)\) for examining the linear stability of the out-of-plane point \( L_6 \).

We linearise the equations of motion (1). We obtain
\[
\begin{align*}
\dot{x} &= 2n^2 \Omega_{2x} x + \Omega_{2y} \Omega_{2z} \\
\dot{y} &= 2n^2 \Omega_{2y} y + \Omega_{2x} \Omega_{2z} \\
\dot{z} &= \Omega_{2z} (x + y) \\
\end{align*}
\]

(2)

where the partial derivatives are evaluated at the equilibrium point and \( \Omega_{2x} = \Omega_{2z} \).

The characteristic equation is given by
\[
\begin{align*}
\lambda^6 + a\lambda^4 + b\lambda^2 + c &= 0 \\
\end{align*}
\]

\[ \begin{align*}
\Omega_{2x} &= \Omega_{2y} + \Omega_{2z} \\
b &= \Omega_{2x} \Omega_{2y} + \Omega_{2x} \Omega_{2z} + \Omega_{2y} \Omega_{2z} - (\Omega_{2y})^2 - 4n^2 (\Omega_{2z})^2 \\
c &= \Omega_{2y} (\Omega_{2z})^2 - \Omega_{2x} \Omega_{2z} \\
\end{align*}
\]

The values of co-efficients \( a, b \) and \( c \) of equation (3) are (using Mathematica)
\[ a = 2n^2 = 2 + 3A_2 \]
Substituting in characteristic equation (3), we find six roots (using Mathematica).

Solution of equation $\lambda^6 + a\lambda^4 + b\lambda^2 + c = 0$ is given (using Mathematica)

$$\lambda_1 \rightarrow \left\{ \begin{array}{l}
\left( \frac{a}{3} \right)^{1/2} \\
\left( -2a^3 + 9ab - 27c + 3\sqrt[3]{a^2b^2 + 4b^3 + 4a^3c - 18abc + 27c^2} \right)^{1/3}
\end{array} \right. $$

$$\lambda_3 \rightarrow \left\{ \begin{array}{l}
\left( \frac{a}{3} \right)^{1/2} \\
\left( -2a^3 + 9ab - 27c + 3\sqrt[3]{a^2b^2 + 4b^3 + 4a^3c - 18abc + 27c^2} \right)^{1/3}
\end{array} \right. $$

$$\lambda_2 \rightarrow \left\{ \begin{array}{l}
\left( \frac{a}{3} \right)^{1/2} \\
\left( -2a^3 + 9ab - 27c + 3\sqrt[3]{a^2b^2 + 4b^3 + 4a^3c - 18abc + 27c^2} \right)^{1/3}
\end{array} \right. $$

$$\lambda_4 \rightarrow \left\{ \begin{array}{l}
\left( \frac{a}{3} \right)^{1/2} \\
\left( -2a^3 + 9ab - 27c + 3\sqrt[3]{a^2b^2 + 4b^3 + 4a^3c - 18abc + 27c^2} \right)^{1/3}
\end{array} \right. $$

$$\lambda_5 \rightarrow \left\{ \begin{array}{l}
\left( \frac{a}{3} \right)^{1/2} \\
\left( -2a^3 + 9ab - 27c + 3\sqrt[3]{a^2b^2 + 4b^3 + 4a^3c - 18abc + 27c^2} \right)^{1/3}
\end{array} \right. $$

$$\lambda_6 \rightarrow \left\{ \begin{array}{l}
\left( \frac{a}{3} \right)^{1/2} \\
\left( -2a^3 + 9ab - 27c + 3\sqrt[3]{a^2b^2 + 4b^3 + 4a^3c - 18abc + 27c^2} \right)^{1/3}
\end{array} \right. $$
\[
\lambda_4 \rightarrow \left\{ \begin{array}{c}
\frac{a}{3} \left( 32/3 \left( -2a^3 + 9ab - 27c + 3\sqrt[3]{-a^2b^2 + 4b^3 + 4a^2c - 18abc + 27c^2} \right) \right)^{1/2} \\
\frac{\sqrt{2}}{b} \left( -2a^3 + 9ab - 27c + 3\sqrt[3]{-a^2b^2 + 4b^3 + 4a^2c - 18abc + 27c^2} \right)^{1/2} \\
\frac{1}{\sqrt{3}} \left( -2a^3 + 9ab - 27c + 3\sqrt[3]{-a^2b^2 + 4b^3 + 4a^2c - 18abc + 27c^2} \right)^{1/2} \\
\frac{a}{3} \left( 32/3 \left( -2a^3 + 9ab - 27c + 3\sqrt[3]{-a^2b^2 + 4b^3 + 4a^2c - 18abc + 27c^2} \right) \right)^{1/2} \\
\end{array} \right.
\]

\[
\lambda_5 \rightarrow \left\{ \begin{array}{c}
\frac{a}{3} \left( 32/3 \left( -2a^3 + 9ab - 27c + 3\sqrt[3]{-a^2b^2 + 4b^3 + 4a^2c - 18abc + 27c^2} \right) \right)^{1/2} \\
\frac{\sqrt{2}}{b} \left( -2a^3 + 9ab - 27c + 3\sqrt[3]{-a^2b^2 + 4b^3 + 4a^2c - 18abc + 27c^2} \right)^{1/2} \\
\frac{1}{\sqrt{3}} \left( -2a^3 + 9ab - 27c + 3\sqrt[3]{-a^2b^2 + 4b^3 + 4a^2c - 18abc + 27c^2} \right)^{1/2} \\
\frac{a}{3} \left( 32/3 \left( -2a^3 + 9ab - 27c + 3\sqrt[3]{-a^2b^2 + 4b^3 + 4a^2c - 18abc + 27c^2} \right) \right)^{1/2} \\
\end{array} \right.
\]

\[
\lambda_6 \rightarrow \left\{ \begin{array}{c}
\frac{a}{3} \left( 32/3 \left( -2a^3 + 9ab - 27c + 3\sqrt[3]{-a^2b^2 + 4b^3 + 4a^2c - 18abc + 27c^2} \right) \right)^{1/2} \\
\frac{\sqrt{2}}{b} \left( -2a^3 + 9ab - 27c + 3\sqrt[3]{-a^2b^2 + 4b^3 + 4a^2c - 18abc + 27c^2} \right)^{1/2} \\
\frac{1}{\sqrt{3}} \left( -2a^3 + 9ab - 27c + 3\sqrt[3]{-a^2b^2 + 4b^3 + 4a^2c - 18abc + 27c^2} \right)^{1/2} \\
\frac{a}{3} \left( 32/3 \left( -2a^3 + 9ab - 27c + 3\sqrt[3]{-a^2b^2 + 4b^3 + 4a^2c - 18abc + 27c^2} \right) \right)^{1/2} \\
\end{array} \right.
\]
We may examine the stability of other equilibrium point \( L_7 \) in the same manner as \( L_6 \). We will find that \( L_7 \) is also unstable. Thus we conclude that non-planar equilibrium points are unstable in linear sense due to positive real part in complex roots.

III. CONCLUSION

We conclude that equilibrium points are unstable due to positive real part in complex roots when they are out of plane.

REFERENCES


