Stability of Equilibrium Points in the Generalised Photogravitational Restricted Three Body Problem When It Is Coplanar

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Abstract: **The Restricted Three Body Problem Is Generalised To Include The Effects Of An Inverse Square Distance Radiation Pressure Force On The Infinitesimal Mass Due To The Primaries, Which Are Both Radiating. In This Paper We Investigate The Stability Of Coplanar Equilibrium Points, Based On Equations In Variations. We Have Found The Characteristic Equation For The Complex Normal Frequencies Which Is A Sixth Order Polynomial .Thus We Conclude That Coplanar Equilibrium Points Are Unstable Due To Positive Real Part In Complex Roots.**

Keywords: *Stability; coplanar points; generalised; photogravitational; RTBP*

I. INTRODUCTION

Radzievskii (1950) showed that in the restricted photogravitational three body problem, allowing for the gravitational attraction and light pressure of primaries, coplanar equilibrium points (L_6, L_7) exist in addition to three collinear and two triangular ones. Chernikov(1970) described the photogravitational restricted three body problem. Perezhogin (1976) discussed the stability of coplanar equilibrium points in the absence of a repulsive force from the smaller of the primaries. An investigation of the stability of collinear and triangular solutions in this problem was made by Kunitsyn and Tureshbaev (1983), (1985). A.T. Tureshbaev (1986) investigated the stability of the relative equilibrium positions (coplanar libration points) for a particle in a gas-dust cloud subject to the gravitational field and radiation pressure of a binary star. Luk'yanov (1987) obtained regions of stability for libration points L_6 and L_7 for any values of three parameters (μ, q_1, q_2). Perezhogin and Tureshbaev (1989) showed that stability for the majority of initial conditions and formal stability occur almost everywhere in the domain of first order stability of coplanar libration points.

Sharma, R.K. and Subba Rao, P.V. (1976) discussed the three dimensional restricted three body problem with oblateness. C.N. Douskos and V.V. Markellos (2006) found the existence of non-planar equilibrium points in the three dimensional restricted three body problem with oblateness.

Hence, we thought to examine the stability of equilibrium points L_6, L_7 in the generalised photogravitational coplanar restricted three body problem. Both the primaries are radiating and smaller primary is supposed to be an oblate spheroid.

We linearise the equations of motion. We have found the characteristic equation. The partial derivatives are evaluated at the equilibrium point L_6. We have found the roots of characteristic equation. We conclude that due to positive real part in complex roots, the out-of-plane equilibrium points are unstable.

II. STABILITY OF EQUILIBRIUM POINTS

The equations of motion (1) of the infinitesimal mass are given by Douskos and Markellos(2006).

$$\ddot{x} - 2n\dot{y} = \frac{\partial \Omega}{\partial x}$$

$$\ddot{y} + 2n\dot{x} = \frac{\partial \Omega}{\partial y} \qquad (1)$$

$$\ddot{z} = \frac{\partial \Omega}{\partial z}$$

where

$$\Omega = \frac{1}{2}n^2\left(x^2 + y^2\right) + \frac{(1-\mu)q_1}{r_1} + \frac{\mu q_2}{r_2}\left(1 + \frac{A_2}{2r_2^2} - \frac{3z^2 A_2}{2r_2^4}\right)$$ an

d $\quad n^2 = 1 + \frac{3}{2}A_2$

$$r_1^2 = (x+\mu)^2 + y^2 + z^2$$

$$r_2^2 = (x+\mu-1)^2 + y^2 + z^2$$

$$x_0 = (1-\mu) + \frac{6\sqrt{3}[(1-\mu)(1-q_1)]A_2^{3/2}}{\mu q_2}$$

$$z_0 = \sqrt{3}\sqrt{A_2} - \frac{9(1-\mu)q_1 A_2^2}{\mu q_2}$$

A_2 = oblateness co-efficient of smaller primary

n = Mean motion

q_1 = radiation co-efficient of bigger primary

q_2 = radiation co-efficient of smaller primary

$$\mu = \frac{m_2}{m_1 + m_2}$$

The equilibrium point L_6 is given by Ishwar et.al(2010) (using Mathematica)

We transfer the origin to equilibrium point (x_0, z_0) for examining the linear stability of the out-of-plane point L_6 .

We linearise the equations of motion (1). We obtain

$$\begin{aligned} \ddot{x} - 2n\dot{y} &= \Omega_{xx}.x + \Omega_{xz}.z \\ \ddot{y} + 2n\dot{x} &= \Omega_{yy}.y \\ \ddot{z} &= \Omega_{zx}.x + \Omega_{zz}.z \end{aligned} \qquad (2)$$

where the partial derivatives are evaluated at the equilibrium point and $\Omega_{zx} = \Omega_{xz}$.

The characteristic equation is given by

$$\lambda^6 + (\Omega_{xx} + \Omega_{yy} + \Omega_{zz})\lambda^4 + \left[\Omega_{xx}\Omega_{yy} + \Omega_{xx}\Omega_{zz} + \Omega_{yy}\Omega_{zz} - (\Omega_{xz})^2 - 4n^2\Omega_{zz}\right]\lambda^2 + \Omega_{yy}\left[(\Omega_{xz})^2 - \Omega_{xx}\Omega_{zz}\right] = 0 \qquad (3)$$

i.e. $\lambda^6 + a\lambda^4 + b\lambda^2 + c = 0$

where

$$a = (\Omega_{xx} + \Omega_{yy} + \Omega_{zz})$$

$$b = \left[\Omega_{xx}\Omega_{yy} + \Omega_{xx}\Omega_{zz} + \Omega_{yy}\Omega_{zz} - (\Omega_{xz})^2 - 4n^2\Omega_{zz}\right]$$

$$c = \Omega_{yy}\left[(\Omega_{xz})^2 - \Omega_{xx}\Omega_{zz}\right]$$

The values of co-efficients a, b and c of equation (3) are (using Mathematica)

$$a = 2n^2 = 2 + 3A_2$$

$$b = \frac{o[q_2]^2}{A_2^3} + \frac{\left(\frac{2\mu}{\sqrt{3}} + \left(-5\sqrt{3}\mu + 5\sqrt{3}\mu^2\right)q_1 + o[q_1]^2\right)q_2 + o[q_2]^2}{A_2^{3/2}} +$$

$$\frac{\left(\left(\frac{92}{3} - \frac{184\mu}{3} + \frac{92\mu^2}{3}\right) + \left(-\frac{184}{3} + \frac{368\mu}{3} - \frac{184\mu^2}{3}\right)q_1 + o[q_1]^2\right) + o[q_2]^2}{A_2} +$$

$$\frac{\left(\sqrt{3}\mu + \left(-\frac{9\sqrt{3}\mu}{2} - \frac{9\sqrt{3}\mu^2}{2}\right)q_1 + o[q_1]^2\right)q_2 + o[q_2]^2}{\sqrt{A_2}} +$$

$$\left(\left(1 + \left(-57 + 159\mu - 102\mu^2\right)q_1 + o[q_1]^2\right) + o[q_2]^2\right) +$$

$$\frac{\left(576\sqrt{3} - \frac{288\sqrt{3}}{\mu} - 288\sqrt{3}\mu\right) + \left(-6264\sqrt{3} + \frac{2280\sqrt{3}}{\mu} + 5688\sqrt{3}\mu - 1704\sqrt{3}\mu^2\right)q_1 + o[q_1]^2}{q_2} +$$

$$\left(\left(\left(-\frac{225\sqrt{3}\mu}{8} + \frac{225\sqrt{3}\mu^2}{8}\right)q_1 + o[q_1]^2\right)q_2 + o[q_2]^2\right)\sqrt{A_2} +$$

$$\left(\frac{1}{q_2^2}\left(\left(-34344 - \frac{5724}{\mu^2} + \frac{22896}{\mu} + 22896\mu - 5724\mu^2\right) + \right.\right.$$

$$\left.\left(137376 + \frac{22896}{\mu^2} - \frac{91584}{\mu} - 91584\mu + 22896\mu^2\right)q_1 + o[q_1]^2\right) +$$

$$\left(3 + \left(1062 - 2097\mu + 1035\mu^2\right)q_1 + o[q_1]^2\right) + o[q_2]^2\Big)A_2 + o[A_2]^{3/2}$$

$$c = -46 + 92\mu - 46\mu^2 + \left(1211 - 3409\mu + 3200\mu^2 - 1002\mu^3\right)q_1 +$$

$$\frac{-\frac{188}{3} + \frac{376\mu}{3} - \frac{188\mu^2}{3} + \left(-\frac{1736}{3} + \frac{5584\mu}{3} - \frac{5960\mu^2}{3} + 704\mu^3\right)q_1}{A_2}$$

$$+ A_2\left(\left(-2898 + 7875\mu - \frac{14049\mu^2}{2} + \frac{14049\mu^3}{2}\right)q_1 + \frac{65448 + \frac{10908}{\mu^2} - \frac{43632}{\mu} - 43632\mu + 10908\mu^2}{q_2^2}\right)$$

$$+ \frac{\left(1133388 + \frac{95886}{\mu^2} - \frac{523062}{\mu} - 1220652\mu + 653958\mu^2 - 139518\mu^3\right)A_2 q_1}{q_2^2} +$$

$$\frac{\left(\frac{30208}{\sqrt{3}} - \frac{7552}{\sqrt{3}\mu} - 15104\sqrt{3}\mu + \frac{30208\mu^2}{\sqrt{3}} - \frac{7552\mu^3}{\sqrt{3}}\right)q_1}{\sqrt{A_2}\,q_2} +$$

$$\frac{\left(16560\sqrt{3} - \frac{4632\sqrt{3}}{\mu} - 21696\sqrt{3}\mu + 12240\sqrt{3}\mu^2 - 2472\sqrt{3}\mu^3\right)\sqrt{A_2}\,q_1}{q_2} + \frac{\left(-\frac{44\mu}{9\sqrt{3}} + \frac{88\mu^2}{9\sqrt{3}} - \frac{44\mu^3}{9\sqrt{3}}\right)q_2}{A_2^{5/2}}$$

$$+\frac{2\mu q_2}{3\sqrt{3}A_2^{3/2}}+\frac{\left(\frac{88\mu}{9\sqrt{3}}-\frac{176\mu^2}{9\sqrt{3}}+\frac{88\mu^3}{9\sqrt{3}}\right)q_1q_2}{A_2^{5/2}}+\frac{\left(\frac{58\mu}{\sqrt{3}}-32\sqrt{3}\mu^2+\frac{38\mu^3}{\sqrt{3}}\right)q_1q_2}{A_2^{3/2}}$$

$$+\frac{\left(-121\sqrt{3}\mu+246\sqrt{3}\mu^2-125\sqrt{3}\mu^3\right)q_1q_2}{\sqrt{A_2}}+\left(\frac{171\sqrt{3}\mu}{8}-\frac{171\sqrt{3}\mu^2}{8}\right)\sqrt{A_2}\,q_1q_2+$$

$$\frac{\frac{-\frac{7552}{\sqrt{3}}+\frac{1888}{\sqrt{3}\mu}+3776\sqrt{3}\mu-\frac{7552\mu^2}{\sqrt{3}}+\frac{1888\mu^3}{\sqrt{3}}}{q_2}+\frac{2\mu q_2}{\sqrt{3}}}{\sqrt{A_2}}+\sqrt{A_2}\left(\frac{192\sqrt{3}-\frac{96\sqrt{3}}{\mu}-96\sqrt{3}\mu}{q_2}+\frac{\sqrt{3}\mu q_2}{2}\right)$$

Substituting in characteristic equation (3), we find six roots (using Mathematica).

Solution of equation $\lambda^6+a\lambda^4+b\lambda^2+c=0$ is given as (using Mathematica)

$$\lambda_1\to-\left(\left(-\frac{a}{3}+\frac{2^{1/3}a^2}{3\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}\right)-\frac{2^{1/3}b}{\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}+\frac{\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}{3\,2^{1/3}}\right)^{1/2}$$

$$\lambda_2\to\left(\left(-\frac{a}{3}+\frac{2^{1/3}a^2}{3\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}\right)-\frac{2^{1/3}b}{\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}+\frac{\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}{3\,2^{1/3}}\right)^{1/2}$$

$$\lambda_3\to-\left(\left(-\frac{a}{3}-\frac{a^2}{3\,2^{2/3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}\right)+\frac{ia^2}{2^{2/3}\sqrt{3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}+\frac{b}{2^{2/3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}-\frac{i\sqrt{3}b}{2^{2/3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}-\frac{\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}{6\,2^{1/3}}-\frac{i\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}{2\,2^{1/3}\sqrt{3}}\right)^{1/2}$$

$$\lambda_4 \to \left(\left(-\frac{a}{3} - \frac{a^2}{3\,2^{2/3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}} \right) + \frac{ia^2}{2^{2/3}\sqrt{3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}} + \frac{b}{2^{2/3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}} - \frac{i\sqrt{3}b}{2^{2/3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}} - \frac{\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}{6\,2^{1/3}} - \frac{i\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}{2\,2^{1/3}\sqrt{3}} \right)^{1/2}$$

$$\lambda_6 \to \left(\left(-\frac{a}{3} - \frac{a^2}{3\,2^{2/3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}} \right) - \frac{ia^2}{2^{2/3}\sqrt{3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}} + \frac{b}{2^{2/3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}} + \frac{i\sqrt{3}b}{2^{2/3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}} - \frac{\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}{6\,2^{1/3}} + \frac{i\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}{2\,2^{1/3}\sqrt{3}} \right)^{1/2}$$

$$\lambda_5 \to -\left(\left(-\frac{a}{3} - \frac{a^2}{3\,2^{2/3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}} \right) - \frac{ia^2}{2^{2/3}\sqrt{3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}} + \frac{b}{2^{2/3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}} + \frac{i\sqrt{3}b}{2^{2/3}\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}} - \frac{\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}{6\,2^{1/3}} + \frac{i\left(-2a^3+9ab-27c+3\sqrt{3}\sqrt{-a^2b^2+4b^3+4a^3c-18abc+27c^2}\right)^{1/3}}{2\,2^{1/3}\sqrt{3}} \right)^{1/2}$$

We may examine the stability of other equilibrium point L_7 in the same manner as L_6 . We will find that L_7 is also unstable. Thus we conclude that non-planar equilibrium points are unstable in linear sense due to positive real part in complex roots.

III. CONCLUSION

We conclude that equilibrium points are unstable due to positive real part in complex roots when they are out of plane.

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