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Square Difference Labeling of Theta Graphs

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***Abstract*:- In this work, we investigate Square Difference labeling of theta graph (T_α), We also discuss SDL in some graph operations namely Fusion, Duplication, Switching, path union and one point union of r copies of T_α graph.**

Keyword:* *Square difference graph (SDG), Square difference labeling (SDL), fusion, duplication, switching, P.r(T_α), one point union.

AMS Classification: 0578

1. INTRODUCTION

Graph labeling was first introduced in the mid sixties. A dynamic survey on graph labeling is regularly updated in [4]. For all other terminology and notations in graph theory follow [3]. The square Sum labeling is previously defined by [1]. The concept of SDL was first introduced by [6]. CDL for some special graphs and some graphs is proved [5, 2].

2. DEFINITIONS

Definition 2.1

A graph G = (p, q) with x vertices and y edges is said to admits a square difference labeling, if there exist a bijection f: V→ $\{0,1,2,\ldots x-1\}$ such that the induced function f*: E →N given by f*(ab) = $|[f(a)]^2 - [f(b)]^2|$ is injective$\forall a, b \in E$. A graph which admits square difference labeling is called square difference graph.

Definition 2.2

A theta graph (T_α) is a segment with two different vertices of degree 3 and all other vertices of degree 2.

Definition 2.3

A vertex p'_k is said to be a duplication of p if all the vertices which are adjacent to p_k are now adjacent to p'_k.

Definition 2.4

Let G be a (p, q) graph. Let u ≠ v be two vertices of G. We replace them with single vertex w and all

Edges incident with u and that with v are made incident with w. If a loop formed is deleted, the new graph has p-1 vertices and q-1 edges are called fusion of graphs [3].

Definition 2.5

A vertex switching G_v of a graph is obtained by taking a vertex c of G, detach all the edges incident with c and connect edges joining c to every vertex which are not adjacent to c in G.

Definition 2.6

Let G = G_1 = G_2 = ... = G_n be graph for n ≥ 2. Then the graph is reduced by adding an edge from G_i to G_{i+1} for i=1 to n-1 is called the path union of G.

3. MAIN RESULTS

Theorem 3.1

The Theta graph (T_α) is a SDG.

Proof:

Let T_α, with center$v_0$and the edge set be

$E(G) = \{v_i v_{i+1}/1 \le i \le 5\} \cup \{v_0 v_1, v_0 v_4\} \cup \{v_1 v_6\}$

Now, $|V(T\alpha)| = 7$ and $|E(T\alpha)| = 8$

Define a vertex labeling as

$f(v_i) = i - 1,\ 1 \le i \le 6,\ f(v_0) = 6$, now the edge labelings are defined as,

$f^*(v_0 v_1) = [f(v_0)]^2$, $f^*(v_0 v_4) = |[f(v_0)]^2 - [f(v_4)]^2|$,$f^*(v_i v_{i+1}) = 2i - 1$ for$i = 1$ to 5 , $f^*(v_1 v_6) = [f(v_6)]^2$

Thus, $f^*(e_i) \ne f^*(e_j)$, $\forall e_i e_j \in E(G)$ and now edge labelings are distinct. Hence the graph $T_\alpha$admits SDL. For instance, $T_6$is given below.

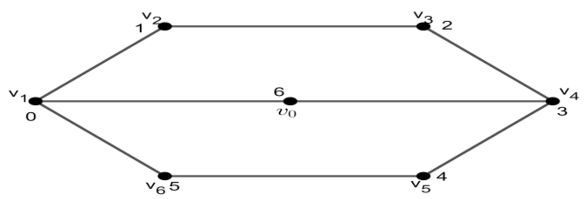

Fig 3.1 T_6 admits SDL

Theorem 3.2

The duplication of any vertex v_i of degree 3 in the cycle of T_α admits square difference graph.

Proof:

Let G be the graph obtained by duplication of any vertex v_i in T_α and v'_i be the duplicating vertex of v_i of degree 3. In T_α only two vertices are of degree 3. *i.e.*,v_1 and v_4.

Consider, $V(G) = \{v_j/0 \le j \le 7\}$and $E(G) = \{v_j v_{j+1}/1 \le j \le 5\} \cup \{v_0 v_1, v_0 v_4, v_1 v_6\}$.

Clearly, $|V(G)| = 8$ and $|E(G)| = 11$

Now, define a vertex label $f: V \to \{0,1,\ldots 7\}$ as

$f(v_j) = j - 1,\ 1 \le j \le 6$

$f(v_0) = 6$

$f(v'_1) = 7$, where v'_1 is the duplicating vertex of v_1. For the above pattern the edges are defined as same as in theorem3.1 and are distinct similarly,

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Case (i) : Duplication of v_1, the edges are labeled as follows $f^*(v_0v_1^1) = 13, f^*(v_2v_1^1) = 48, f^*(v_6v_1^1) = 24$. Thus, all the edge labeling are distinct and are illustrated in figure 3.1(a)

Example 3.1

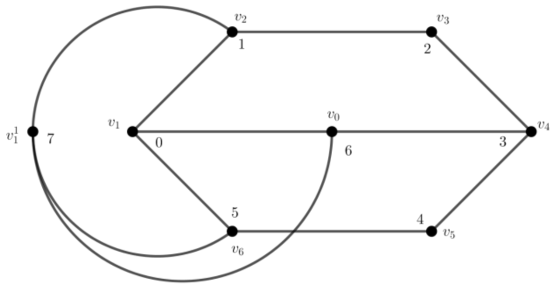

Fig 3.1(a) Duplication of v_1 in T_6 admits CDL

Case (ii) : Duplication of v_4, the edges are labeled distinct as $f^*(v_0v_4^1) = 13, f^*(v_3v_4^1) = 45, f^*(v_5v_4^1) = 33$ and an example is illustrated in figure 3.1(b).

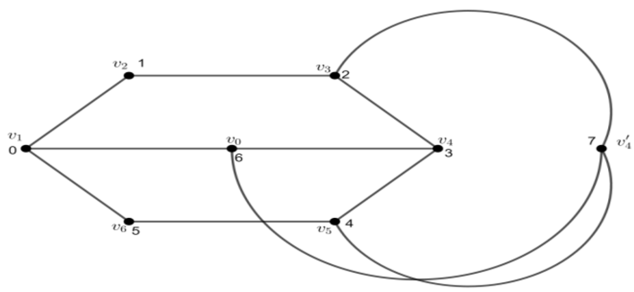

Fig 3.1(b) Duplication of v_4 in T_6 admits SDL

Theorem 3.3

The P.r(T_α) is a SDG.

Proof:

Assume the graph r(T_α) with the vertex set V = $u_i^{(j)}$, 0≤ $i \le 5$, 1≤ $j \le r$ and the edge set $E(G) = E_1 \cup E_2 \cup E_3$, where

$E_1 = \left\{u_i^{(j)}u_{i+1}^{(j)}/1 \le i \le 5\right\}$

$E_2 = \left\{u_0^{(j)}u_1^{(j)}, u_0^{(j)}u_4^{(j)}/1 \le j \le r\right\}$

$E_3 = \left\{u_3^{(j)}u_2^{(j+1)}/1 \le j \le r-1\right\}$ and $|v|$ =7r and $|e|$ =9r-1.

Vertex function is defined as h: $v \to \{0,1, \dots 7r-1\}$

$h(u_0^{(j)}) = 7j-1$

$h(u_i^{(j)}) = i+7j-8$

The edge set E is specified as E_1 and E_2.

E_1: The edges formed have an increasing sequence of even integers, if the end of the two vertices have either odd or even.

E_2: The edges formed have an increasing sequence of odd integers, if one end of vertex has odd integer and other has even integer.

Hence the condition is satisfied. Therefore the graph T_α is square difference graph. The example for the above graph is shown below.

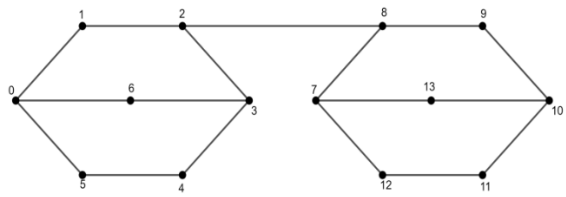

Fig 3.3 The path union of 2(T_6) is SDL

Theorem 3.4.

The one point union of r copies of theta graph admits SDL.

Proof:

Consider G = (*V*, *E*) be the one point union of $r(T_\alpha)$.

Now define $V(G) = \left\{u_i^{(j)}, w/0 \le i \le 5, 1 \le j \le r\right\}$ and

$E(G) = \left\{u_i^{(j)}u_{i+1}^{(j)}\right\} \cup \left\{wu_i^{(j)}\right\} \cup \left\{wu_5^{(j)}\right\} \cup \left\{u_0^{(j)}u_3^{(j)}\right\} \cup \left\{u_0^{(j)}w\right\}$

Clearly, $|V(G)|$= *6r+1* and $|E(G)|$= *8r*

Define the vertex function as

$f(u_0^{(j)})$ = *6j-1*, $f(u_i^{(j)})$ = *i+6j-7 for* $1 \le i \le 5, 1 \le j \le r$, $f(w)$ = *6r*

The edge set *E* is classified as same as mentioned in the *theorem 3.3*. Thus $f^*(e_i) \ne f^*(e_j)$, $\forall e_i e_j \in E(G)$. Hence the theorem is verified. For instance, the example of *4*(T_α) given below.

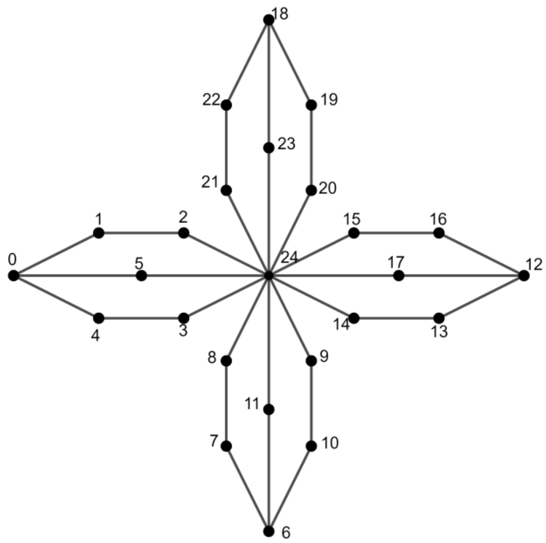

Fig 3.4. SDL of One point union of 4(T_6)

Theorem 3.5

The fusion of any two vertices in the cycle of T_α is SDL.

Proof:

Let T_α be the graph with centre v_0, V = $\{u_0, u_1, \dots u_6\}$ and $E = E_1 \cup E_2$, where $E_1 = \{u_iu_{i+1}/1 \le i \le 6\}$ and $E_2 = \{u_0u_1, u_0u_4, u_1u_6\}$

Also the cardiality of vertices and edges is noted as 7 and 8 respectively.

Now, we obtain a graph G by fusing two vertices u_5 and u_6 in T_α and we name it as u_5. After the fusion

$|V(G)|$ = *6* and $|E(G)|$ = *7*. Define the function $f: v \to \{0,1, \dots 5\}$ as

$f(u_0) = 5$
$f(u_i) = i-1$
Then the edge yields the labeling as,
$f^*(u_0u_1) = [f(u_0)]^2$, $f^*(u_0u_4) = |[f(u_0)]^2 - [f(u_4)]^2|$, $f^*(u_iu_{i+1}) = 2i-1$ for i = 1 to 4, $f^*(u_1u_5) = [f(u_5)]^2$.
It is easily observed that all the edge labels are distinct. Hence the graph *G* admits *SDL*. For the above graph, the example mentioned in *fig 3.5*

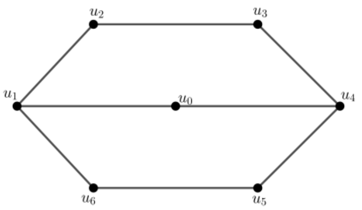

Fig 3.5 (a) Theta graph

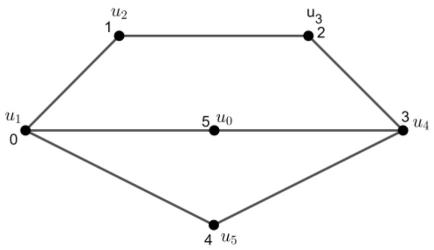

Fig 3.5 (b) the fusion of u5 and u6 in theta graphs is SDG

Theorem 3.6
The switching of a central vertex in T_α is square difference graph.
Proof
Let the graph *G* is obtained by switching the central vertex x_0 in T_α with the vertices $x_0, x_1, \ldots, x_6$ and the edges $\{x_jx_{j+1} / 1 \le j \le 5\} \cup \{x_0x_2, x_0x_3, x_0x_5, x_0x_6\} \cup \{x_1x_6\}$
The cardinality of vertices and edges are 7 and 10 resp.,
Consider the one to one function $f: V \to \{0,1,\ldots 6\}$ as
$f(x_0) = 6$
$f(x_j) = j-1$
We label the edge as
$f^*(x_jx_{j+1}) = 2j-1$, $1 \le j \le 5$, $f^*(x_1x_6) = 25$, $f^*(x_0x_2) = 35$, $f^*(x_0x_3) = 32$, $f^*(x_0x_5) = 20$,
$f^*(x_0x_6) = 11$.

Clearly, the entire 10 edge labels are distinct. Therefore, the graph *G* is square difference graph.

Example 3.2

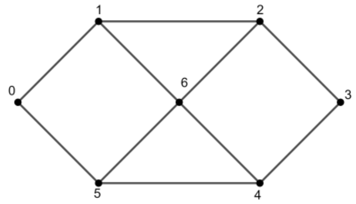

Fig 3.6 The switching of x_0 in theta graph is SDG

CONCLUSION:

From this work we conclude that T_α and its associated graphs are Square difference graph.

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