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Square Difference for Some Path Union and Duplication of Graphs

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***Abstract*:- In this paper, we prove that cycle union of r copies of $H_n \odot K_2$, open star of r copies of $H_n \odot K_2$, corona of H_n with $\overline{K_2}$, path union of corona of H_n with $\overline{K_2}$ are Square Difference graph (SDG).**

Keywords:- Square difference graph, duplication, path union, cycle union, open star
AMS classification : 05C78

1. INTRODUCTION

Throughout this work, we use finite, undirected, simple graph and we follow [1,6]. In [4, 5] proved some pyramid graphs and H - graphs for square difference. Square sum labelling for pyramid graph, Square Difference labeling of theta graphs and PCL of corona of $H_n \odot K_2$ are proved by Subashini et. al. [7, 8, 9]. Prime Cordial Labeling of H-graph and its related graphs are established in [9]. Thousands of labeling are surveyed and revised be Gallian [3]. Cube difference labelling for H graph were proved by [2]. Motivated be their work, in this paper we prove the union and duplication of some graphs.

2. MAIN RESULTS

2.1. Union and open star of corona graphs

In my previous work, I proved that H_n graph $(n \geq 3)$, P(r, H_n), C(r, H_n), S(r, H_n), $H_n \odot K_1$ etc., [5]. By continuing that, in this paper we prove some H_n related graphs for SDG. For definition of path union, cycle union and open star refer [5].

Definition 2.1.1.[4]

A graph G = (p, q) is said to be a square difference graph if it admits a bijective function g: V → {0, 1, 2, … p–1}

such that the induced function $g^*: E(G) \rightarrow N$ given by $g^*(xy) = |[g(x)]^2 - [g(y)]^2|$ are all distinct, $\forall xy \in E(G)$.[6].

Definition 2.1. 2.[9]

An H_n $(n \geq 3)$ graph is obtained by the two paths P_n^1 and P_n^2 with the vertices $u_1, u_2, \ldots u_n$ and $v_1, v_2, \ldots v_n$ respectively and joining the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ by an edge, if n is odd otherwise $u_{\frac{n}{2}+1}$ and $v_{\frac{n}{2}}$.

Theorem 2.1.1

The graph $H_n \odot \overline{K_2}$ $(n \geq 3)$ is a SD graph.

Proof:

Consider the graph $H_n \odot \overline{K_2}$ with

$V = \{x_i, y_i, x_{i,j}, y_{i,j} / 1 \leq i \leq n;\ 1 \leq j \leq 2\}$

$E = E_1 \cup E_2 \cup E_3$ where,

$E_1 = \{x_i\, x_{i+1}, y_i\, y_{i+1} / 1 \leq i \leq n\text{-}1\}$

$$E_2 = \begin{cases} x_{\frac{n+1}{2}} y_{\frac{n+1}{2}}, n - odd \\ x_{\frac{n}{2}+1} y_{\frac{n}{2}}, n - even \end{cases}$$

$E_3 = \{x_i x_{i,j}\ ,\ y_i y_{i,j} / 1 \leq i \leq n;\ 1 \leq j \leq 2\}$

Clearly, the cardinality of the vertices and edges are 6n and 6n-2 respectively. Now, define the function f as

$f(x_i) = 2(i - 1)$,

$f(y_i) = 2i - 1$

$f(x_{i,j}) = 2n + 4i + 2j - 5$

$f(y_{i,j}) = 2n + 4i + 2j - 6$

and we receive the edge labels f^* as follows:

$f^*(x_i\, x_{i+1}) = 8i - 4,\ 1 \leq i \leq n$

$f^*(y_i\, y_{i+1}) = 8i,\ 1 \leq i \leq n$

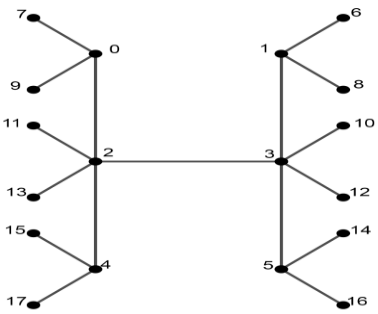
$f^*(x_{\frac{n+1}{2}}\ y_{\frac{n+1}{2}}) = 2n - 1 \equiv 1 (mod\ 4)$

$f^*(x_{\frac{n}{2}+1}\ y_{\frac{n}{2}}) = 2n - 1 \equiv 3 (mod\ 4)$

Thus, the entire 6n – 2 edges are distinct. Hence the theorem.

Example 2.1.1.

Square difference labeling for $H_3 \odot \overline{K_2}$ and $H_4 \odot \overline{K_2}$.



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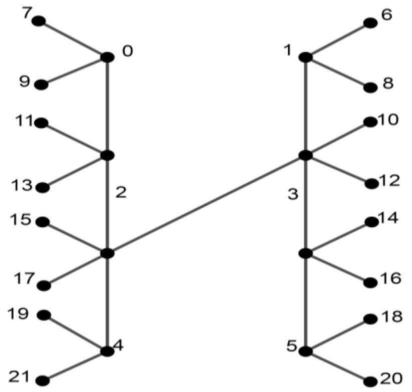

Figure 2.1. SDL for $H_3 \odot \overline{K_2}$ and $H_4 \odot \overline{K_2}$

Theorem 2.1.2.

The path union of $H_n \odot \overline{K_2}$ $(n \geq 3)$ admits SDL.

Proof:

Let $H_n \odot \overline{K_2}$ $(n \geq 3)$ be the corona graph of H_n with $\overline{K_2}$ with the vertex set,

$V = V_1 \cup V_2$, where,

$V_1 = \{x_i^{(k)}, y_i^{(k)} \,/\, 1 \leq i \leq n,\ 1 \leq k \leq r\}$ and

$V_2 = \{ x_{i,j}^{(k)}, y_{i,j}^{(k)} \,/ 1 \leq i \leq n, 1 \leq j \leq 2; 1 \leq k \leq r\}$

and the edges $E = \cup_{k=1}^{4} E_k$, where,

$E_1 = \left\{x_i^{(k)} x_{i+1}^{(k)}, y_i^{(k)} y_{i+1}^{(k)} \,/ 1 \leq i \leq n-1; 1 \leq k \leq r\right\}$;

$$E_2 = \begin{Bmatrix} x_{\frac{n+1}{2}}^{(k)} y_{\frac{n+1}{2}}^{(k)}, n-odd \\ x_{\frac{n}{2}+1}^{(k)} y_{\frac{n}{2}}^{(k)}, n-even, 1 \leq k \leq r \end{Bmatrix}$$

$E_3 = \left\{x_i^{(k)} x_{i,j}^{(k)}, y_i^{(k)} y_{i,j}^{(k)} \,/ 1 \leq i \leq n; 1 \leq j \leq 2; 1 \leq k \leq r\right\}$

$E_4 = \left\{y_1^{(k)} x_1^{(k+1)} \,/ 1 \leq k \leq r-1\right\}$

It is obvious that, the number of vertices and edges are $6nr$ and $8nr-1$ resp.,

Also, define the vertex labeling function as follows:

For $1 \leq i \leq n,\ 1 \leq j \leq 2,\ 1 \leq k \leq r$,

$f(x_i^{(k)}) = 2(i-1) + 2n(k-1)$

$f(y_i^{(k)}) = 2i - 1 + 2n(k-1)$

$f(x_{i,j}^{(k)}) = f(y_n^{(r)}) + 4i + 2j - 4 + 4n(k-1)$

$f(y_{i,j}^{(k)}) = f(y_n^{(r)}) + 4i + 2j - 5 + 4n(k-1)$

Thus, the induced function $f^*: E(H_n \odot \overline{K_2}) \rightarrow N$ satisfies the condition of SD labeling and the edges of $H_n \odot \overline{K_2}$ receives label as,

For $1 \leq k \leq r$,

$f^*(x_i^{(k)} x_{i+1}^{(k)}) = 8i - 4 + 8n(k-1)$

$f^*(y_i^{(k)} y_{i+1}^{(k)}) = 8i + 8n(k-1)$

$f^*(x_{\frac{n+1}{2}}^{(k)} y_{\frac{n+1}{2}}^{(k)}) = 2n - 1 + 4n(k-1) \equiv 1 \pmod 4$(n is odd)

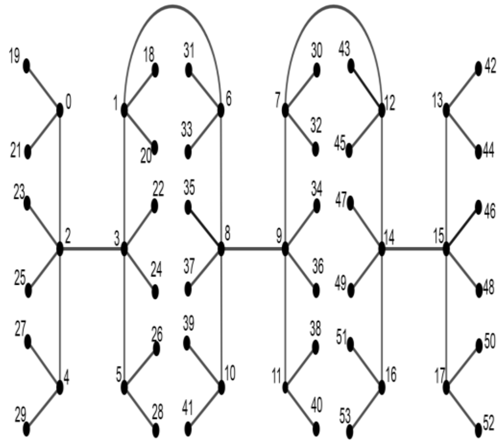

Figure 2.2(a). SDL for P($H_3 \odot \overline{K_2}$)

$f^*(u_{\frac{n}{2}+1}^{(k)} v_{\frac{n}{2}}^{(k)}) = 2n - 1 + 4n(k-1) \equiv 3 \pmod 4)$

$f^*(u_{i,1}^{(k)} u_{i,2}^{(k)}) = f^*(v_{n-1}^{(r)} v_n^{(r)}) + 16n(k-1) + 16i, 1 \leq i \leq n$

$f^*(v_{i,1}^{(k)} v_{i,2}^{(k)}) = f^*(v_{n-1}^{(r)} v_n^{(r)}) + 16n(k-1) + 16i - 4, 1 \leq i \leq n$

$f^*(v_1^{(k)} u_1^{(k+1)}) = 4n^2 - 1 + (8n^2 - 4n)(k-1)$

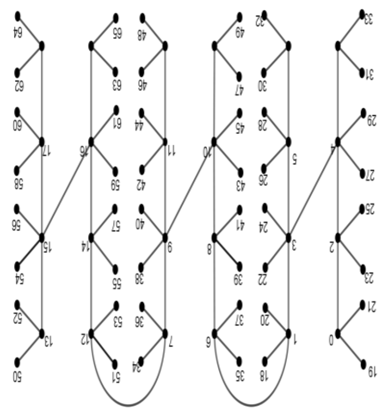

Figure 2.2(b). SDL for P($H_4 \odot \overline{K_2}$)

Thus, all the edge labeling are distinct *i.e.*, $f^*(e_i) \neq f^*(e_j), \forall e_i \neq e_j \in E(G)$. Hence P(r, $H_n \odot \overline{K_2}$), $(n \geq 3)$ graph admits Square difference labeling.

Theorem 2.1.3.

C(r, $H_n \odot \overline{K_2}$)) is SDG.

Proof

Consider, G = C(r, $H_n \odot K_2$) be the graph.

Let $V = V_1 \cup V_2$, where,

$V_1 = \{g_i^{(k)}, l_i^{(k)} \,/\, 1 \leq i \leq n,\ 1 \leq k \leq r\}$ and

$V_2 = \{ g_{i,j}^{(k)}, l_{i,j}^{(k)} \,/ 1 \leq i \leq n, 1 \leq j \leq 2; 1 \leq k \leq r\}$

and the edges $E = \cup_{k=1}^{6} E_k$, where,

$E_1 = \{g_i^{(k)} g_{i+1}^{(k)}, l_i^{(k)} l_{i+1}^{(k)} \,/\, 1 \le i \le n-1; 1 \le k \le r\}$;

$E_2 = \left\{ \begin{array}{l} g_{\frac{n+1}{2}}^{(k)} l_{\frac{n+1}{2}}^{(k)}, n - odd \\ g_{\frac{n}{2}+1}^{(k)} l_{\frac{n}{2}}^{(k)}, n - even, 1 \le k \le r \end{array} \right\}$

$E_3 = \{g_i^{(k)} g_{i,j}^{(k)}, l_i^{(k)} l_{i,j}^{(k)} \,/\, 1 \le i \le n; 1 \le j \le 2; 1 \le k \le r\}$

$E_4 = \{g_{i,1}^{(k)} g_{i,2}^{(k)}, l_{i,1}^{(k)} l_{i,2}^{(k)} \,/\, 1 \le i \le n; 1 \le k \le r\}$

$E_5 = \{l_1^{(k)} g_1^{(k+1)} \,/\, 1 \le k \le r-1\}$

$E_6 = \{l_1^{(r)} g_1^{(1)}\}$

Clearly,

$|V(G)| = 6nr$ and

$|E(G)| = 8nr$

Also, we receive vertex and edge labeling as

For $1 \le i \le n$; $1 \le j \le 2$; $1 \le k \le r$

$f(g_i^{(k)}) = 2(i-1) + 2n(k-1)$

$f(l_i^{(k)}) = 2i - 1 + 2n(k-1)$

$f(g_{i,j}^{(k)}) = f(l_n^{(r)}) + 4i + 2j - 4 + 4n(k-1)$

$f(g_{i,j}^{(k)}) = f(l_n^{(r)}) + 4i + 2j - 5 + 4n(k-1)$

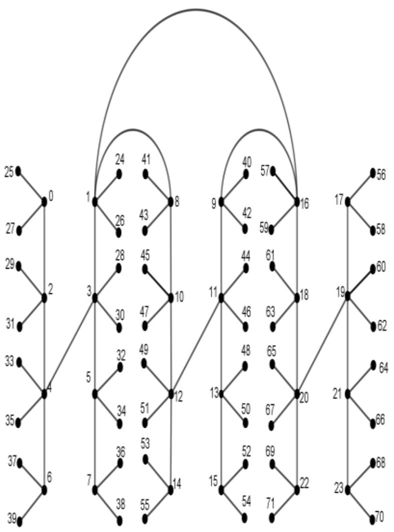

Figure 2.3(a) SDL of C(3, $H_4 \odot K_2$)

Using this induced function f^*, the edges of G receives labeling as same as mentioned in theorem 3.4.4 and added to

$f^*(l_1^{(r)} l_1^{(1)}) = [f((l_1^{(r)})]^2 - 1$

Hence, all the edge labeling are distinct and strictly increasing. Hence, the theorem is proved.

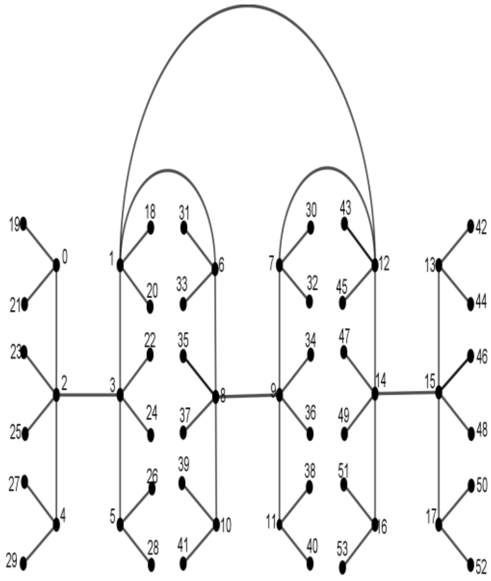

Figure 2.3(b) SDL of C(3, $H_3 \odot K_2$)

Theorem 2.1.4.

The graph S(r, $H_n \odot K_2$) admits SDL.

Proof:

Let G = S(r, $H_n \odot K_2$) with $V = V_1 \cup V_2$, where,

$V_1 = \{g_i^{(k)}, l_i^{(k)} \,/\, 1 \le i \le n, 1 \le k \le r\}$ and

$V_2 = \{ g_{i,j}^{(k)}, l_{i,j}^{(k)} \,/\, 1 \le i \le n, 1 \le j \le 2; 1 \le k \le r\}$

And $E = \bigcup_{k=1}^{5} E_k$, where,

$E_1 = \{g_i^{(k)} g_{i+1}^{(k)}, l_i^{(k)} l_{i+1}^{(k)} \,/\, 1 \le i \le n-1; 1 \le k \le r\}$;

$E_2 = \left\{ \begin{array}{l} g_{\frac{n+1}{2}}^{(k)} l_{\frac{n+1}{2}}^{(k)}, n - odd \\ g_{\frac{n}{2}+1}^{(k)} l_{\frac{n}{2}}^{(k)}, n - even, 1 \le k \le r \end{array} \right\}$

$E_3 = \{g_i^{(k)} g_{i,j}^{(k)}, l_i^{(k)} l_{i,j}^{(k)} \,/\, 1 \le i \le n; 1 \le j \le 2; 1 \le k \le r\}$

$E_4 = \{g_{i,1}^{(k)} g_{i,2}^{(k)}, l_{i,1}^{(k)} l_{i,2}^{(k)} \,/\, 1 \le i \le n; 1 \le k \le r\}$

$E_5 = \{w l_1^{(k)} \,/\, 1 \le k \le r\}$

We know that, the cardinality of vertices and edges are $6nr + 1$ and $8nr$ resp.,

And the vertex valued function are as same as mentioned in the above theorem and added to $f(w) = f(l_n^{(r)}) + 1$.

The induced function f^* receives the edge labels as

$f^*(g_i^{(k)} g_{i+1}^{(k)}) = 8i - 4 + 8n(k-1)$

$f^*(l_i^{(k)} l_{i+1}^{(k)}) = 8i + 8n(k-1)$

$f^*(g_{\frac{n+1}{2}}^{(k)} \; l_{\frac{n+1}{2}}^{(k)}) = 2n - 1 + 4n(k-1) \equiv 1 (\mathrm{mod}\ 4)$(n is odd)

$f^*(g_{\frac{n}{2}+1}^{(k)} \; l_{\frac{n}{2}}^{(k)}) = 2n - 1 + 4n(k-1) \equiv 3 (\mathrm{mod}\ 4)$

$f^*(g_{i,1}^{(k)} g_{i,2}^{(k)}) = f^*(l_{n-1}^{(r)} l_n^{(r)}) + 16n(k-1) + 16i, 1 \le i \le n$

$f^*(l_{i,1}^{(k)} l_{i,2}^{(k)}) = f^*(l_{n-1}^{(r)} l_n^{(r)}) + 16n(k-1) + 16i - 4, 1 \le i \le n$

$f^*(w g_1^{(k)}) = 0 (\mathrm{mod}\ 2)$

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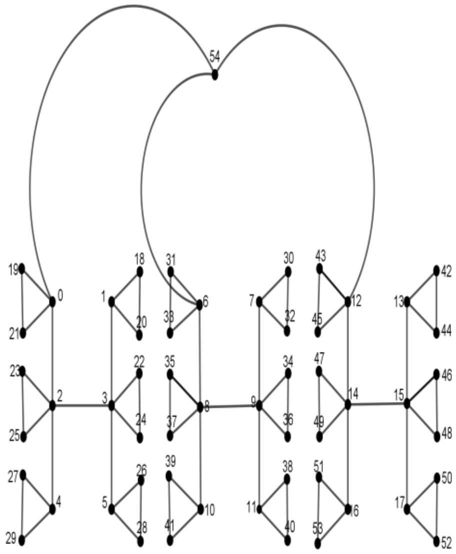

Figure 2.4(a) SDL of S(3, $H_3 \odot K_2$)

and $f^*(g_i^{(k)} g_{i,j}^{(k)}, l_i^{(k)} l_{i,j}^{(k)})$ is in the form of an increasing order of odd integer when its one end vertex is odd integer and the other end vertex is even integer.

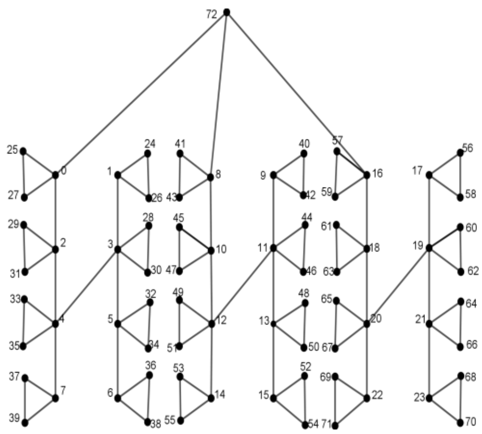

Figure 2.4(b) SDL of S(3, $H_4 \odot K_2$)

From the above, $f^*(e_i) \neq f^*(e_j), \forall e_i \neq e_j \in E(G)$. Hence S(r, $H_n \odot K_2$), ($n \geq 3$) graph admits Square difference labeling.

2.2. Duplication of a pendant vertex of pyramid and hanging pyramid graph

Definition 2.2.1.

A vertex v_k' is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v_k'.

Example 2.2.1.

Duplication of vertex by a vertex of C_3.

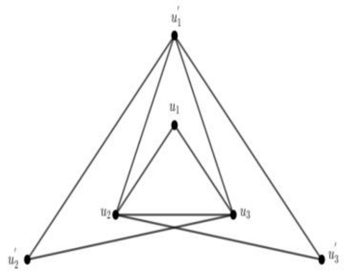

Figure 2.5. Duplication of vertex by a vertex of C_3

Theorem 2.2.1.

The Duplication of any pendant vertex of pyramid graph J_n ($n \geq 3$), is SDG.

Proof:

Let G be the graph obtained by duplication of any pendant vertex of J_n. $u_{i,j}'$ be the duplication vertex of $u_{i,j}$ of degree one. In J_n, only two vertices are pendant vertices. i.e. $u_{n,1}$, and $u_{n,n}$.

Consider,

V(G) = $\{u_{i,j} \,/\, 1 \leq i \leq n;\ 1 \leq j \leq i\} \cup \{u_{n,1}', u_{n,n}'\}$ and the edge set

E(G) = $\{u_{i,j}u_{i+1,j},\ u_{i,j}u_{i+1,j+1} \,/\, 1 \leq i \leq n-1;\ 1 \leq j \leq i\} \cup \{u_{n,1}'\, u_{n-1,1}',\ u_{n,n}',\ u_{n-1,n-1}'\}$

Then,

$$|V(G)| = \frac{(n^2+n)}{2} + 2 = p \text{ and}$$

$$|E(G)| = (n^2 - n) + 2 = q$$

Let the function f : V → {0,1, 2 --- p-1} defined as follows:

$f(u_{i,j}) = \frac{1}{2}i(i-1) + (j-1)$ for $1 \leq i \leq n$; $1 \leq j \leq i$

$f(u_{n,1}') = f(u_{n,n}) + 1$

$f(u_{n,n}') = f(u_{n,n}) + 2$

Clearly, the above given labeling satisfies the condition of SDL and receives the distinct edge labeling.

Hence the theorem is verified.

Example 3.1.2.

The Duplication of pendant vertex of J_5 is SDG.

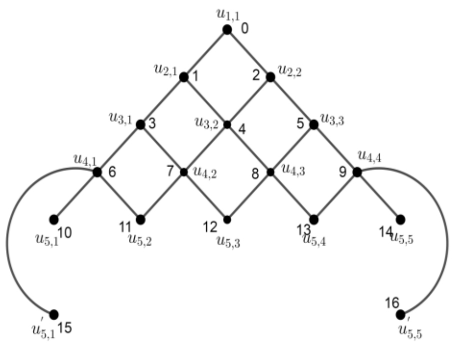

Figure 2.6. SDL for duplication of pendant vertex of J_5.

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Theorem 2.2.2.

The duplication of any pendant vertex of hanging pyramid graph admits SDL.

Proof:

Consider the graph G procured by duplicating the pendant vertex of hanging pyramid graph.

In HJ_n, the vertices u_0, $u_{n,1}$, $u_{n,n}$ are the pendant vertices.

The vertex set and edge set are same as theorem 2.4.1. added to $\{u_0'\}$ and $\{u_0u_0'\}$ respectively.

Then,

$$|V(G)| = \frac{(n^2+n)}{2} + 3 \text{ and}$$

$$|E(G)| = (n^2 - n) + 3$$

Let the vertex valued function *f* are defined as,

$f(u_0) = 0$

$$f(u_{i,j}) = \frac{1}{2}(i^2 - i) + j, \; 1 \le i \le n; \; 1 \le j \le i$$

$f(u_0') = f(u_{n,n}) + 3$

Then, the induced edge function f^* for the above labelling pattern are distinct. *i.e.*, $f^*(e_i) \neq f^*(e_j) \; \forall e_i \neq e_j \in E(G)$.

Therefore, the duplication of pendant vertex of HJ_n admits SDL.

Example 2.4.2

The duplication of pendant vertex of HJ_4 is SDL.

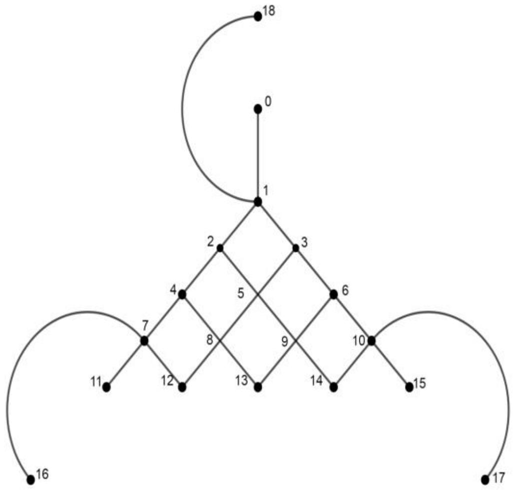

Figure 2.7. SD for duplication of pendant vertex of hanging pyramid graph.

CONCLUSION

In this work, we investigated that the path union, cycle union, open star and duplication of some graphs admits Square difference labelling.

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