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Split, Boost, Regenerate: A Solar-Fed Energy Loop, Its Efficiency Limit and a Validation Plan for Testing Perpetual-Motion-Style Claims

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Abstract - Machines that claim to give out more energy than they take in, called perpetual motion machines of the first kind (PMM1), break the first law of thermodynamics. A common design idea is to split a one-time energy input, boost one part, and loop it back. This paper presents a conceptual study of a split–boost–regenerate system and tests it with energy accounting. Two architectures are modelled: a split–boost loop (A) and a loop that recovers otherwise lost energy (B). For both, a closed-form efficiency is derived and shown to stay at or below 100% for every split fraction and boost gain. A solar-fed version is then examined, where the Sun supplies energy from outside, so the system is sustainable rather than perpetual. In a worked example the loop lifts the useful efficiency from 0.70 to 0.903, and about 19% of the incident sunlight becomes useful work. These are analytic design estimates and have not been verified experimentally. The paper documents the model, a staged measurement plan with uncertainty analysis, performance metrics, limitations and risks. No experimental results are reported.

Index Terms - *perpetual motion, first law of thermodynamics, energy regeneration, solar energy, exergy, loop efficiency, concept study.*

Nomenclature

Table 1. Abbreviations and symbols used in this paper.

Symbol / term	Meaning	Symbol / term	Meaning
PMM1	Perpetual motion machine of the first kind	η_w	Work-stage efficiency
E_s, E_n	Source energy, node energy (J)	η_r	Round-trip efficiency of return path
a	Fraction sent to the work stage	u, β	Useful and recoverable fractions
G	Voltage or force gain of boost stage	η_{sys}	System efficiency
η_b	Boost-stage efficiency	S_{gen}, X_d	Entropy generated, exergy destroyed
COP	Coefficient of performance	GUM	Guide to uncertainty in measurement

1 Introduction

The first law of thermodynamics states that energy is neither created nor destroyed. James Joule measured this law in the 1840s [6], and Emmy Noether later showed that it follows from the fact that the laws of physics do not change with time [1]. A machine that gives out more work than the energy it receives is called a perpetual motion machine of the first kind (PMM1), and it is ruled out by this law [5]. Claims of such machines have been made for centuries. Careful tests have shown that the missing energy was lost as heat, sound or deformation, or was never created in the first place.

A popular design idea is to give a machine energy once, split it into two parts, use one part to do work, boost the other part, and send some of it back so that the loop never stops. Boosting sounds like making new energy. In practice, levers, gearboxes and transformers only trade one quantity for another [2, 22]. The present study asks what such a loop can and cannot achieve.

The concept is organised around three functions: Split, Boost and Regenerate. Split divides the energy at a node. Boost changes one share of the energy, for example voltage for current. Regenerate returns recovered energy to the node. The loop can also be fed by sunlight, which is an outside source of energy [11].

The novelty claimed here is not a new energy source. Energy loops, converters, regenerative braking and solar panels are established technology. The contribution is a clear model of the loop, a proof of its efficiency limit, and a validation plan that lets anyone test a prototype honestly. Whether such a loop gives a worthwhile gain in a real device remains to be established experimentally.

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Vol. 15 Issue 11 , November - 2026

2 Problem Statement and Motivation

The work addresses four concerns: (i) the claim that a boost stage can add energy; (ii) the claim that a loop can keep running from a one-time energy input; (iii) the question of how much a recovery loop can really improve a solar-fed system; and (iv) how to measure a prototype so that the result can be trusted. A practical concern is that claims of "free energy" often come with measurements that miss an energy input.

Potential readers include students, teachers, makers and engineers who must judge such claims. The validation pathway proposed is a bench prototype first, using a calibrated DC supply that imitates a solar panel. Outdoor tests with a real panel should follow only after the bench results are understood.

3 Related Work

The first law and its origin are well established [1, 6, 16, 19]. Carnot's work on heat engines led to the second law, which rules out a PMM2 [8], and the long history of perpetual motion claims is documented in [5]. In general relativity there is no global energy conservation in an expanding universe, but the energy lost by stretching light cannot be delivered to a local machine [20]. Collapse models predict a tiny energy gain [17], and tests of Lorentz-violating theories leave no room for large effects [18]. None of these gives a usable mechanism, so they are background only.

For the solar input, the exergy of sunlight is about 93% of its energy [7, 14], and a single-junction solar cell is limited to about 33% [9]. The latest efficiency tables list the best large silicon cell near 28% [10], while commercial modules reach about 20–23% [11]. These figures are background information and were not measured by the author.

Heat pumps are often mistaken for PMM1 because their coefficient of performance (COP) is above 1, but the extra heat comes from the surroundings [3]. The cited material supports the study of individual parts but does not give a proof for the loop studied here. The research question is therefore one of energy accounting: how much can a split–boost–regenerate loop help, and what is its limit? A systematic prior-art and patent search is required before any definitive novelty claim.

4 Aim and Objectives

The aim is to develop and evaluate a concept for a solar-fed split–boost–regenerate system and to find the upper limit of its efficiency. The objectives are:

1. build an energy-accounting model of the split–boost loop (Architecture A);
2. build a model in which recoverable losses are returned to the loop (Architecture B);
3. prove an upper limit on the system efficiency of each architecture;
4. estimate the useful share of sunlight for a solar-fed system and its sensitivity to each parameter;
5. design a metered test plan with uncertainty analysis; and
6. state clearly which results are analytic estimates and which need experiments.

Table 2. What common "multipliers" multiply, and what they cost.

Device	Gains	Pays	Net energy
Lever, gear train	force	distance	out $\leq$ in
Hydraulic press	force	piston travel	out $\leq$ in
Transformer, boost converter	voltage	current	out $\leq$ in
Flywheel	peak power	spin-up time	stores only
Heat pump (COP 3–5)	heat moved	heat taken in	out = in + heat gathered

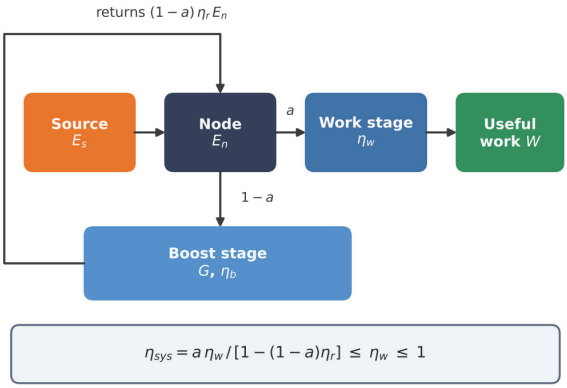


Figure 1. Line diagram of Architecture A, the split–boost loop. The gain G does not appear in the efficiency bound. Schematic, not to scale.

5 System Architecture

Two architectures are studied. Table 2 first shows why a boost stage cannot create energy: every apparent gain is paid for in another quantity.

In Architecture A (Fig. 1), the source feeds a node. A fraction a of the node energy goes to the work stage. The rest, $1-a$, goes through a boost stage and returns to the node. The boost gain G multiplies voltage or force, but it also divides current or speed by G, so it never enters the energy balance. Only the boost efficiency η_b does.

In Architecture B (Fig. 2), the work stage turns its input into three parts: useful work (fraction u), recoverable energy such as braking energy or captured waste heat (fraction β), and energy lost for good (the rest). The recoverable part returns to the node through a regeneration path with efficiency η_r.

For the solar-fed version, the source is a solar module followed by a DC–DC converter (Fig. 7). The sun,

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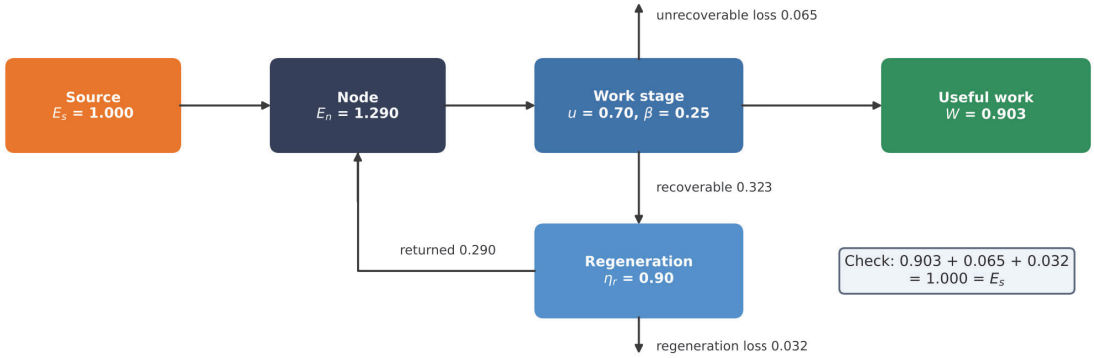


Figure 2. Energy balance of Architecture B for the worked example $u = 0.70$, $\beta = 0.25$, $\eta_r = 0.90$. Numbers are energy per 1.000 supplied by the source (analytic, not measured).

not the loop, supplies the energy, so the first law is respected at the system boundary.

6 Operating Principle

Let E_s be the energy supplied by the source per cycle and $\eta_r = \eta_b \eta_t$ the round-trip efficiency of the boost path, where $\eta_t \leq 1$ is the efficiency of carrying the energy back to the node. In steady state, the node balance of Architecture A is

$$E_n = E_s + (1-a)\,\eta_r\,E_n \;\Rightarrow\; E_n = \frac{E_s}{1-(1-a)\,\eta_r} \quad (1)$$

The useful work is $W = a\,\eta_w\,E_n$, so the system efficiency is

$$\eta_{sys} = \frac{W}{E_s} = \frac{a\,\eta_w}{1-(1-a)\,\eta_r} \quad (2)$$

Each trip around the loop adds a smaller amount, so E_n is a geometric series that adds up to a finite number because $(1-a)\eta_r < 1$.

Theorem 1 (loop limit). For every $0 < a \leq 1$ and all efficiencies η_r, η_w between 0 and 1, $\eta_{sys} \leq \eta_w \leq 1$. In simple words, the loop can never beat sending all the energy straight to the work stage.

Proof. Since $\eta_r \leq 1$, we have $(1-a)\eta_r \leq (1-a)$. So the bottom part of Eq. (2) is at least a, and $a/[1-(1-a)\eta_r] \leq 1$. Hence $\eta_{sys} \leq \eta_w$. □

The loop only sends around energy that the source has already supplied, so any loss in the loop makes the result worse (Fig. 3).

For Architecture B, the recovered energy $\beta\eta_r E_n$ returns to the node, so $E_n = E_s/(1-\beta\eta_r)$ and

$$\eta_{sys} = \frac{u}{1-\beta\,\eta_r} \quad (3)$$

Theorem 2 (regeneration limit). If $u + \beta \leq 1$ and $0 \leq \eta_r \leq 1$, then $u \leq \eta_{sys} \leq 1$. The value 1 is reached only if $\eta_r = 1$ and nothing is lost for good. In simple words, regeneration lifts efficiency from u toward 100%, but never above it.

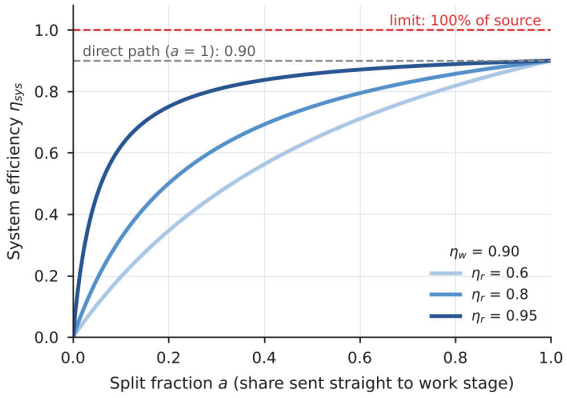


Figure 3. System efficiency of Architecture A against split fraction a for three values of η_r (Eq. 2, $\eta_w = 0.90$). Every curve stays at or below the direct path.

Proof. Lower limit: $1 - \beta\eta_r \leq 1$, so $\eta_{sys} \geq u$. Upper limit: $u \leq 1-\beta$, and $1-\beta \leq 1-\beta\eta_r$ because $\eta_r \leq 1$. So $\eta_{sys} \leq (1-\beta)/(1-\beta\eta_r) \leq 1$. □

If the source is removed after one single input E_0, and each cycle has efficiency $\eta < 1$, the energy left after n cycles is

$$E_n = \eta^{\,n}\,E_0 \quad (4)$$

The energy leaving the loop in cycle n is $(1-\eta)\eta^n E_0$; these terms sum to E_0, so the total work never exceeds E_0 (Fig. 5). A machine that runs forever from a one-time input is therefore impossible.

Each cycle also generates entropy $S_{gen} \geq 0$, which destroys useful energy (exergy) by

$$X_d = T_0\,S_{gen} \quad (5)$$

where T_0 is the surrounding temperature [3, 4, 15]. Splitting and joining energy adds conversion steps, and each step adds loss. Architecture B helps only when the energy it recovers is more than the extra loss from its added hardware.

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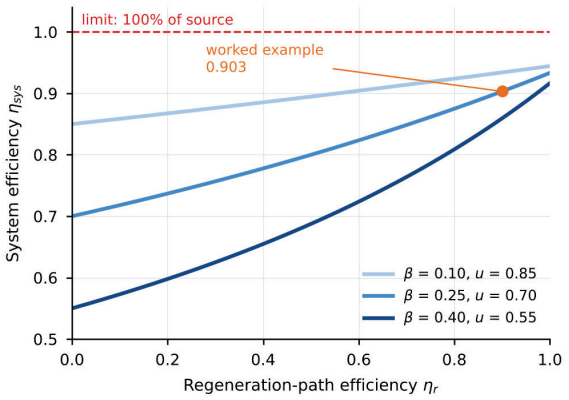


Figure 4. System efficiency of Architecture B against regeneration efficiency η_r (Eq. 3) with the unrecoverable loss fixed at 0.05. The left end of each curve is the efficiency without regeneration.

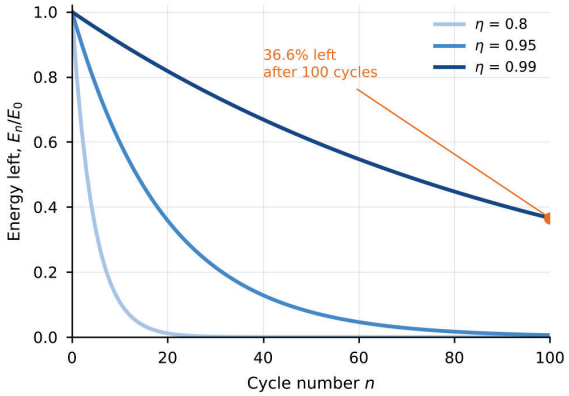


Figure 5. Energy left after n cycles without a source (Eq. 4). Even a 99% loop keeps only 36.6% after 100 cycles.

7 Design Estimates and Feasibility

Table 3 gives worked examples of Eq. (3). They are analytic planning values, not measurements. The last row is the ideal limit: a lossless regeneration path with no permanent loss returns exactly the input, never more.

For a solar-fed system, the source limits matter more than the loop. At the top of the atmosphere, sunlight delivers about 1361 W/m^2, and solar panels are rated at 1000 W/m^2 and 25 °C [11]. The exergy of blackbody radiation at $T_s \approx 5778$ K, with surroundings at $T_0 = 300$ K, is

$$\Psi = 1 - \tfrac{4}{3}\tfrac{T_0}{T_s} + \tfrac{1}{3}\left(\tfrac{T_0}{T_s}\right)^4 \approx 0.93 \qquad (6)$$

which is the physical ceiling on the useful share of sunlight [7, 14]. The end-to-end efficiency is

$$\eta_{total} = \eta_{pv}\,\eta_{conv}\,\eta_{sys} \qquad (7)$$

With $\eta_{pv} = 0.22$, $\eta_{conv} = 0.97$ and $\eta_{sys} = 0.903$ (second row of Table 3), $\eta_{total} = 0.22 \times 0.97 \times 0.903 \approx 0.193$. About 19% of the sunlight would become useful work. Regeneration raises the useful share by a factor of $0.903/0.70 \approx 1.29$ compared with the same system without it, but the total stays far below 100% because the solar panel is the biggest limit, not the loop.

Table 3. Worked examples of Architecture B (Eq. 3, analytic).

u	β	η_r	η_{sys}
0.60	0.30	0.80	0.789
0.70	0.25	0.90	0.903
0.80	0.20	0.95	0.988
0.80	0.20	1.00	1.000

Table 4. Sensitivity of η_{sys} (analytic).

Parameter	Rate of change	Effect of +0.01
u	1.29	+0.013
β	1.05	+0.010
η_r	0.29	+0.003

Table 4 shows how η_{sys} changes when one number changes a little, using $\partial\eta/\partial u = 1/(1-\beta\eta_r)$, $\partial\eta/\partial\eta_r = u\beta/(1-\beta\eta_r)^2$ and $\partial\eta/\partial\beta = u\eta_r/(1-\beta\eta_r)^2$ at $u = 0.70$, $\beta = 0.25$, $\eta_r = 0.90$.

The useful fraction u matters most. So the first design step is to reduce direct losses, and the second is to recover what is left. Table 5 compares all configurations.

No configuration is above 100%. Only the ideal limit of Architecture B reaches it.

8 Proposed Validation Methodology

This section describes a proposed validation plan; no experiments have been completed. Testing should begin on a bench with a calibrated DC supply that imitates a solar panel. A real panel should be used outdoors only after the bench results are understood. Fig. 6 shows the overall method and Fig. 7 the measurement setup. Fig. 8 shows a conceptual bench layout for planning.

Measurement. Use four-wire voltage and current meters at every port, logging at 10 Hz or faster. Add a pyranometer for sunlight, thermocouples for temperature, and a torque and speed sensor on the load. Meter every energy crossing of the boundary as $V \times I \times$ time.

Energy balance. Measure the input and output of each stage and calculate each stage efficiency. Close the balance $E_{in} - (W + Q + \Delta E_{stored}) =$ residual. Repeat at least 30 runs for each setup and report the mean and standard deviation.

Decay test. Remove the source, record how the stored energy falls, and compare it with Eq. (4).

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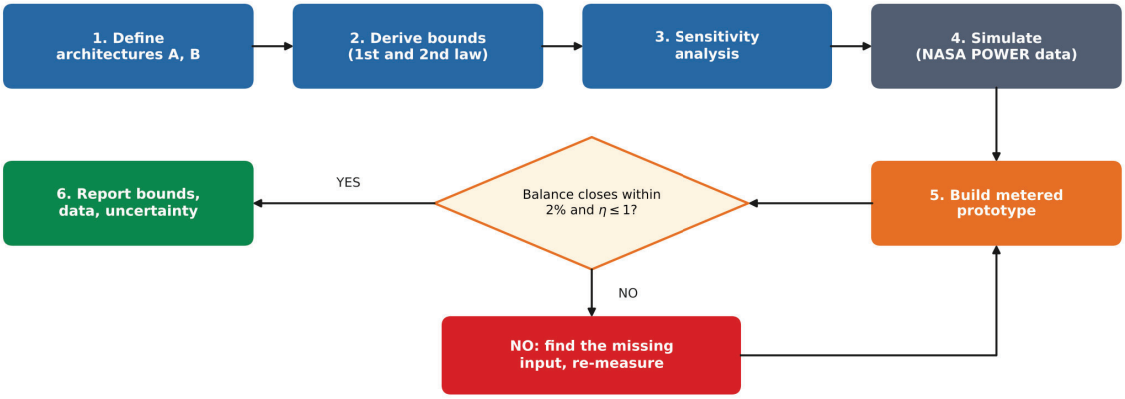


Figure 6. Methodology and validation logic. Steps 1–3 are analytic and done in this paper. Steps 4–6 are planned. If the energy balance does not close, the test is repeated after finding the missing input. Conceptual.

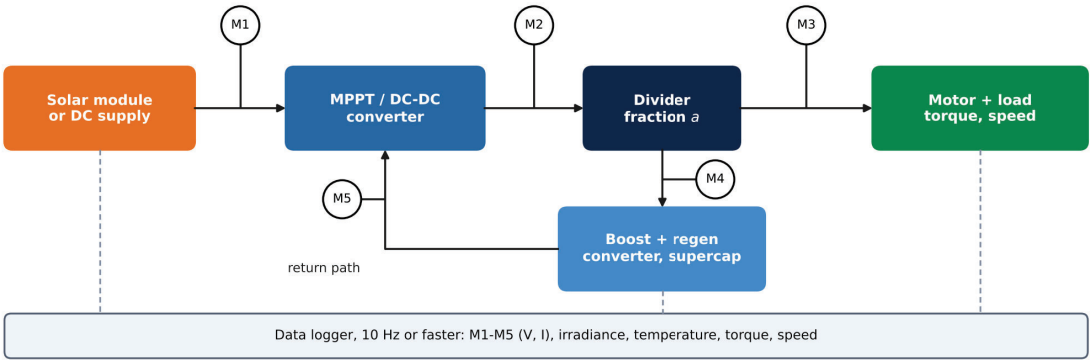


Figure 7. Planned experimental setup with five in-line power meters (M1–M5) and one data logger. Dashed lines are sensor signals. Planned, not yet built.

Table 5. Comparison of configurations (analytic, example parameters).

Configuration	Example parameters	η_{sys}
Direct path ($a = 1$)	$\eta_w = 0.90$	0.900
Architecture A	$a = 0.5$, $\eta_w = 0.90$, $\eta_r = 0.90$	0.818
Architecture B	$u = 0.70$, $\beta = 0.25$, $\eta_r = 0.90$	0.903
Architecture B, ideal limit	$u = 0.80$, $\beta = 0.20$, $\eta_r = 1.00$	1.000
Loop, no source, $n = 100$	$\eta = 0.99$	0.366

Uncertainty. For $\eta = P_{out}/P_{in}$, the combined relative uncertainty follows the GUM method [13]:

$$\frac{u_c(\eta)}{\eta} = \sqrt{\left(\frac{u(P_{out})}{P_{out}}\right)^2 + \left(\frac{u(P_{in})}{P_{in}}\right)^2} \qquad (8)$$

A result can be trusted only if the balance residual is smaller than the instrument uncertainty (target: below 2%) and any $\eta > 1$ appears again in independent runs with different instruments. In practice, an apparent $\eta > 1$ almost always means a missed energy input, such as stored charge or heat from the room, or a measurement error.

Simulation. A time simulation uses hourly sunlight data from NASA POWER [21] for a chosen place and year, a solar panel model with temperature correction, and Eq. (3) for the loop. The script in Appendix A checks the limit of Theorem 2.

Safety. Use fuses and a current-limited supply. Supercapacitors and batteries store energy and can cause burns, so they need protected wiring and discharge procedures. Testing should be supervised by a qualified person.

9 Performance Metrics and Data Analysis

Metrics should be reported with units, uncertainty and test conditions: input energy E_s (J); useful work W (J); stage efficiencies η_b, η_w, η_r; system efficiency η_{sys} and end-to-end efficiency η_{total}; energy balance residual (%); component temperatures (°C); and decay of stored energy against η^n. Each protocol should specify instrument range, calibration status, sampling frequency, number of repeats and the data-reduction method. Comparisons must use

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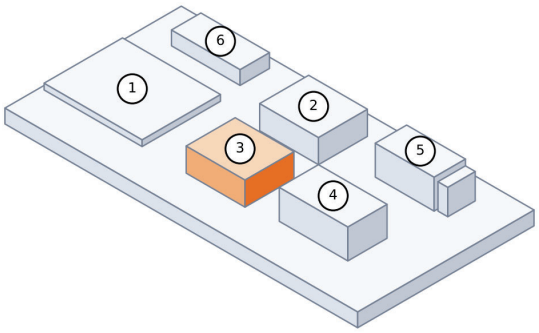


Figure 8. Conceptual isometric bench layout, not a dimensioned CAD model. 1 solar module or DC supply; 2 MPPT / DC–DC converter; 3 boost + regen converter (stage under test); 4 supercapacitor bank; 5 motor, load and sensors; 6 data logger and meters.

matched operating conditions. Raw data and instrument records should be kept for independent checking.

10 Expected Outcomes

The expected outcome is a built and metered prototype evaluated against defined technical and safety criteria. The first target is to measure the stage efficiencies and to see whether the measured η_{sys} stays below the limits of Theorems 1 and 2. A second outcome is a measured decay curve to compare with Eq. (4), and a measured energy balance with uncertainty. Negative or inconclusive findings will be reported alongside positive ones.

11 Limitations and Risks

The hardware has not been built or tested. No claim can currently be made about measured efficiency, useful power or the share of sunlight that is converted. The model is steady and simplified. Start-up, battery charge level and changes in converter efficiency with load are not included, and u, β and η_r are treated as fixed numbers although real converters change with load. Sunlight changes with clouds and night, so storage is needed, and storage adds loss. The efficiency figures for sunlight and converters come from cited background sources, not from experiments. Electrical faults and stored-energy hazards require design controls. These constraints are treated as core design requirements.

12 Environmental and Social Relevance

The study supports clear thinking about energy claims. A solar-fed loop is sustainable because the Sun supplies the energy, but this paper does not demonstrate any environmental benefit. Its social value lies in helping students and makers to test “free energy” claims with a proper energy balance, so that effort goes into cutting losses and recovering waste energy instead. A full assessment of a real device would also need to consider material and battery impacts, lifetime and cost.

13 Future Work

Work should proceed in stages: (1) finish the time simulation with real sunlight data; (2) build the bench prototype of Fig. 7; (3) characterise each stage independently; (4) integrate the stages and compare the measured η_{sys} with the model; (5) test outdoors with a real solar panel; and (6) choose the best split fraction for changing loads. A systematic prior-art and patent search should also be completed.

14 Conclusion

This paper presented a solar-fed split–boost–regenerate system and tested it with energy accounting. Architecture A is limited by the work-stage efficiency (Theorem 1), and Architecture B by 100% of the source (Theorem 2). With sunlight as input, the system is sustainable, not perpetual. In the worked example, regeneration lifts the useful efficiency from 0.70 to 0.903, and about 19% of the sunlight becomes useful work. These values are analytic design estimates or values from cited background sources, not experimental results. The right goal is to cut losses and recover what is left, not to chase more than 100%. The essential next step is a metered bench test with a full uncertainty analysis.

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An AI system (Claude, Anthropic) assisted with the analysis, figures and drafting of this manuscript. The author reviewed the content and is responsible for it.

Author Contributions

S. S. Gomase: concept development (original handwritten note on split-and-multiply energy), research,

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modelling and manuscript preparation.

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Conflict of Interest

The author declares that he has no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

Data Availability

No experimental dataset is reported in this concept-stage manuscript. The verification script is given in Appendix A. Data and test protocols will be made available from the corresponding author on reasonable request after experimental validation.

Appendix A: Verification Script

```
def eta_sys(u, beta, eta_r):
    assert u + beta <= 1 + 1e-9
    return u / (1 - beta * eta_r)

best = max(eta_sys(u / 100, b / 100, 1.0)
           for u in range(30, 91)
           for b in range(0, 51) if u + b <= 100)
print(best <= 1 + 1e-9)   # True: never above 100%
```

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