Speed Control of a DC Motor Using Fractional order Proportional Derivative (FOPD) Control

Anish Thomas[1], Subha Hency Jose[2],Asha Prasad[3],Ajith Ravindran[4]
[1]Assistant Professor, Saintgits College of Engineering, Kottayam, [2]Assistant Professor, Karunya University, Coimbatore. [3]Assistant Professor, Government Engineering College, Sreekrishnapuram [4]Assistant Professor, Saintgits College of Engineering, Kottayam

Abstract

This paper presents speed control of a DC Motor using Fractional order Proportional Derivative control. Fractional order Proportional Derivative control and Fractional Order Proportional Integral Derivative control has wide range of applications. Many analysis and design methods has been proposed for fractional order controllers. Here, a new tuning method for fractional order proportional and derivative (FO-PD) controller is used for speed control of a Permanent Magnet DC motor. Tuning is performed in such a manner that the given gain crossover frequency and phase margin is fulfilled and the phase derivative with respect to the frequency is zero making the closed loop system robust to gain variations. Tuned Fractional order Proportional Derivative Controller is compared with conventional controller PID controller. Real-time results show that the FOPD controller exhibits better response compared to a PID controller.

.

1. Introduction

The DC motor is a power actuator, which converts direct current electrical energy into rotational mechanical energy. The DC motors are still widely used in industry and in numerous control applications, robotic manipulators and commercial applications such as disk drive, tape motor as well. Nowadays different methods for speed control are available based on the application. The DC Motor setup is shown in Fig.1. It consists of a DC Motor with an optical encoder attached to the shaft of the DC Motor. PID controllers were normally used for DC Motor Speed Control due to their advantages like less overshoot, no steady state error etc. But here we employ a nonconventional control technique which is known as a fractional-order control. Mentioned technique was developed during last few decades and there are various practical applications as for example flexible spacecraft attitude control , car suspension control , temperature control , motor control, etc. Clearly, for closed-loop control systems, there are four situations: (1) integer order (IO) plant with IO controller; (2) IO plant with fractional-order (FO) controller; (3) FO plant with IO controller, and (4) FO plant with FO controller. In control practice, the fractional-order controller is usually more common, because the plant model can be obtained as an integer-order model in the classical sense. Improving or optimizing the performance is the major concern as well as the primary target. Hence, our objective is to apply the fractional-order control (FOC) to enhance the (integer order) dynamic system control performance.

In most cases, our objective of using fractional calculus is to apply the FOC to enhance the system control performance. The fractional order controllers can be realized using analogue circuits. A $PI^{\lambda}D^{\mu}$ controller can be used in a wide variety of applications and it also exhibits better control performance when compared with the classical proportional–integral–differential (PID) controller because of extra real parameters λ and μ involved. In general, there is no systematic way for setting the fractional order parameters λ and μ. However, we may be able to get practical and simple FOC parameters tuning methods for certain specific plants. In this paper, design method of fractional order PD^{μ} for typical second-order plant is discussed. Aiming at the specific class of second-order plants, the major contributions of this paper are that the new FO-PD tuning method proposed is simple, practical, systematic, and can achieve favorable dynamic performance as well as robustness. The fractional order proportional and derivative controller FO-PD has the following form of transfer function:

$$C(s) = K_p(1+K_d s^{\mu}) \qquad (1)$$

Where, $\mu \in (0,1)$. Clearly, this is a specific form of the most common controller which involves an integrator of order λ (equal to 0, in this paper) and a differentiator of order μ . The derivative action increases the stability of the system and tends to emphasize the effects of noise at high frequencies. In the time domain a decrease in overshoot and the settling time is observed. In the complex plane , the derivative action produces a displacement of the root locus of the system towards the left half-plane. In the frequency domain, this action produces a constant phase lead of $\pi/2$ rad and an increase of 20 dB/dec in the slopes of the magnitude curves. Proper selection of the value of μ helps to attain better control as compared to the conventional control schemes.

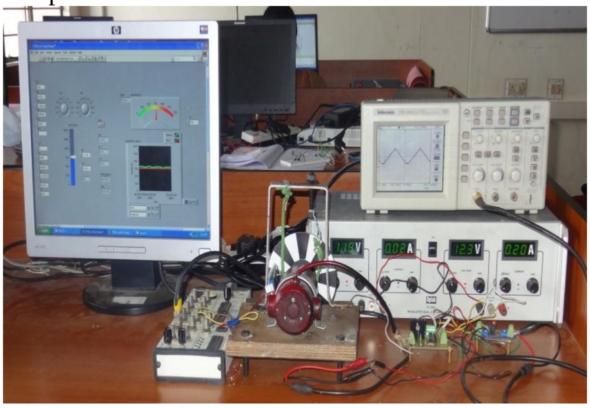

Figure.1 DC Motor Setup

The remaining part of this paper is organized as follows. In Section 2, the system is described briefly with functional block diagram and mathematical modeling. In Section 3, a new tuning method for FOC design is proposed for a class of second-order plants. Implementation of the FOPD controller and PID controller with their comparison is given in Section 4. Finally, conclusion is presented in Section 5.

2. System Description

2.1 Functional Block Diagram

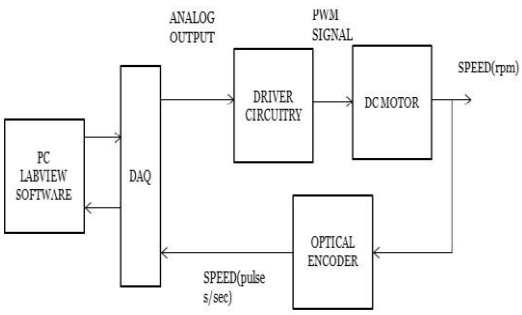

Figure.2 Functional Block Diagram

The functional block diagram of a DC Motor is shown in Fig.2. Analog output voltage generated by the DAQ is given to the driver circuit. The driver circuit generates a PWM signal, which is fed to the DC Motor, which in turn controls the speed of the DC Motor. The Motor speed is monitored by an optical encoder which is attached to the shaft of the DC Motor. Encoder output which is in the form of pulses is given to the DAQ.

2.2 Mathematical Modelling

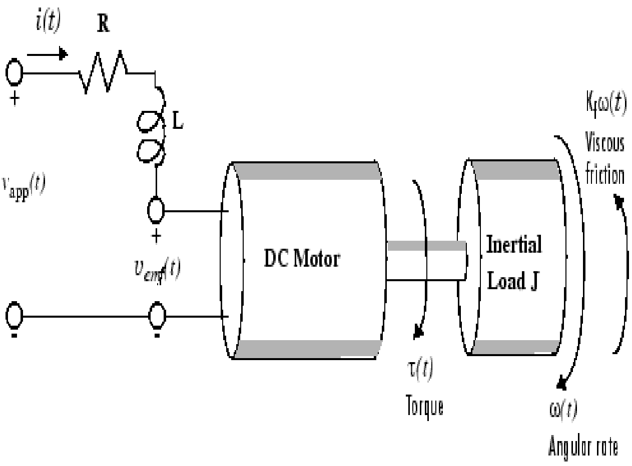

Figure.3 General Model of a DC Motor

General model of a DC Motor is shown in Fig.3. The speed of a D.C motor is directly proportional to armature voltage and inversely proportional to the flux in the field winding. This speed control system is electro mechanic system. The electrical system consists of armature and field excited by a constant voltage.

Let, Ra-Resistance of the armature(Ω).
La- Inductance of the armature(H).
T-Torque developed(N\m).
J- Moment of inertia(Kg\m2).
Ia - Armature current(A).
Va- Armature voltage(V).
Eb-Back emf(V).
f0-viscous friction.

The back emf is proportional to angular displacement and is given by,

$$E_b = K_b\omega(s) \quad (2)$$

The difference equation of armature is,

$$L\frac{dIa}{dt} + RaIa + Eb = Va \quad (3)$$

By taking Laplace transform of the above equation, it gives

$$LsIa + RaIa + Eb = Va \quad (4)$$

Torque equation is,

$$J\omega(s) = -Kfs(s) + KtIa \quad (5)$$

From Equ(4),

$$I_a = \frac{Va - Eb}{(Ls + Ra)} \quad (6)$$

From Equ(5),

$$\omega = \frac{KtIa}{Js2+Kfs} \quad (7)$$

Substitute (6) in (7)

$$\omega = \frac{Kt}{(Js+Kfs)}\frac{(Va-Kbs)}{Js+Ra} \quad (8)$$

On solving the equ(8) gives

$$\frac{\omega(s)}{V(s)} = \frac{Kt}{(Ra+sLa)(Js+f0)+(KbKt)} \quad (9)$$

Equ(9) gives the transfer function of the DC motor.

The DC motor rating used here is 12 volt, 1.5A,1500rpm.The DC motor parameters obtained through experiments are

R= 8.8 Ω
L=3.005mH
K_b = 0.0777 V.sec/rad
K_t = 0.0777 Nm/A
J = 0.000132 kg.m^2.

The transfer function of the DC motor is obtained by substituting the values given above in equ (9) and can be written as:

$$G(s) = \frac{\omega(s)}{v(s)} = \frac{0.0777}{0.000000397s^2+0.001162s+0.00649} \quad (10)$$

3. Fractional Order Control

3.1 Introduction

Fractional order(non-integer order) controllers have a better control performance when compared to classical PID control because of extra parameters $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ .This helps in disturbance rejection. An integer order control is expressed by:

$$G_c(s) = K_p + T_i s^{-1} + T_d \quad (11)$$

Whereas in a fractional order we have additional parameters $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ and can be given as:

$$G_c(s) = K_p + T_i s^{-\lambda} + T_d s^{\mu} \quad (12)$$

Clearly, this is a specific form of the most common controller which involves an integrator of order $\boldsymbol{\lambda}$ (equal to 0, in this paper) and a differentiator of order $\boldsymbol{\mu}$. The derivative action increases the stability of the system and tends to emphasize the effects of noise at high frequencies. Proper selection of the value of $\boldsymbol{\mu}$ helps to attain better control as compared to the conventional control schemes

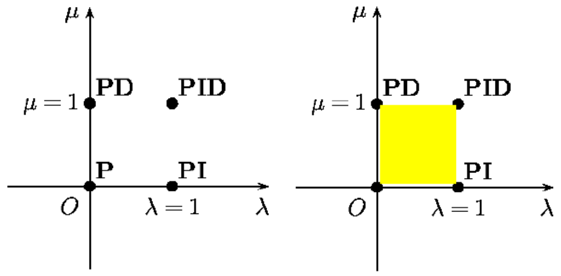

Figure.4 Parameter Allocation Region

When $\boldsymbol{\lambda}$=0 and $\boldsymbol{\mu}$=1, we obtain a PD controller. When $\boldsymbol{\lambda}$=1 and $\boldsymbol{\mu}$=0, we obtain a PI controller. When $\boldsymbol{\lambda}$=1 and $\boldsymbol{\mu}$=1, we obtain a PID controller. Fractional order region lies in the yellow region as shown in Fig.4. In a Fractional Order Proportional and Derivative control, the transfer function has the following form:

$$C(s) = K_p(1+K_d s^{\mu}) \quad (13)$$

The controller block diagram is shown in Fig.5. The advantages of the controller are:

- No- steady state error
- Robustness to high frequency Noise
- Robustness to variation in the gain of the plant
- Good output disturbance rejection due to 3 tuning parameters

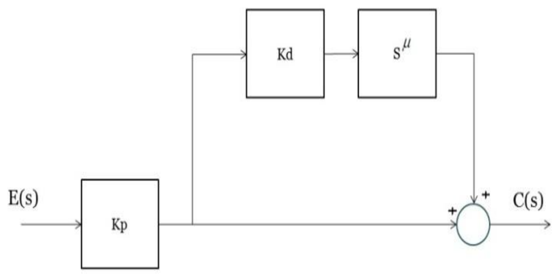

Figure.5 Controller Block Diagram

3.2 Controller Design

For the design of the FOC PD$^{\mu}$ controller, three interesting specifications which need to be met was proposed. Depending upon phase margin and gain crossover frequency specification we obtain the following :

(i) Phase margin specification

$$Arg[G(j\omega_c)]=Arg[C(j\omega_c)P(j\omega_c)]=-\pi+\phi_m \quad (14)$$

(ii) Robustness to variation in gain of the plant

$$\left(\frac{d(Arg[C(j\omega c)P(j\omega c)])}{d\omega}\right)=0 \quad (15)$$

(iii) Gain crossover frequency specification

$$|G(j\boldsymbol{\omega}_c)|dB = |C(j\boldsymbol{\omega}_c)P(j\boldsymbol{\omega}_c)|\ dB = 0 \qquad (16)$$

According to Specification (i) about the phase of G(s) , the relationship between and can be established as follows:

$$K_d = \frac{1}{\omega_c^{\mu}} \tan\left[\frac{\pi}{2} + \boldsymbol{\phi}_m + \tan^{-1}(0.18\boldsymbol{\omega}_c) - \frac{\mu\pi}{2}\right] \text{X} \cos\frac{(1-\mu)\pi}{2} - \frac{1}{\omega_c^{\mu}}\sin\frac{(1-\mu)\pi}{2} \qquad (17)$$

According to Specification (ii) about the robustness to gain variations in the plant, we can establish an equation about in the following form:

$$K_d = \frac{C-D}{E} \qquad (18)$$

Where

$$C = -2A\omega_c^{\mu}\sin\frac{(1-\mu)\pi}{2} + \boldsymbol{\mu}\omega_c^{\mu-1}\cos\frac{(1-\mu)\pi}{2} + \sqrt{2A\omega_c^{\mu}\sin\frac{(1-\mu)\pi}{2} - \mu\omega_c^{\mu-1}\cos\frac{(1-\mu)\pi}{2}}$$

$$D = \frac{4\omega_c^{2}\mu}{\sqrt{0.00689+0.000225\,\omega_c^{2}}}$$

$$E = \frac{2\omega_c^{2}\mu}{\sqrt{0.00689+0.000225\,\omega_c^{2}}}$$

$$A = \frac{T}{1+(\omega_c T)^2}$$

$$B = 2A\omega_c^{\mu}\sin\frac{(1-\mu)\pi}{2} - \omega_c^{\mu-1}\cos\frac{(1-\mu)\pi}{2}$$

According to Specification (iii), we can establish an equation about K_p:

$$|G(j\boldsymbol{\omega}_c)| = \frac{K_p\sqrt{(1+K_d\omega_c^{\mu}\cos\frac{\mu\pi}{2})^2+(K_d\omega_c^{\mu}\sin\frac{\mu\pi}{2})^2}}{\omega_c\sqrt{1+(\omega_c T)^2}} = 1 \qquad (19)$$

The design procedure is given as follows: It can be observed from (17) and (19) that $\boldsymbol{\mu}$, K_d can be obtained jointly. Fortunately, a graphical method can be used as a practical and simple way to get $\boldsymbol{\mu}$ and K_d because of the plain forms about (17) and (19). The procedures to tune the PD^{μ} fractional order controller are as follows:

1) Given $\boldsymbol{\omega}_c$, the gain crossover frequency;
2) Given $\boldsymbol{\phi}_m$, the desired phase margin;
3) Plot the curve 1, K_d w.r.t $\boldsymbol{\mu}$, according to (17);
4) Plot the curve 2, K_d w.r.t $\boldsymbol{\mu}$, according to (18);
5) Obtain $\boldsymbol{\mu}$ and K_d from the intersection point of the above two curves
6) Calculate K_p the from (19).

Here gain crossover frequency was set as 10 rad/sec, and the desired phase margin as $70^{o.}$ According to the numerical method given above, the following values were obtained

Kp=0.22
Kd= 0.87
$\boldsymbol{\mu}$=0.525

4. Real-time Implementation

4.1 FOPD Controller Implementation

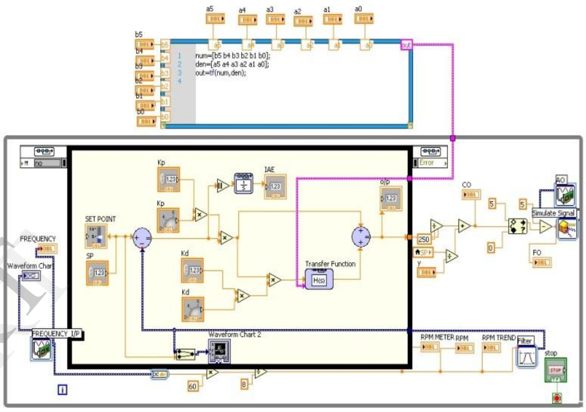

Figure.6 LabVIEW Block Diagram-FOPD Control

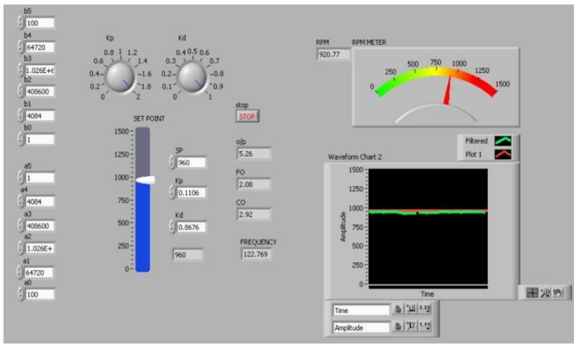

Figure.7 LabVIEW Front Panel-FOPD Control

For FOPD Simulation rationalization of the irrational term s^{μ} has to be done. For that Oustaloup approximation algorithm is used. Taking $\boldsymbol{\mu}$=0.525 ,w_l=0.001,w_h=1000 we get approximation of fractional function as:

$$s = \frac{100s^5+64720s^4+1026000s^3+408600s^2+4084s+1}{s^5+4084s^4+408600s^3+1026000s^2+64720s+100}$$

Kp, Kd & $\boldsymbol{\mu}$ values obtained by tuning are substituted to the respective blocks. The LabVIEW Block diagram and Front Panel for an FOPD controller is shown in Fig.6 and Fig.7 respectively

.

4.2 PID Controller Implementation

For PID control implementation the Kp, Ki & Kd values obtained by tuning are substituted into the respective blocks to constitute a PID Controller.The LabVIEW Block diagram and Front Panel for an PID controller is shown in Fig.8 and Fig.9 respectively.

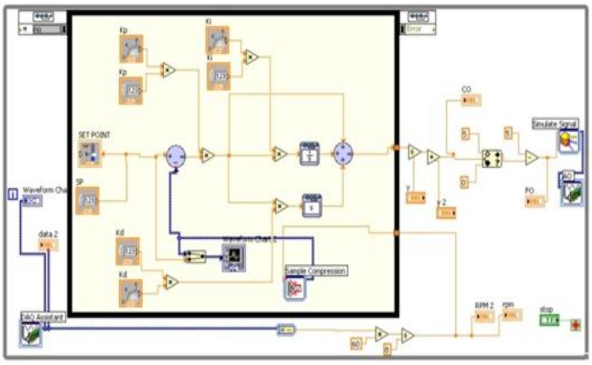

Figure.8 LabVIEW Block Diagram-PID Control

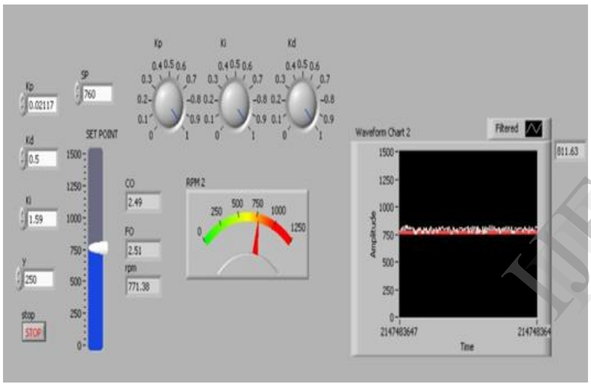

Figure.9 LabVIEW Front Panel-PID Control

4.3 Results

In Real-time FOPD Controller exhibits impeccable performance when compared to PID controller. The FOPD controller is faster than PID and settles very fast. The overshoot is less when compared to a PID controller. The Integral Absolute Error is also very less for a FOPD controller. Comparison of the two controllers is shown in Table I.

Table.1 Controller Comparison

Characteristics	FOPD	PID
Rise time	1s	1.5s
Settling time	4s	5s
Overshoot	Less	More
IAE	212	407

5. Conclusion & Futurework

In this paper, a fractional order proportional derivative controller was tuned using a new tuning method and controller parameters were obtained. Tuning is done such that the given gain crossover frequency and phase margin is fulfilled. Real-time implementation of the controller was done and the FOPD controller exhibits better performance than PID controller in real-time.A FO[PD] and FO-MRAC Controller can be implemented for the speed control of a DC Motor. Both the controllers mentioned above will definitely exhibit better responses than the conventional controllers. A distributed motion control system can be also built in which multiple systems can communicate with one another using networking features of LabVIEW.

References

L. [1] HongSheng Li, Ying Luo, and YangQuan Chen, “A Fractional Order Proportional and Derivative (FOPD) Motion Controller: Tuning Rule and Experiments” IEEE Transactions on Control Systems Technology, Vol. 18, No. 2, March 2010,pp 516-520

[2] Ying Luo , YangQuan Chen , Youguo Pi, “Experimental study of fractional order proportional derivative controller synthesis for fractional order systems” www.elsevier.com/locate/mechatronics 21(2011), pp 204-214

[3] Ivo Petras, “Fractional order Feedback control of a DC Motor,” in Journal of Electrical Engineering, Vol. 60, No. 3, 2009, pp 117–128

[4] I.Podlubny, “Fractional-order systems and $PI^{\lambda}D^{\mu}$ controller,” IEEE Trans. Autom. Control, vol. 44, no. 1, Jan. 1999, pp. 208–214

[5] Ying Luo, Yang Quan Chen, Chun Yang Wang, Yongshun Jin, “LabVIEW Based Experimental Validation of Fractional Order Motion Controllers,” in Control and Decision Conference, 2009. 17-19 June 2009, pp 323 – 328.