

Spare Parts Management for Corrective Maintenance of the Complex System under Uncertainty of Failures

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Abstract— Spare part is one of the main factors in the maintainability. Therefore, it is very importance to appropriately supply the spares. If spares are acquired more than necessary then it would make the inventory cost, which is a sunk cost incurred. If insufficient supply of spares, it will cause a lack of spare parts for maintenance actions. And this will result in cost of lost opportunity to make profit during machine stopped while waiting for spares. Importantly, the appropriated purchase and storage of parts in inventory will be helpful to save costs. This study is mainly focused and applied the principle of payoff matrix using for the spare parts' procurement for a number of appropriations. The main idea is to think of the supply for spare parts on the basis of multiple machines with applying these with payoff matrix principle. This study also takes the opportunity cost when machine stopped production in the absence period of spares to be added into the equation. The principle and concept of this study is based on a balance and trade-off between the inventory cost of spare parts and the cost of losing in opportunity. In addition, the raised sample of this research was basically supposed under conceptual of general form which is able to apply this principle to all types of machines.

Keywords—*spare part, maintenance, inventory, opportunity cost*

I. INTRODUCTION

Spare part is one of the main components for maintenance actions. It can probably be grouped by type of using or unit of measurement, these are 2 types: (1) the use of a piece or discrete unit as such pulley, switch, motor, so on; and, (2) the use of not in a piece or unit of measurement is continuous such as conveyor belt, chain, electrical cable, etc.

This research mainly focuses on spares of the discrete unit or as a piece. The consideration for the inventory of these parts is applied the payoff matrix principle that proposes of finding the re-order point and stocking level. And, machine failures are applied the Binomial distribution. In our point of view, we see that distribution is appropriated for this research.

Furthermore, the equation of total cost is also included the opportunity cost which is effected from machine downtime due to the absence spares in inventory into the equation. The total cost is the combination of the cost of lost in opportunity to make the profit during downtime and inventory cost. The trade-off between two portions is issued in term of effective total cost. This idea is to satisfy the total cost equation of a staph even more than in past research.

II. RETERATURES REVIEW AND RELEVANT RESEARCHES

There are many researches and articles which are focused on spare parts' inventory management for the corrective maintenance actions. Almost of them mainly apply the EOQ approach such as [1], [2], [3], [4], etc. However, many articles as well have adopted the payoff matrix which are applied to find the re-order point and number of stocking spares such as [5], [6], [7] but there are articles not many like [8], [9] have considered by the cost of losing chance of stopping production in case of the lack of spare parts included in the equation of the total cost structure. And most of the researches in this approach are rather based on the principle of EOQ more than apply the payoff matrix for answers but mostly of them did not consider any of the trade-off between the cost of losing in opportunity and cost of storage. The overall of cost structure did not fulfill these into the equation as well. As a result, the previous researches are mainly focused on three main costs as the cost of materials, cost of purchase, and storage costs, primarily in finding out about the inventory of spare parts for maintenance. The result is a lack of completeness in the equation for the total cost structure.

Thus, this research study is conducted on this concept, and principles to fulfill this niche of the previous researches. The equation of total cost for spare parts' inventory management for maintenance is more complete and can be used as a study guide for research in the future.

III. DEFINITION OF VARIABLES

All variables appearing in this paper should be clarified before using in order to easy understand. So, the variables are:

TC is the grand total cost or total cost for all periods.

$E[TC(n)]$ is expected cost of sequence n .

TC_t is total cost at period t .

IC_t is total spare parts' cost at period t .

UC is spare part unit cost.

Q_t is spare parts purchasing at period t .

SC_t is total ordering cost at period t .

OC is ordering cost per one time.

S_t is policy of ordering at period t ; $S_t \in [0,1]$, Integer;

by: $S_t = 0 \rightarrow$ not buy, $S_t = 1 \rightarrow$ buy at period t

HC is holding cost per unit per period.

HC_t is total holding cost a period t .

h is holding cost fraction (percentage of unit cost).

Q_t^r is remaining of spares at period t .

LC is the lost cost during machine stoppage per machine per period.

LC_t is total lost cost at period t .

PC is production lost cost during spares waiting time.

EC is expedited cost in case of express purchase when there is lack of spares in inventory.

M_{DT} is maintenance downtime.

P_R is production rate of each machine per period.

P_U is unit cost to produce the product.

e is the multiplication time of express purchase compare with ordinary purchase.

SP_t is the number of spare parts shortage at period t .

Φ_j is joint probability at sequence j .

j is running number of types of failures from period 1 to period T (or considered period), and $j = 1, 2, 3, \dots, k$

n is running number of possible sets of solutions, and $n = 1, 2, \dots$, total numbers of possible sets of solution.

k is total numbers of types of failures or states of nature, and $k = (M+1)^T$

M is amount of machines in the system.

T is machine life time or considered period.

X is number of failures.

X_t is number of failure at period t .

IV. COST STRUCTURE

For this study, already includes the effect of losing cost in opportunity during machine stoppage (in case of spares shortage) into the equation. So the structure of total cost is composed of four sub-cost facilities as:

1) *Spare parts cost*, this is the total cost of materials or unit price multiplied by the number of pieces of spare parts used. So, the cost equation is:

$$IC_t = UC \cdot Q_t \quad (1)$$

2) *Ordering cost*, this is the total cost to purchase spare parts. This includes relevant costs such as the cost of transportation, packaging, etc., So the cost equation is:

$$SC_t = OC \cdot S_t \quad (2)$$

3) *Holding cost*, this is the total cost of storing the parts in stock. This includes relevant costs such as the cost of storage facilities, interest, depreciation, etc. So the cost equation is:

$$HC = h \cdot UC \quad (3)$$

And:
$$HC_t = h \cdot UC \cdot Q_t^r$$

$$= HC \cdot Q_t^r \quad (4)$$

4) *The lost cost of machine stoppage*, this cost is composed of two portions. The opportunity cost from stopping production (in case of parts shortage) and cost to purchase spare parts fast. So, the cost equation is:

$$LC = PC + EC \quad (5)$$

By:
$$PC = M_{DT} \cdot \pi \cdot P_R \cdot P_U \quad (6)$$

And:
$$EC = e \cdot UC \quad (7)$$

In case of shortage of spares, this cost will occur. Suppose parts shortages at period t of SP_t pieces. From equation (6), the lost cost from the lack of spare parts on hand at period t is:

$$LC_t = SP_t \cdot LC \quad (8)$$

5) *Total cost*, The total cost includes the cost of subsection 4 of the foregoing. The equation as mentioned, including at period t is:

$$TC_t = IC_t + SC_t + HC_t + LC_t \quad (9)$$

Or:
$$TC_t = (UC)Q_t + (OC)S_t + (HC)Q_t^r + (LC)Q_t^r \quad (10)$$

And the equation for the total cost of all times from period 1 to considered period T is:

$$TC = \sum_{t=1}^T TC_t \quad (11)$$

For Q_t^r is the number of remaining parts at period t , this is equal to the number of parts ordered at period t plus the amount of parts left from the period $t - 1$ subtract by the number of machine failures at period t .

$$\text{Or: } Q_t^r = Q_t + Q_{t-1}^r - X_t \quad (12)$$

By: $\text{Max}[Q_t^r, 0]$ when $Q_t^r > 0$; Holding cost will occur.
 $-\text{Min}[Q_t^r, 0]$ when $Q_t^r < 0$; Lost cost will occur.

$$\text{For: } Q_t^r < 0; -Q_t^r = SP_t$$

From equation (11), if machine has probability of failures that is not continuous. The total cost equation needs to be changed to be the total expected cost. And rewriting as a form of the expected value is:

$$E(TC(n)) = \sum_{j=1}^k \sum_{t=1}^T \phi_j TC(n)_t \quad (13)$$

V. JOINT PROBABILITY

From equation (13), joint probability Φ_j is the probability that the machines will fail according to each type sequence j . These are the format of all types of machines (in the system) fail. Regarding to this study, machines failures are discrete probabilistic which the Binomial distribution, if probability that one machine is broken (probability of failure) which is equal to P . So, the joint probability is used in the case of M machines will fail equal to X machines that is:

$$P[X] = \binom{M}{X} P^X P^{(M-X)} \quad (14)$$

In case of each machine is independent. And failures of each period are also independent. Therefore, the joint probability of failures from period 1 to period T is:

$$P(X_1, X_2, \dots, X_T) = P(X_1) \cdot P(X_2) \dots \cdot P(X_T) \\ = \prod_{t=1}^T P(X_t) \quad (15)$$

Then, the joint probability of machine failures for each type is:

$$\Phi_j = \prod_{t=1}^T P(X_t) \quad (16)$$

Comparing equations (13) and (16), the sequence n is run from number 1 to total number of possible sets of solution. The sequence j is run from 1 to $(M+1)^T$ or equal to total number for all types of machine failures.

VI. PAYOFF MATRIX

The payoff matrix is also known as decision table. This is thought on the basis of the calculation and find answers to all cases (or complete search), then find the best one for the answer. The purchasing strategy or format of ordered spares which gives the lowest total expected cost will be selected. The equation of the expected cost with multiple machines is:

$$E(Q_{i1}, \dots, Q_{iT}) = \sum_{j=1}^K \sum_{t=1}^T \Phi_j F(Q_{it}, X_{tj}) \quad (17)$$

Where, $E(Q_{i1}, \dots, Q_{iT})$ is expected cost for all purchasing patterns or possible sets of solution. And $F(Q_{it}, X_{tj})$ is the total cost function (this is already mentioned in section IV).

Note: see table 1.

VII. NUMERICAL RESULTS

Assume that the number of machines in the system is 2 machines and there are 2 considered period, with the following conditions:

- Prob. of failures = 0.2
- Spare part unit price is 10,000 Baht.
- Ordering cost is 1,000 Baht per time.
- Holding cost is 1,200 baht per unit per period.
- Opportunity cost 500,000 Baht per machine per one period).

Note: 1. The results and numerical calculation are shown in table 2.

2. Thai currency is Baht and 32.5 Baht is equal to 1 US\$ (on December 2014).

From tables 2, the lowest total expected cost is 37,361.9 Baht when purchases spares on period 1 equal to 2 units and purchases spares on period 2 equal to 1 unit.

The examples shown in point 1 of Table 2, this is the joint probability of the machine failures on period 1 equal to 2 machines and on period 2 equal to 1 machine, that is equal to 0.013. In point 2 in Table 2, the total expected cost is equal to 6,825 Baht. For the case of buying spares on period 1 equal to 1 unit and period 2 equal to 2 units when machine fail on period 1 equal to 2 machines and on period 2 equal to 1 machines, this total cost is equal to 55,456 Baht.

VIII. CONCLUSION AND FUTURE RESEARCH

Application of payoff matrix advantage is to guarantee that the value is the optimal and dependability of 100%, but the payoff matrix for multiple machines and for many considered periods; these will incur the problem of magnitude for the matrix size. By increasing the number of machines and the considered period will cause of incurring of rapid growth of matrix size. As a result, the payoff matrix is used to calculate the answer cannot be done directly. Due to the size of the problem is much more than that calculated by this method directly.

So, the simulation will be alternative in order to reduce the total number of failures statuses or decrease in the numbers of states of nature. And to calculate by using the payoff matrix, then it can increase the number of machines and the number of considered periods.

However, this approach also has limitations. Since it cannot reduce the number of possible sets of solution to be so when increasing the number of machines and the number of periods considered up to a certain level, the simulation and calculation with the payoff matrix, it will cause of problems such as calculating the payoff matrix directly. If required, the calculated to supply spares for a system with multiple machines and have a lot of periods to consider. The appropriate method is to create the heuristics used to calculate the results. Although, the heuristics can to tackle even the size of the matrix size and can be calculated easily and quickly for supply spares than the payoff matrix, but the heuristics method has drawbacks. As the results, it does not guarantee that the results are the best or optimal. But the

results from this method, it is close to the best that is good enough to use it in practice.

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TABLE I. PAYOFF MATRIX

Parts Stocking (Q_{i1}, \dots, Q_{iT}) or Purchasing Strategy	Prob. of failure	$\Phi_1 \Phi_2 \dots \Phi_j \dots \Phi_K$	Expected Cost $E(Q_{i1}, \dots, Q_{iT})$
	T_1	0 1 M 0 1 M 0 1M	
	T_2	0 0 0 1 1 1 M MM	
	No. of failures (X_{i1}, \dots, X_{ij})	: : : : :	
	(State of nature)	T_1 0 0 0 0 0 M MM	
$T_1 T_2 \dots T_T$	T_T 0 0 0 0 0 M MM		
$Q_{11} Q_{12} \dots Q_{1T}$	$a_{11} a_{12} a_{13} \dots a_{1j} \dots a_{1K}$	$E(Q_{11}, \dots, Q_{1T})$	
$Q_{21} Q_{22} \dots Q_{2T}$	$a_{21} a_{22} a_{23} \dots a_{2j} \dots a_{2K}$	$E(Q_{21}, \dots, Q_{2T})$	
: : :	: : :	:	
$Q_{i1} Q_{i2} \dots Q_{iT}$	$a_{i1} a_{i2} a_{i3} \dots a_{ij} \dots a_{iK}$	$E(Q_{i1}, \dots, Q_{iT})$	
: : :	: : :	:	
$Q_{B1} Q_{B2} \dots Q_{BT}$	$a_{B1} a_{B2} a_{B3} \dots a_{Bj} \dots a_{BK}$	$E(Q_{B1}, \dots, Q_{BT})$	

TABLE II. NUMERICAL RESULT

Prob. of failure		0.41	0.205	0.026	0.205	0.1024	0.013	0.026	0.013	0.002	1.00	
Possible sets of solution	Period	States of nature										Total expected cost
	1	0	1	2	0	1	2	0	1	2		
Period	2	0	0	0	1	1	1	2	2	2		
1	2	Row No.	Expected cost									
0	0	1	0	1E+05	25600	1E+05	102400	19200	25600	19200	3200	400,000.0
0	1	2	4997	1E+05	25912	2253	52326	12941	13082	12941	2418	231,768.0
0	2	3	9585	1E+05	26199	4547	53473	13084	537.6	6669	1634	222,920.0
1	0	4	5489	2253	13082	2499	52326	12941	13112	12941	2418	117,059.5
1	1	5	10486	4751	13394	4997	2252.8	6682	593.9	6682	1635	51,473.3
1	2	6	15073	7045	13681	7291	3399.7	6825	880.6	409.6	851.2	55,456.0
2	0	7	10568	4792	537.6	5038	2273.3	6669	599	6684	1634	38,794.6
2	1	8	15565	7291	849.9	7537	3522.6	409.6	911.4	425	851.2	37,361.9
2	2	9	20152	9585	1137	9830	4669.4	553	1198	568.3	67.2	47,760.0
3	0	10	15647	7332	855	7578	3543	412.2	916.5	427.5	851.5	37,561.9
3	1	11	20644	9830	1167	10076	4792.3	568.3	1229	583.7	69.12	48,960.0
4	0	12	20726	9871	1172	10117	4812.8	570.9	1234	586.2	69.44	49,160.0