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Space - Time as a Complete Measure Manifold of Dimension-4

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Abstract: **- In this paper we introduce the structure of space-time as a complete measure manifold of dimension-4 endowed with a partial order relation $\prec$ defined by causal connections. This new approach to space-time structure gives us insight into the dark reason of a space-time that is still measurable.**

Key words: Causal connectedness, complete measure manifold, measure manifold, sequential connectedness.

Subject Classification: 57N13, 58C35.

1. INTRODUCTION

Space–time is the 4-dimensional manifold in which all physical events take place. An event is a point in space- time specified by its space and time co-ordinates. In physics space- time is a mathematical model that combines space and time into a single manifold called Minkowski space time. In cosmology the concept of space-time combines space and time to a single abstract universe. The common practice to study Minkowski space–time is by selecting Lorentz metric or Minkowski metric that measures the interval between two events in space-time that is, $ds^2=dx^2+dy^2+dz^2+(icdt)^2$. The interval ds^2 may be classified into three different types, time-like ($ds^2<0$), light like ($ds^2=0$) and space like ($ds^2>0$).

For some physical applications, a space-time continuum is mathematically a 4-dimensional smooth connected Lorentzian manifold (M, g), g is a Lorentz metric with signature (3, 1). The causal structure of Lorentzian manifold describes the causal relation between the events in the manifold. Hawking, S. W. Ellis, G.F.R(1973)[4], Penrose R. (1972)[7] have studied causal structure in terms of causal future, causal past, chronological future , chronological past.

The major distinction in the study of physics is the difference between local and global structures, measurement in physics are done in the local neighborhood of the events in the space- time, leading to the study of local structure of space-time in general relativity. The study of global space- time structure is vital in cosmological problems.

Generally, over lapping coordinate charts covers a manifold. The intersection of two charts represents a region of space–time in which two observers can measure physical quantities and compare their results. The concepts of co-ordinate charts as local observer who perform their measurements to collect and compare their results in the non empty intersection of charts locally, is vital to our mathematical model. The concept of connectedness serves this purpose. Without connectedness this would not be possible.

In this paper we study the concept of causal connected μ_1-a.e., sequentially connected μ_1-a.e. and maximally connected μ_1-a.e., on complete measure manifold of dimension-4 introduced by S. C. P. Halakatti ([9], [10], [11], [12], [13],[14]). We have proved some results on space-time manifold $(M^4,\mathcal{T}_1,\Sigma_1,\mu_1)$. The significance of these results are, sequentially connected μ_1-a.e. is an invariant property under measurable homeomorphism and measure invariant function F. Also, we have developed the concept of maximal connectedness on $(M^4,\mathcal{T}_1,\Sigma_1,\mu_1)$ using sequential connected μ_1-a.e. property. It is interesting to see that if $\mathbb{A}_i$ is sequentially connected μ_1-a.e. to $\mathbb{A}_j$ and $\mathbb{A}_j$ is sequentially connected μ_1-a.e. to $\mathbb{A}_l$, under the composition of two measurable homeomorphic and measure invariant function $G\circ F\circ\gamma$ on $\mathbb{A}_i \cup \mathbb{A}_j \cup \mathbb{A}_l \in A^k(M^4)$ then $(M^4,\mathcal{T}_1,\Sigma_1,\mu_1)$ is maximally connected.

The following concepts are introduced and developed by S. C. P. Halakatti to generate a causally connected network manifold, whose applications are in the field of neural network, brain structure, in the study of large scale structures and engineering science.

2. PRELIMINARIES

Some basic definitions referred are as follows

Definition 2.1: Convergence point wise almost everywhere on $(M, \mathcal{T}_1, \Sigma_1, \mu_1)$

Let $f_n \to f$ point wise almost everywhere in $(\mathbb{R}^n, \mathcal{T}, \Sigma, \mu)$ and if any measure manifold $(M, \mathcal{T}_1, \Sigma_1, \mu_1)$ is measurable homeomorphic to $(\mathbb{R}^n, \mathcal{T}, \Sigma, \mu)$ then for every $x \in A_n \in (\mathbb{R}^n, \mathcal{T}, \Sigma, \mu)$ $\exists\ \phi^{-1}(x) = p \in \phi^{-1}(A_n)$ denoted by,
$S = \phi^{-1}(A_n) \in (U, \phi) \in (M, \mathcal{T}_1, \Sigma_1, \mu_1)$ and $(f_n \circ \phi) \to f \circ \phi$ point wise a.e. in $S \in (U, \phi) \in (M, \mathcal{T}_1, \Sigma_1, \mu_1)$ such that,
$S = \phi^{-1}(A_n) =\{p=\phi^{-1}(x)\in(M, \mathcal{T}_1, \Sigma_1, \mu_1): |(f_n \circ \phi)(p) - (f \circ \phi)(p) | < \epsilon , \forall n \in N\}$ on the chart (U, ϕ) satisfying the following conditions:

(i) $\mu_1(S) > 0$, if $| (f_n \circ \phi)(p) - (f \circ \phi)(p) | < \epsilon , \forall n \in N$,

(ii) $\mu_1(S) = 0$, if $| (f_n \circ \phi)(p) - (f \circ \phi)(p) | \geq \epsilon , \forall n \geq N$, that is, $\mu_1(S) = 0$ as $n \to \infty$.

Definition 2.2: Convergence μ_1-a.e. on (M, $\mathcal{T}_1$, Σ_1, μ_1)

Let $f_n \xrightarrow{\mu_1-a.e} f$ in $A_n \in (\mathbb{R}^n, \mathcal{T}, \Sigma, \mu)$ and if any measure manifold (M, $\mathcal{T}_1$, Σ_1, μ_1) is measurable homeomorphic to ($\mathbb{R}^n$, $\mathcal{T}$, Σ, μ) then for every $A_n \in (\mathbb{R}^n, \mathcal{T}, \Sigma, \mu)$ $\exists$ S $\in$ (U,ϕ) $\in$ (M,$\mathcal{T}_1$,Σ_1, μ_1) such that, $(f_n \circ \phi) \xrightarrow{\mu_1-a.e} (f \circ \phi)$, $\forall$ x on S $\in$ (U,ϕ) $\in$ (M, $\mathcal{T}_1$, Σ_1, μ_1) and

S = { p ∈ (M, $\mathcal{T}_1$, Σ_1, μ_1) : $|(f_n \circ \phi)(p) - (f \circ \phi)(p)| < \epsilon$, ∀ n ∈ N} with,

(i)$\mu_1(S) > 0$, if $|(f_n \circ \phi)(p) - (f \circ \phi)(p)| < \epsilon$, ∀ n ∈ N,

(ii)$\mu_1(S) = 0$, if $|(f_n \circ \phi)(p) - (f \circ \phi)(p)| \geq \epsilon$, ∀ n ≥ N, that is, $\mu_1(S) = 0$.

Definition 2.3: Complete Measure Manifold

If (M, $\mathcal{T}_1$, Σ_1, μ_1) is a measure manifold of dimension n and suppose that for every measure chart (U, ϕ) ⊆ (M, $\mathcal{T}_1$, Σ_1, μ_1), μ_1(U)=0 and every V ⊂ (U, ϕ), if μ_1(V) = 0, then (M, $\mathcal{T}_1$, Σ_1, μ_1) is called as a complete measure manifold.

Let (M, $\mathcal{T}_1$, Σ_1, μ_1) be a complete measure manifold of dimension n which is measurable homeomorphic to a measure space ($\mathbb{R}^n$, $\mathcal{T}$, Σ, μ). Let {f_n}, {g_n} be measurable real valued functions converging to f and g respectively in ($\mathbb{R}^n$, $\mathcal{T}$, Σ, μ).

Since ϕ is measurable homeomorphism from (M, $\mathcal{T}_1$, Σ_1, μ_1) to ($\mathbb{R}^n$, $\mathcal{T}$, Σ, μ) for every {f_n} and {g_n} on ($\mathbb{R}^n$, $\mathcal{T}$, Σ, μ) there exist corresponding measurable real valued functions {$f_n \circ \phi$} and {$g_n \circ \phi$} converging to $f \circ \phi$ and $g \circ \phi$ on (M, $\mathcal{T}_1$, Σ_1, μ_1).

The ordered pair ({$f_n \circ \phi$}, $f \circ \phi$) induces a Borel subset S ∈ (U, ϕ) ∈ (M, $\mathcal{T}_1$, Σ_1, μ_1) satisfying the following condition:

S = {p ∈ (M, $\mathcal{T}_1$, Σ_1, μ_1): $|(f_n \circ \phi)(p) - (f \circ \phi)(p)| < \epsilon$, ∀ n ∈ N} on the chart (U, ϕ) for which $\mu_1(S) > 0$.

Definition 2.4: Locally path connected μ_1 – a.e. on complete measure manifold

The Borel subset S is locally path connected μ_1 –a.e. if ∃ a C^∞- map

$\gamma : [0, 1] \rightarrow S \in (U, \phi)$ such that

$\gamma(0) = p \in S$,

$\gamma(1) = q \in S$, such that $\mu_1(S) > 0$.

That is, p is locally path connected μ_1 –a.e. to q in S ⊂ (U, ϕ) ∈ A ∈ A^k(M).

That is, locally path connectedness μ_1 –a.e. is between two points in the same chart ∈ A ∈ A^k(M).

If $\mu_1(S) = 0$, then there does not exist a path γ between p and q.

Definition 2.5:

If $\mu_1(S) = 0$ where $(f_n \circ \phi) \nrightarrow f \circ \phi$, then S ⊂ (U, ϕ) ⊂ (M, $\mathcal{T}_1$, Σ_1, μ_1) is called as a dark region in the chart (U, ϕ).

Let (M, $\mathcal{T}_1$, Σ_1, μ_1) be a complete measure manifold on which {$f_n \circ \phi$} and {$g_n \circ \phi$} are sequence of real valued measurable functions converging to $f \circ \phi$ and $g \circ \phi$ pointwise μ_1 – a.e. on (U, ϕ) and (V, ψ) belonging to the atlas $\mathbb{A}$ respectively. The ordered pairs ({$f_n \circ \phi$}, $f \circ \phi$) and ({$g_n \circ \phi$}, $g \circ \phi$) induce two Borel subsets S ∈ (U, ϕ) ∈ A ∈ A^k(M) and R ∈ (V, ψ) ∈ $\mathbb{A}$ ∈ A^k(M) satisfying the following condition:

S = {p ∈ (M, $\mathcal{T}_1$, Σ_1, μ_1): $|(f_n \circ \phi)(p) - (f \circ \phi)(p)| < \epsilon$, ∀ n ∈ N} on the chart (U, ϕ) for which $\mu_1(S) > 0$.

R = {q ∈ (M, $\mathcal{T}_1$, Σ_1, μ_1): $|(g_n \circ \phi)(q) - (g \circ \phi)(q)| < \epsilon$, ∀ n ∈ N} on the chart (V, ψ) for which $\mu_1(R) > 0$.

Note: We denote the Borel subsets $\phi^{-1}(A_n)$ = S ∈(U,ϕ)∈ (M,$\mathcal{T}_1$,Σ_1,μ_1) and $\phi^{-1}(B_n)$ = R ∈ (V, ψ) ∈ (M, $\mathcal{T}_1$, Σ_1, μ_1).

Definition 2.6: Interconnected μ_1 – a.e on complete measure manifold

The Borel subset S ∈ (U, ϕ) ∈ A ∈ (M, $\mathcal{T}_1$, Σ_1, μ_1) is interconnected to the Borel subset R ∈ (V, ψ) ∈ A ∈ (M, $\mathcal{T}_1$, Σ_1, μ_1) μ_1 –a.e. if ∃ a C^∞- map

$\gamma : [0, 1] \rightarrow S \cup R \in \mathbb{A} \in A^k(M)$ such that

$\gamma(0) = p \in S \in \mathbb{A}$,

$\gamma(1) = q \in S \in \mathbb{A}$, such that $\mu_1(S) > 0$ and $\mu_1(R) > 0$.

That is, p is interconnected μ_1 – a.e. to q in S ∪ R ∈ A ∈ A^k(M).

That is, interconnectedness μ_1 – a.e. is between two charts in the same atlas ∈ A^k(M).

If $\mu_1(S)$ =0 and $\mu_1(R)$ =0 then ∄ a path between p and q.

Definition 2.7:

If μ_1(S)=0 where {$f_n \circ \phi$} ↛ $f \circ \phi$ in (U,ϕ) and μ_1(R)=0 where {$g_n \circ \phi$} ↛ $g \circ \phi$ in (V, ψ), then S is called as dark region in the chart (U, ϕ) and R is called as dark region in the chart (V, ψ) belonging to the same atlas A in A^k(M).

Let ($\mathbb{R}^n$, $\mathcal{T}$, Σ, μ) be a measure space and {f_n}, {g_n}and {h_n}are sequences of measurable functions on ($\mathbb{R}^n$,$\mathcal{T}$,Σ,μ) converging to f, g and h point wise almost everywhere on ($\mathbb{R}^n$, $\mathcal{T}$, Σ, μ). The ordered pairs ({f_n},f), ({g_n}, g) and ({h_n}, h) induce the following Borel subsets A_n, B_n and C_n.

We define Borel subsets

A_n = { x ∈ ($\mathbb{R}^n$, $\mathcal{T}$, Σ, μ) : $|f_n(x) - f(x)| < \epsilon$ } , ∀ n ∈ N, where

(i) $\mu(A_n) > 0$, if $|f_n(x) - f(x)| < \epsilon$, ∀ n ∈ N,

(ii) $\mu(A_n) = 0$, if $|f_n(x) - f(x)| \geq \epsilon$, ∀ n ≥ N, that is, $\mu(A_n) = 0$ as n→∞.

Similarly,

(i)B_n = { y ∈ ($\mathbb{R}^n$, $\mathcal{T}$, Σ, μ) : $|g_n(y) - g(y)| < \epsilon$ }, ∀ n ∈ N, where

$\mu(B_n) > 0$, if $|g_n(y) - g(y)| < \epsilon$, ∀ n ∈ N,

(ii)$\mu(B_n) = 0$, if $|g_n(y) - g(y)| \geq \epsilon$, ∀ n ≥ N, that is, $\mu(B_n) = 0$ as n→∞ and

C_n = { z ∈ ($\mathbb{R}^n$, $\mathcal{T}$, Σ, μ) : $|h_n(z) - h(z)| < \epsilon$ }, ∀ n ∈ N, where

(i) $\mu(C_n) > 0$, if $|h_n(z) - h(z)| < \epsilon$, ∀ n ∈ N,

(ii) $\mu(C_n) = 0$, if $|h_n(z) - h(z)| \geq \epsilon$, ∀ n ≥ N, that is, $\mu(C_n) = 0$ as n→∞.

If (M, $\mathcal{T}_1$, Σ_1, μ_1) is a complete measure manifold that is measurable homeomorphic and measure invariant to ($\mathbb{R}^n$, $\mathcal{T}$, Σ, μ). Then ∃ a measurable homeomorphism and measure invariant transformation
ϕ: (M, $\mathcal{T}_1$, Σ_1, μ_1)→ ($\mathbb{R}^n$, $\mathcal{T}$, Σ, μ), such that, $\{f_n\circ \phi\}$, $\{g_n\circ \phi\}$ and $\{h_n\circ \phi\}$ are sequences of real valued measurable functions converging to f∘ ϕ, g∘ ϕ and h∘ ϕ point wise μ_1-a.e. on (U, ϕ), (V, ψ) and (W, χ) belonging to the atlases $\mathbb{A}_i$, $\mathbb{A}_j$, $\mathbb{A}_l$ respectively. Also, for every induced Borel subsets A_n, B_n and C_n in ($\mathbb{R}^n$, $\mathcal{T}$, Σ, μ), ∃ the corresponding induced Borel subsets namely
S ∈ (U, ϕ) ∈ $\mathbb{A}_i$ ∈ $A^k(M)$, R ∈ (V, ψ) ∈ $\mathbb{A}_j$ ∈ $A^k(M)$ and Q ∈ (W, χ) ∈ $\mathbb{A}_l$ ∈ $A^k(M)$ on (M, $\mathcal{T}_1$, Σ_1, μ_1).
Now, we define S, R and Q as follows:
S = {p ∈ (M, $\mathcal{T}_1$, Σ_1, μ_1): | $(f_n \circ \phi)(p) - (f \circ \phi)(p)$ | < ϵ , ∀ n ∈ N} on the chart (U, ϕ) ∈ $\mathbb{A}_i$ ∈ $A^k(M)$, for which $\mu_1(S) > 0$,
R = {q ∈ (M, $\mathcal{T}_1$, Σ_1, μ_1): | $(g_n \circ \phi)(q) - (g \circ \phi)(q)$ | < ϵ , ∀ n ∈ N} on the chart (V, ψ) ∈ $\mathbb{A}_j$ ∈ $A^k(M)$, for which $\mu_1(R) > 0$ and
Q = {r ∈ (M, $\mathcal{T}_1$, Σ_1, μ_1): | $(h_n \circ \phi)(r) - (h \circ \phi)(r)$ | < ϵ , ∀ n ∈ N} on the chart (W, χ) ∈ $\mathbb{A}_l$ ∈ $A^k(M)$, for which $\mu_1(Q) > 0$.
Note: We denote the Borel subsets $\phi^{-1}(A_n)$ = S ∈ (U, ϕ)∈ $\mathbb{A}_i$ ∈ $A^k(M)$, $\phi^{-1}(B_n)$= R∈(V, ψ) ∈ $\mathbb{A}_j$∈$A^k(M)$ and $\phi^{-1}(C_n)$=Q∈ (W, χ)∈ $\mathbb{A}_l$ ∈ $A^k(M)$.

Definition 2.8: Maximal connected μ_1-a.e on complete measure manifold

Let (M, $\mathcal{T}_1$, Σ_1, μ_1) be a complete measure manifold and let A_i, $\mathbb{A}_j$ and $\mathbb{A}_l$ ∈ $A^k(M)$ be atlases on (M, $\mathcal{T}_1$, Σ_1, μ_1). Let S, R and Q be Borel subsets of $\mathbb{A}_i$, $\mathbb{A}_j$ and $\mathbb{A}_l$. Then, we say that $A^k(M)$ ∈ (M, $\mathcal{T}_1$, Σ_1, μ_1) is maximally connected if ∃ a map γ:[0,1]→ S ∪ R ∪ Q ∈ $\mathbb{A}_i$ ∪ $\mathbb{A}_j$ ∪ $\mathbb{A}_l$ ∈ $A^k(M)$ such that, γ(0) = p ∈ S ∈ (U, ϕ) ∈ $\mathbb{A}_i$ ∈ $A^k(M)$ for which $\mu_1(S) >$ 0,
$\gamma(\frac{1}{2})$ = q ∈ R ∈ (V, ψ) ∈ $\mathbb{A}_j$ ∈ $A^k(M)$ for which $\mu_1(R) > 0$ and γ(1) = r ∈ Q ∈ (W, χ) ∈ $\mathbb{A}_l$ ∈ $A^k(M)$ for which $\mu_1(Q) > 0$.
That is, for each p ∈ (U, ϕ) ∈ $\mathbb{A}_i$ is path connected to each q ∈ (V, ψ) ∈ $\mathbb{A}_j$, for $\mathbb{A}_i$ ∪ $\mathbb{A}_j$ ∈ $A^k(M)$, $\mu_1(\mathbb{A}_i \cup \mathbb{A}_j) > 0$ for each q ∈ (V, ψ) ∈ $\mathbb{A}_j$ is path connected to each r ∈ (W, χ) ∈ $\mathbb{A}_l$ ∈ $A^k(M)$ and for , $\mathbb{A}_j$ ∪ $\mathbb{A}_l$ ∈ $A^k(M)$, $\mu_1(\mathbb{A}_j \cup \mathbb{A}_l) > 0$. Then, if for each p ∈ (U, ϕ) ∈ $\mathbb{A}_i$ is path connected to each r ∈ (W, χ) ∈ $\mathbb{A}_l$ ∈ $A^k(M)$ and for $\mathbb{A}_i$ ∪ $\mathbb{A}_l$ ∈ $A^k(M)$, $\mu_1(\mathbb{A}_i \cup \mathbb{A}_l) > 0$ then ($\mathbb{A}_i$∪ $\mathbb{A}_j$∪ $\mathbb{A}_l$) ∈ $A^k(M)$ ∈ (M, $\mathcal{T}_1$, Σ_1, μ_1) is maximally path connected if $\mu_1(\mathbb{A}_i \cup \mathbb{A}_j \cup \mathbb{A}_l) > 0$ on complete measure manifold.

If $\mu_1(S) = 0$ and $\mu_1(R) = 0$ then there does not exist a path γ between p ∈ S and q ∈ R.

Definition 2.9:

If $\mu_1(S) = 0$ where $\{f_n\circ \phi\} \nrightarrow f \circ \phi$ in (U, ϕ) and $\mu_1(R) = 0$ where
$\{g_n\circ \phi\} \nrightarrow g\circ \phi$ in (V, ψ) and $\mu_1(Q) = 0$ where $\{h_n\circ \phi\} \nrightarrow h\circ \phi$ in (W, χ), then S is called as *dark region* in the chart (U, ϕ) ∈ $\mathbb{A}_i$, R is called as *dark region* in the chart (V, ψ) ∈ $\mathbb{A}_j$ and Q is called as *dark region* in the chart (W, χ) ∈ $\mathbb{A}_l$ in $A^k(M)$.

3. DIFFERENT VERSIONS OF CONNECTEDNESS ON SPACE-TIME MANIFOLD OF DIMENSION - 4

Let (M^4, $\mathcal{T}_1$, Σ_1, μ_1) be a complete measure manifold of dimension- 4.
Let $p_i = (x_i, y_i, z_i, t_i)$ and $p_j = (x_j, y_j, z_j, t_j)$ are events in (M^4,$\mathcal{T}_1$,Σ_1,μ_1), where (x, y, z) are space coordinates and t is the time co-ordinate.
Let f_n and f be measurable functions on the chart (U, ϕ) of a complete measure manifold (M^4, $\mathcal{T}_1$, Σ_1, μ_1) such that
f_n : U →ℝ converges to f: U →ℝ μ_1-a.e., ∀ p_i ∈(U, ϕ)∈ (M^4,$\mathcal{T}_1$,Σ_1,μ_1) then, the ordered pair ({f_n}, f) induces a set S of all events p_i, that is S ={p_i ⊂ (U_i, ϕ_i)∈ (M^4,$\mathcal{T}_1$,Σ_1,μ_1) : $|f_n(p_i) - f(p_i)| < \epsilon$, i=1,2….,n}with the following conditions:
(i) $\mu_1(S) > 0$ if $|f_n(p_i) - f(p_i)| < \epsilon$ ∀ n ∈ N , i=1,2…
(ii) $\mu_1(S) = 0$ if $|f_n(p_i) - f(p_i)| > \epsilon$ ∀ n ≥ N ,i=1,2…..
We introduce Causal Connectedness μ_1-a.e., sequentially connected μ_1-a.e. and maximally connectedness μ_1-a.e. on (M^4,$\mathcal{T}_1$,Σ_1,μ_1).

Definition 3.1: Causally connected μ_1-a.e.

Let (U, ϕ) ⊂ (M^4, $\mathcal{T}_1$,Σ_1,μ_1) and let p_i and p_j , ∀ i < j, i, j=1,2,…,n are events in (U, ϕ) then we say that (U, ϕ) is causally connected μ_1-a.e. if there exists a C^∞ map γ : [0,1]→S ⊂ U ⊂ M^4 between p_i and p_j :
$\gamma(0) = p_i$ ∈ S ⊂ U ⊂ (M^4, $\mathcal{T}_1$,Σ_1,μ_1), $\mu_1(S) > 0$,
$\gamma(1) = p_j$ ∈ S ⊂ U ⊂ (M^4,$\mathcal{T}_1$,Σ_1,μ_1), $\mu_1(S) > 0$, ∀ i, j ∈ I, where $p_i < p_j$ for $t_i < t_j$, ∀ i≠j∈ I. Then a relation '<' is called as a causal connection μ_1-a.e between p_i and p_j. p_i is called causally connected μ_1-a.e. to p_j in S ∈ (U,ϕ) ⊂ (M^4,$\mathcal{T}_1$,Σ_1,μ_1), if $\mu_1(U) > 0$, $\mu_1(S) > 0$.

Note:
(i) If $\mu_1(S) > 0$ then the events are separated by time-like interval.
(ii) If $\mu_1(U) = 0$, then the events are not causally connected in (U, ϕ) and they are separated by space-like interval. Then S can be recognized as dark region of (U, ϕ).

Definition 3.2: Sequentially connected μ_1-a.e.

Let (M^4, $\mathcal{T}_1$,Σ_1,μ_1) be a complete measure manifold and let $\mathbb{A}$ be a atlas in $A^k(M^4)$, where (U_i, ϕ_i) ∈ $\mathbb{A}$ then we say $\mathbb{A}$ is sequentially connected
μ_1-a.e. if ∃ a C^∞-map γ : [0, 1]→ $\mathbb{A}$ ⊂ $A^k(M)$ such that,
$\gamma(0) = p_1$ ∈ S ∈ (U_1, ϕ_1) ∈ $\mathbb{A}$ ⊂ (M^4, $\mathcal{T}_1$, Σ_1, μ_1), $\mu_1(S) > 0$,
$\gamma(\frac{k}{2^n}) = p_i$ ∈ S ∈ (U_i, ϕ_i) ∈ $\mathbb{A}$ ⊂ (M^4, $\mathcal{T}_1$, Σ_1, μ_1), ∀ $\frac{k}{2^n}$ ∈ (0, 1), $k < 2^n$
∀ i ∈ I, $\mu_1(U_i) > 0$ and $\mu_1(S) > 0$.
$\gamma(1) = p_n$ ∈ S ∈ (U_n, ϕ_n) ∈ $\mathbb{A}$ ⊂ (M^4, $\mathcal{T}$, Σ_1, μ_1), $\mu_1(S) > 0$ satisfying a causal relation '<' such that $p_1 < … < p_i < …. < p_n$ for $t_1 < … < t_i < …. < t_n$, ∀ i < j.
Then the relation '<' is called a sequentially connected μ_1-a.e. on S ∈
(U, ϕ)∈$\mathbb{A}$ ⊂ (M^4,$\mathcal{T}_1$,Σ_1,μ_1), if $\mu_1(S) > 0$, $\mu_1(U) > 0$, where U = $\cup_{i=1}^{n}(U_i, \phi_i)$.

Let $(M^4, \mathcal{T}_1, \Sigma_1, \mu_1)$ be a complete measure manifold and let $\mathbb{A}_i$, $\mathbb{A}_j$ and $\mathbb{A}_l \in A^k(M^4)$ and $\{f_n\}$, $\{g_n\}$, $\{h_n\}$ are measurable functions converging to real valued functions f, g , h on $\mathbb{A}_i$, $\mathbb{A}_j$, $\mathbb{A}_l$. Then the ordered pairs $(\{f_n\}, f)$, $(\{g_n\}, g)$, $(\{h_n\}, h)$ induces the following sets S_1, S_2, S_3 on $\mathbb{A}_i$, $\mathbb{A}_j$, $\mathbb{A}_l$ respectively, where $\mathbb{A}_i$ is sequentially connected to $\mathbb{A}_j$ and $\mathbb{A}_j$ is sequentially connected to $\mathbb{A}_l$ by the functions F and G respectively.

$S_1 = \{p_i \in (U_i, \phi_i) \in \mathbb{A}_i \in A^k(M^4): |f_n(p_i) - f(p_i)| < \epsilon \ \forall\, n \in N, i=1,\dots n\}$,

$S_2 = \{q_i \in (V_i, \Psi_i) \in \mathbb{A}_j \in A^k(M^4): |g_n(q_i) - g(q_i)| < \epsilon \ \forall\, n \in N, i=1,\dots n\}$,

where $g_n = f_n \circ F^{-1}$ on $\mathbb{A}_j$ and

$S_3 = \{r_i \in (W_i, \chi_i) \in \mathbb{A}_l \in A^k(M^4): |h_n(r_i) - h(r_i)| < \epsilon \ \forall\, n \in N, i=1,\dots n\}$,

where $h_n = (g_n \circ f_n) \circ F^{-1}$ on $\mathbb{A}_l$.

Since $\mathbb{A}_i$ and $\mathbb{A}_j$ are sequentially connected by measurable homeomorphism and measure invariant map F and $\mathbb{A}_j$ and $\mathbb{A}_l$ are sequentially connected by measurable homeomorphism and measure invariant map G, if $\mathbb{A}_i$ is sequentially connected to $\mathbb{A}_l$ by measurable homeomorphism and measure invariant map G∘F then $(M^4, \mathcal{T}_1, \Sigma_1, \mu_1)$ is maximally connected. Therefore, one can develop the following definition.

Definition 3.3: Maximal Connectedness μ_1-a.e. on $(M^4, \mathcal{T}_1, \Sigma_1, \mu_1)$

Let $(M^4, \mathcal{T}_1, \Sigma_1, \mu_1)$ be a complete measure manifold and $\mathbb{A}_i$, $\mathbb{A}_j$, $\mathbb{A}_l \in A^k(M^4)$ are mutually sequentially connected by measurable homeomorphism and measure invariant maps F and G and G∘F respectively, then $(M^4, \mathcal{T}_1, \Sigma_1, \mu_1)$ is maximally connected μ_1-a.e.

4. SOME RESULTS ON SPACE-TIME MANIFOLD

It is necessary to note that if any atlas $\mathbb{A}_i$ is sequentially connected μ_1-a.e. then we show that any other atlas $\mathbb{A}_j$, $i \neq j \in N$ is also sequentially connected μ_1-a.e. that is, sequentially connected μ_1-a.e. is an invariant property under measurable homeomorphism and measure invariant function F.

Theorem 4.1:

Let $(M^4, \mathcal{T}_1, \Sigma_1, \mu_1)$ be a complete measure-manifold, where $\mathbb{A}_i$, $\mathbb{A}_j \in A^k(M^4)$. Let $F: \mathbb{A}_i \to \mathbb{A}_j$ be a C^∞ measurable homeomorphism and measure invariant map. If $\mathbb{A}_i$ is sequentially connected μ_1-a.e., then $\mathbb{A}_j$ is also sequentially connected μ_1-a.e.in $A^k(M^4)$.

Proof: Let $(M^4, \mathcal{T}_1, \Sigma_1, \mu_1)$ be a complete measure manifold and $\mathbb{A}_i$ and $\mathbb{A}_j \in A^k(M^4)$. Let $F: \mathbb{A}_i \to \mathbb{A}_j$ be a C^∞ measurable homeomorphism and measure invariant map. We show that if $\mathbb{A}_i$ is sequentially connected μ_1-a.e. then $\mathbb{A}_j$ is also sequentially connected μ_1-a.e.

Let $\mathbb{A}_i$ be sequentially connected μ_1-a.e.on $S \in (U_i, \phi_i) \in \mathbb{A}_i \in A^k(M^4)$.

By definition of sequentially connectedness, the induced set S is defined as, $S = \{ p_i \in (U_i, \phi_i) \in \mathbb{A}_i \in A^k(M^4) : |f_n(p_i) - f(p_i)| < \epsilon, \forall\, i=1,\dots,n\}$ satisfying, $\mu_1(S) > 0$ in $(U_i, \phi_i) \in \mathbb{A}_i$ and $\exists$ C^∞-map $\gamma : [0,1] \to \mathbb{A}_i \in A^k(M^4)$, such that,

$\gamma(0) = p_1 \in (U_1, \phi_1) \in \mathbb{A}_i \in A^k(M^4)$, $\mu_1(S) > 0$, $\mu_1(U_1) > 0$,

$\gamma(\frac{k}{2^n}) = p_i \in (U_i, \phi_i) \in \mathbb{A}_i \in A^k(M^4)$, $\forall\, (\frac{k}{2^n}) \in (0,1)$, $k < 2^n$, $1 < i < n$, $\mu_1(S) > 0$ and $\mu_1(U_i) > 0$,

$\gamma(1) = p_n \in S \in (U_n, \phi_n) \in \mathbb{A}_i \in A^k(M^4)$, $\mu_1(S) > 0$ and $\mu_1(U_n) > 0$ satisfying a Causal relation '<' in $S \in (U_1, \phi_1) \in \mathbb{A}_i \in A^k(M^4)$ such that, $p_1 < \dots < p_i < \dots < p_n$ for $t_1 < \dots < t_i < \dots < t_n$, $\forall\, i < j$, where $\mu_1(S) > 0$, $\mu_1(U_i) > 0$ where $\mathbb{A}_i = \bigcup_{i=1}^{n}(U_i, \phi_i)$.

Since $F: \mathbb{A}_i \to \mathbb{A}_j$ is a measurable homeomorphism and measure invariant map, for every $(U_i, \phi_i) \in \mathbb{A}_i \in A^k(M^4)$ $\exists$ the corresponding $F(U_j) = V_j \in \mathbb{A}_j \in A^k(M^4)$, where (V_j, ψ_j) is a chart in the atlas $\mathbb{A}_j \in A^k(M^4)$.

Also, for every S in $\mathbb{A}_i$ $\exists$ a corresponding set F(S) in $\mathbb{A}_j$ defined as,

$F(S) = \{q_j \in (V_j, \psi_j) \in \mathbb{A}_j \in A^k(M^4) : | g_n(q_j) - g(q_j) | < \epsilon, j=1,\dots,n\}$ satisfying, $\mu_1(F(S)) > 0$ in $(V_j, \psi_j) \in \mathbb{A}_j$ and $\exists$ C^∞-map $F \circ \gamma : [0,1] \to \mathbb{A}_j \in A^k(M^4)$, such that,

$F\circ\gamma(0) = q_1 \in F(S) \in (V_1, \psi_1) \in \mathbb{A}_j \in A^k(M^4)$, $\mu_1(F(S)) > 0$, $\mu_1(V_1) > 0$,

$F\circ\gamma(\frac{k}{2^n}) = q_i \in F(S) \in (V_i, \psi_i) \in \mathbb{A}_j \in A^k(M^4)$, $\mu_1(F(S)) > 0$, $\mu_1(V_i) > 0$,

$F\circ\gamma(1) = q_n \in F(S) \in (V_n, \psi_n) \in \mathbb{A}_j \in A^k(M^4)$, $\mu_1(F(S)) > 0$, $\mu_1(V_n) > 0$

such that, $q_1 < \dots < q_i < \dots < q_n$ for $t_1 < \dots < t_i < \dots < t_n$, $\forall\, i < j$,

where $\mu_1(F_1(S)) > 0$, $\mu_1(V_i) > 0$ and $\mu_1(\mathbb{A}_j) > 0$ where $\mathbb{A}_j = \bigcup_{i=1}^{n}(V_i, \psi_i)$.

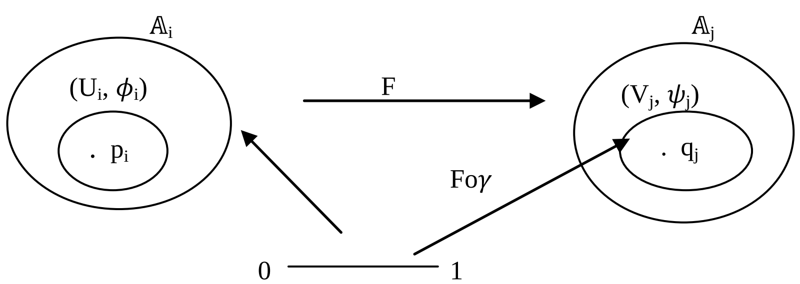

Fig 1

Therefore, the events in $\mathbb{A}_j$ are sequentially connected μ_1-a.e. in $(M^4, \mathcal{T}_1, \Sigma_1, \mu_1)$ if $\mu_1(S) > 0$, $\mu_1(V_i) > 0$ and $\mu_1(\mathbb{A}_j) > 0$.

Therefore, if $\mathbb{A}_i$ is sequentially connected μ_1-a.e. in $S \in (U_i, \phi_i) \in \mathbb{A}_i \in A^k(M^4)$ then $\mathbb{A}_j$ is also sequentially connected μ_1-a.e. in $F(S) \in (V, \psi) \in \mathbb{A}_j \in A^k(M^4)$ under measurable homeomorphism and measure invariant transformation.

In other words, sequentially connectedness μ_1-a.e. on $(M^4, \mathcal{T}_1, \Sigma_1, \mu_1)$ is invariant under measurable homeomorphism and measure invariant function F.

Theorem 4.2:
Let (M^4, $\mathcal{T}_1$, Σ_1, μ_1) be a complete measure manifold that is maximally path connected μ_1-a.e. where $\mathbb{A}_i$, $\mathbb{A}_j$, $\mathbb{A}_l$ $\in A^k(M^4)$. If $\mathbb{A}_i$ is sequentially connected μ_1-a.e to $\mathbb{A}_j$ by measurable homeomorphism and measure invariant map F and if $\mathbb{A}_j$ is sequentially connected μ_1-a.e to $\mathbb{A}_l$ by measurable homeomorphism and measure invariant map G then $\mathbb{A}_i$ is sequentially connected μ_1-a.e. to $\mathbb{A}_l$ by measurable homeomorphism and measure invariant map $G \circ F$, $\forall$ i, j, l $\in$ N, $t_i < t_j < t_l$.

Proof: Let (M^4, $\mathcal{T}_1$, Σ_1, μ_1) be a complete measure manifold which is maximally path connected μ_1-a.e. and $\mathbb{A}_i$, $\mathbb{A}_j$ $\in A^k(M^4)$. Let F: $\mathbb{A}_i \longrightarrow \mathbb{A}_j$ and G: $\mathbb{A}_j \longrightarrow \mathbb{A}_l$ be C^∞ measurable homeomorphism and measure invariant maps such that $\mathbb{A}_i$ is sequentially connected μ_1-a.e. to $\mathbb{A}_j$ and $\mathbb{A}_j$ is sequentially connected μ_1-a.e. to $\mathbb{A}_l$. Now, to show that $\mathbb{A}_i$ is sequentially connected μ_1-a.e. to $\mathbb{A}_l$ $\exists$ a C^∞ measurable homeomorphism and measure invariant map $G \circ F$: $\mathbb{A}_i \longrightarrow \mathbb{A}_l$, since F and G are measurable homeomorphism and measure invariant maps therefore composite function $G \circ F$ is measurable homeomorphism and measure invariant.

According to definition 3.2, $\mathbb{A}_i$ is sequentially path connected μ_1-a.e to $\mathbb{A}_j$ and $\mathbb{A}_j$ is sequentially path connected μ_1-a.e to $\mathbb{A}_l$ in $A^k(M^4)$, then we show that $\mathbb{A}_i$ is sequentially path connected μ_1-a.e to $\mathbb{A}_l$ in $A^k(M^4)$:
Let $S_1 = \{p_i \in (U_i, \phi_i) \in \mathbb{A}_i \in A^k(M^4)$: $|f_n(p_i) - f(p_i)| < \epsilon$ $\forall$ n $\in$ N, i=1,…n$\}$,
$S_2 = \{q_i \in (V_i, \Psi_i) \in \mathbb{A}_j \in A^k(M^4)$: $|g_n(q_i) - g(q_i)| < \epsilon$ $\forall$ n $\in$ N , i=1,…n$\}$,
where $g_n = f_n \circ F^{-1}$ on $\mathbb{A}_j$ and
$S_3 = \{r_i \in (W_i, \chi_i) \in \mathbb{A}_l \in A^k(M^4)$: $|h_n(r_i) - h(r_i)| < \epsilon$ $\forall$ n $\in$ N , i=1,…n$\}$, where $h_n = (g_n \circ f_n) \circ F^{-1}$ on $\mathbb{A}_l$ be the induced sets on $\mathbb{A}_i$, $\mathbb{A}_j$ and $\mathbb{A}_l$ respectively. Since $\mathbb{A}_i$ is sequentially connected μ_1-a.e. to $\mathbb{A}_j$, according to definition 3.2 and theorem 4.1, sequentially connectedness is invariant under measurable homeomorphism and measure invariant function. Since, for every C^∞ map γ: $[0,1] \rightarrow \mathbb{A}_i \in A^k(M^4)$ that connects all the events in sequential way $p_1 <\ldots< p_i <\ldots p_n$ for $t_1 <\ldots t_i \ldots t_n$, $\forall$ i < j where $\mu_1(S_1) > 0$, $\mu_1(U_i) > 0$, $\exists$ $F \circ \gamma$: $[0,1] \rightarrow \mathbb{A}_j$ such that, it connects all the events in $\mathbb{A}_j$ in sequential way $q_1 <\ldots< q_i <\ldots q_n$ for $t_1 <\ldots t_i \ldots t_n$, $\forall$ i< j where $\mu_1(F(S_1)) > 0$, $\mu_1(V_i) > 0$, $\mu_1(\mathbb{A}_j) > 0$.

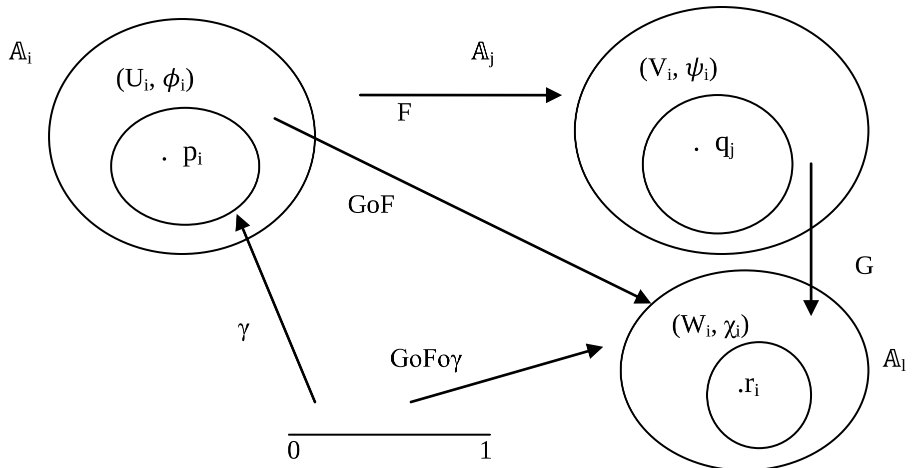

Fig. 2

Similarly, if for every $F \circ \gamma$: $[0,1] \rightarrow \mathbb{A}_j$ $\exists$ $G \circ F \circ \gamma$: $[0,1] \rightarrow \mathbb{A}_l$ where
$G \circ F \circ \gamma$ connects all the events sequentially in $\mathbb{A}_l$, such that,
$G \circ F \circ \gamma\,(0) = r_1 \in S \in (W_1, \chi_1) \in \mathbb{A}_l \in A^k(M^4)$, $\mu_1(S_3) > 0$, $\mu_1(W_1) > 0$,
$G \circ F \circ \gamma(\frac{k}{2^n}) = r_i \in S \in (W_i, \chi_i) \in \mathbb{A}_l \in A^k(M^4)$, $\forall$ $(\frac{k}{2^n}) \in (0,1)$, $k < 2^n$, $1< i <n$, $\mu_1(S_3) > 0$ and $\mu_1(W_i) > 0$,
$G \circ F \circ \gamma\,(1) = r_n \in S \in (W_n, \chi_n) \in \mathbb{A}_l \in A^k(M^4)$, $\mu_1(S_3) > 0$ and $\mu_1(W_n) > 0$ satisfying a Causal relation '<' in $S_3 \in (W_1, \chi_1) \in \mathbb{A}_l \in A^k(M^4)$, such that
$r_1 <\ldots< r_i <\ldots < r_n$ for $t_1 < t_2 <\ldots < t_i < \ldots< t_n$, $\forall$ i < j , where $\mu_1(S_3) > 0$, $\mu_1(W_i) > 0$ where $\mathbb{A}_l = \bigcup_{i=1}^{n}(W_i, \chi_i)$.
Hence, $\mathbb{A}_i$ is sequentially connected μ_1-a.e to $\mathbb{A}_l$, $\forall$ i, j, l $\in$ I, $t_i <\ldots t_j <\ldots t_l$.

Therefore, if $\mathbb{A}_i$ is sequentially connected μ_1-a.e to $\mathbb{A}_j$ and if $\mathbb{A}_j$ is sequentially connected μ_1-a.e to $\mathbb{A}_l$, we have shown that $\mathbb{A}_i$ is sequentially connected μ_1-a.e. to $\mathbb{A}_l$ under the composition of measurable homeomorphism and measure invariant function $G \circ F$. This means if sequentially connectedness is invariant under measurable homeomorphism and measure invariant transformation on $A^k(M^4)$, $\forall$ $\mathbb{A}_i$, $\mathbb{A}_j$ and $\mathbb{A}_l$, $\forall$ i, j, l $\in$ I then M^4 is maximally connected. ■

A maximally connectedness property on (M^4, $\mathcal{T}_1$, Σ_1, μ_1) defines a causal structure on space-time of dimension 4.

Definition 4.3:
A complete measure manifold (M^4, $\mathcal{T}_1$, Σ_1, μ_1) endowed with a causal structure induces a partial ordered relation '$\prec$' that generates a network manifold of dimension 4.

5 CONCLUSION

A measure manifold (M^4, $\mathcal{T}_1$, Σ_1, μ_1) admitting a partial ordered relation '$\prec$' on it denoted by (M^4, $\mathcal{T}_1$, Σ_1, μ_1, $\prec$) generates a network manifold of dimension-4. This approach provides a new vision to the space-time as a 4-dimensional complete measure manifold. The advantage of such approach is to generate a causally connected network manifold, whose applications are in the field of neural network, brain structure, in the study of large scale structures and engineering science.

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