International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 10 , October - 2026

Some Results on Fuzzy Signed Labeling Graphs

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Abstract - In this study, we introduce and analyze the concept of fuzzy signed labeling graphs. In this paper we establish necessary and sufficient condition for the existence of fuzzy signed labeling graphs and also examine Cycle graph, wheel graph , Petersen graph satisfy the FSL conditions.

Key words: Fuzzy Labeling, Signed Labeling, fuzzy signed Labeling, linear graph, Cycle graph, wheel graph and petersen graph.

INTRODUCTION

In graph theory, graph labeling is a crucial field that, under specific circumstances, gives values to a graph's vertices and/or edges. A variety of labeling styles, including graceful, harmonious, and signed labeling, have been developed over time to simulate relationship and interaction issues in the actual world. Signed graphs are particularly helpful in depicting systems with cooperative or antagonistic interactions because they include the idea of positive and negative links.

To simulate uncertainty and ambiguity in the actual world, fuzzy labeling gives soft, graded membership values between 0 and 1 to system elements, such as vertices and edges in a graph.

Under certain balancing or counting criteria, signed labeling (also known as signed graph labeling) gives positive and negative values to the vertices or edges of a graph, usually from the set {-1, +1}.

Fuzzy signed labeling models ambiguity, uncertainty, and positive or negative relationships in networks by combining fuzzy set theory, signed structures, and graph labeling.

2.PRELIMINARIES

Definition.2. 1

Let C^* = (V, E) be a simple graph. Then C^* = (φ, ψ) is called a fuzzy graph on C^*, if φ: V →[0, 1] and ψ: E → [0, 1] and for all $x, y \in E$, ψ (x, y) ≤ min [φ(x), φ (y)].A fuzzy graph C= (φ, ψ)) on C^* is called a fuzzy labeling graph if φ and ψ one to one maps for all $x, y \in E$.

Definition. 2.2

A fuzzy graph G (σ, μ) is said to be a fuzzy signed graph if there is a mapping η: E → {+, −} such that each edge signed to {+, −} or all nodes and edges assigned to {+, −} based on the fuzzy labeling conditions .When we assign {+ o r −} to each edge representing the relationship between the two vertices like trust or distrust etc.

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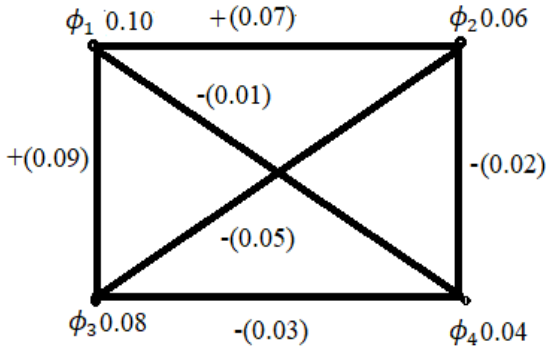
Definition. 2.3

Let C^* = (V, E) be a simple graph. Then C^* = (φ, ψ,s) is called a fuzzy signed graph on C^*, if φ: V →[0, 1] and ψ: E → [0, 1] and S → {-1, 1} for all *x, y* Є*E*. Then the signed values of edges are $^{+}\psi$ (x, y) ≤ max [φ(x), φ (y)] and $^{-}\psi$ (x, y) ≤ min [φ(x), φ (y)] given by the edge relationship. A fuzzy graph C= (φ, ψ,s) on C^* is called a fuzzy signed labeling graph if φ and ψ one to one maps for all *x, y* Є*E*.

Example1:

For n=4

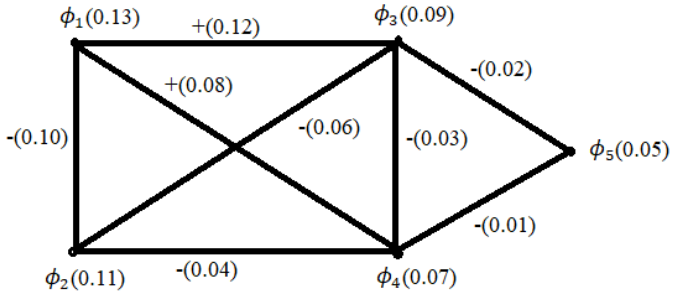
Consider a graph G(V,E,).The vertices and edges are denoted by φ and $^{-}\psi$. Let G(φ, $^{-}\psi$,S) be a signed fuzzy labeling graph , if φ: V →[0, 1] and ψ: E → [0, 1] and S → {-1, 1} .Let us label the vertices and edges are as follows



$\phi_1 = 0.10, \phi_2 = 0.06, \phi_3 = 0.08, \phi_4 = 0.04$: $\phi_1\,\phi_2 = +0.07, \phi_1\,\phi_3 = +0.09$

$\phi_1\,\phi_4 = -0.01, \phi_2\,\phi_3 = -0.05, \phi_2\,\phi_4 = -0.02, \phi_3\,\phi_4 = -0.03$.Here the product of all edge signs are positive. Hence it is a balanced fuzzy signed graph.

For n=5



$\phi_1 = 0.13, \phi_2 = 0.11, \phi_3 = 0.09, \phi_4 = 0.07;\ \phi_5 = 0.05$: $\phi_1\,\phi_2 = -0.10, \phi_1\,\phi_3 = +0.12, \phi_1\,\phi_4 = +0.08, \phi_2\,\phi_3 = -0.06, \phi_2\,\phi_4 = -0.04, \phi_3\,\phi_4 = -0.03, \phi_3\,\phi_5 = -0.02, \phi_4\,\phi_5 = -0.01$

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$\phi_1\,\phi_4 = -0.01, \phi_2\,\phi_3 = -0.05, \phi_2\,\phi_4 = -0.02, \phi_3\,\phi_4 = -0.03$

Here the product of all edge signs are positive. Hence it is a balanced fuzzy signed graph.

Definition. 2.4

A wheel graph is created by joining all of the vertices of a single cycle (the rim) to a single center vertex .

Definition. 2.5

In the field of graph theory Petersen graph is an undirected graph with 10 vertices and 15 edges.

Definition. 2.6

The fuzzy signed labeling graph H (ξ, η,s) is called fuzzy labelling subgraph of G (φ, ¯ψ,S) if ξ(x) ≤ φ(x) for all x∈ V and η∞(x,y) ≤ ¯ψ∞(x ,y), for all x,y∈ V.

3.SOME PROPERTIES ON FUZZY SIGNED LABELING GRAPHS

Properties:1

If (ξ, η,s) be a FSL subgraph of FSL of (φ, ¯ψ,S) then η(x,y) ≤ ψ (x,y) for x,y∈ V.

Proof:

Let G (φ, ¯ψ,S) be a FSLG. And H(ξ, η,s) be its subgraph. Let (x,y) be any path in G then its strength be ψ∞(x,y). Since H is a sub graph, ξ(x) ≤ φ(x) and η,(u,v) ≤ ¯ψ(x,y). which implies η∞(x,y) ≤ ¯ψ∞(x,y) ,for all x,y∈ V.

Property: 2

Edge Sign property of FSL Graph.

Proof:

Let Let G(φ, ¯ψ,S) be a signed fuzzy labeling graph with φ vertices and ¯ψ edges. The edge membership values are assigned by $^{+}\psi$ (x, y) ≤ max [φ(x), φ (y)] ie) positive sign is assigned for truth or trust relationship and the membership values of the truth relationship is less than or equal to the maximum values of the two vertices or members. The negative sign is assigned by falsity or distrust of two members and the membership values of the falsity relations evaluated by ¯ψ (x, y) ≤ min [φ(x), φ (y)].In a fuzzy signed labeling graph all edge membership values are distinct.

Property :3

Every fuzzy signed labeling cycle graph has balanced if the product of edges are positive.

Proof:

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Let C (φ, ˉψ,S) be a FSL of a cycle graph. Let us label the membership values of the edges of FSL graph is by the property 2.By the definition of signed labeling property the product of edge signs are positive ,the graph is balanced graph unless the graph is unbalanced. By using this property to FSL cycle graph the product of the negative edges are positive the FSL of cyclic graph is balanced.

4.SOME RESULTS ON FUZZY SIGNED LABELING GRAPHS

Preposition 4.1

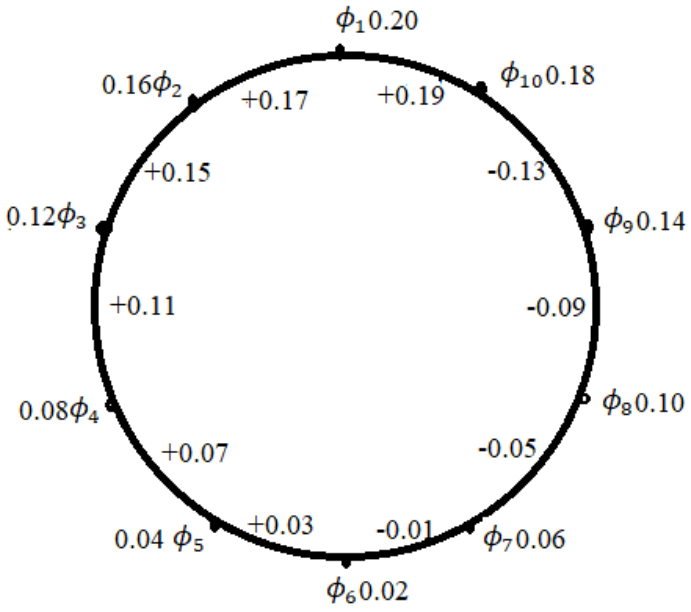
Any cycle graph admits a fuzzy signed labeling graph

Proof:

Consider a graph G(V,E,).The vertices and edges are denoted by φ and ˉψ . Let G(φ, ˉψ,S) be a signed fuzzy labeling graph , if φ: V →[0, 1] and ψ: E → [0, 1] and S → {-1, 1} .Let us label the vertices and edges are as follows

$$\phi_i = 0.20 - 0.02k, k\ is\ even, i = 1,2..n$$

$\Psi_i = \phi_i\,\phi_{i+1}$ =0.20-0.01k, k is odd



FSL of Cycle Graph C_{10}

The edge values are note as follows

$\phi_1 = 0.20,\ \phi_2 = 0.16,\ \phi_3 = 0.12,\ \phi_4 = 0.08, \phi_5 = 0.04, \phi_6 = 0.02, \phi_7 = 0.06,\ \phi_7 = 0.10,\ \phi_8 = 0.10,\ \phi_9 = 0.14,\ \phi_{10} = 0.18$; $\phi_1\,\phi_2 = +0.17,\ \phi_2\,\phi_3 = +0.15,\ \phi_3\phi_4 = +0.11,\ \phi_4\,\phi_5 = +0.07,\ \phi_5\,\phi_6 = +0.03$, $\phi_6\,\phi_7 = -0.01,\ \phi_7\phi_8 = -0.05,\ \phi_8\,\phi_9 = -0.09,\ \phi_9\,\phi_{10} = -0.13,\ \phi_{10}\,\phi_1 =$

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+0.19. In this graph the product of all edge signs are all positive. Hence the cycle graph C_{10} admits the fuzzy signed graph labeling and it also a balanced signed labeling graph.

Preposition 4.2

For n≥4 ,a wheel graph W_n admits fuzzy signed labeling graph.

Proof:

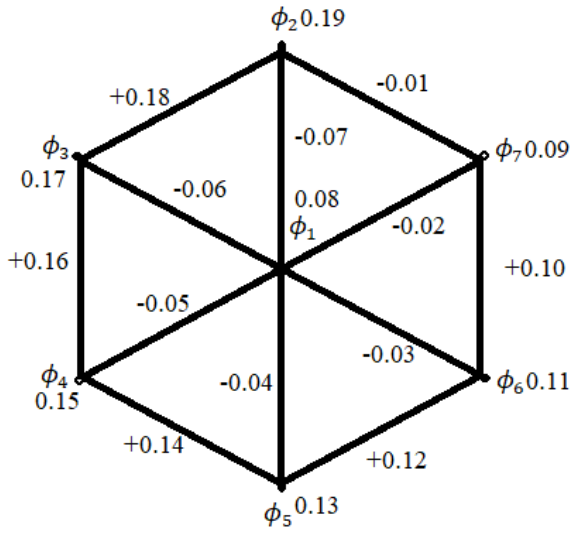
Consider a graph G(V,E,). The vertices and edges are denoted by φ and ˉψ. Let G(φ, ˉψ,S) be a signed fuzzy labeling graph, if φ: V →[0, 1] and ψ: E → [0, 1] and S → {-1, 1}. Let us label the vertices and edges are as follows ,

Fix ϕ_1=0.08 and for n=7

ϕ_{n+1}=(3n-2k)0.01 ; for i=2 to 6 and k=1,2,3…

$\phi_1\phi_{i+1}$=-0.08-(0.01)k ; k=1,2,3… ; i=1,2,3..

$\phi_{i+1}\phi_{i+2}$=0.18-(0.02)k ; k=0,1,2,3,4; i=1,2,3… and $\phi_7\phi_2$=-0.01



Wheel Graph W_7

$\phi_1 = 0.08,\ \phi_2 = 0.19,\ \phi_3 = 0.17,\ \phi_4 = 0.15, \phi_5 = 0.13, \phi_6 = 0.11, \phi_7 = 0.09:\ \phi_1\,\phi_2 = -0.07,$
$\phi_1\,\phi_3 = -0.06,$

$\phi_1\,\phi_4 = -0.05, \phi_1\,\phi_5 = -0.04, \phi_1\,\phi_6 = -0.03, \phi_1\,\phi_7 = -0.02, \phi_2\,\phi_3 = +0.18.\ \phi_3\,\phi_4 = +0.16.$
$\phi_4\,\phi_5 = +0.14, \phi_5\,\phi_6 = +0.12, \phi_6\,\phi_7 = +0.10, \phi_7\,\phi_2 = -0.01.$ Here the product of all edge signs are negative. Hence wheel graph W_7 admits a fuzzy signed labeling graph but it is a unbalanced fuzzy signed graph.

Preposition 4.3

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Petersen graph admits a fuzzy signed labeling .

Proof:

Consider a graph G(V,E,) be a Petersen graph. The vertices and edges are denoted by φ and ¯ψ . Let G(φ, ¯ψ,S) be a signed fuzzy labeling graph, if φ: V → [0, 1] and ψ: E → [0, 1] and S → {-1, 1}. Let us label the vertices and

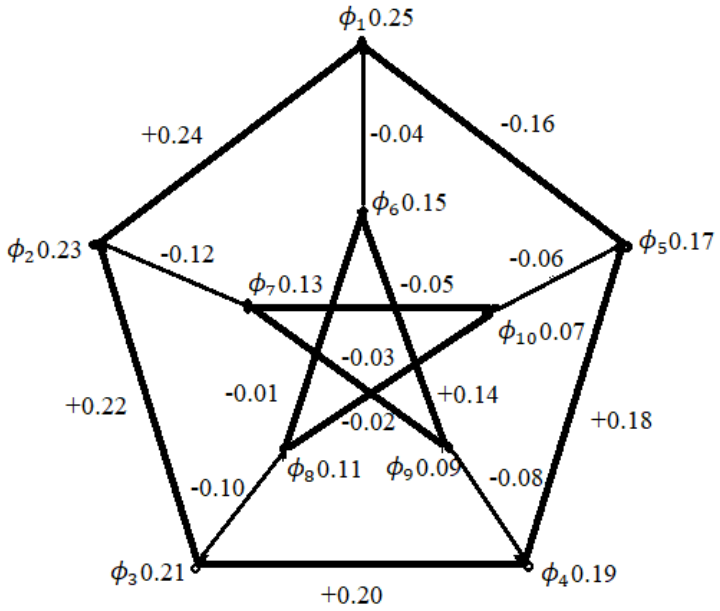
ϕ_i =0.25+(n-1)0.01; n is odd; i=1,2,3..10

$\phi_i\ \phi_{i+1}$=0.24 - 0.01n; n is even; i=1,2,3,4.

$\phi_i\ \phi_{i+5}$=-0.12+(0.02) n; n=0,1,2,3; i=2,3,4,5

$\phi_i\ \phi_{i+2}$=-0.01n; n=1,2,3; n=6,7,8.

$\phi_i\ \phi_{i-3}$=0.14-0.19n; n=0,1; i=9,10; $\phi_1\ \phi_5$=-0.16; $\phi_1\ \phi_6$=-0.04.



$\phi_1 = 0.25, \phi_2 = 0.23, \phi_3 = 0.21, \phi_4 = 0.19, \phi_5 = 0.17, \phi_6 = 0.15, \phi_7 = 0.13, \phi_8 = 0.11, \phi_8 = 0.09, \phi_9 = 0.09, \phi_{10} = 0.07$: $\phi_1\ \phi_2 = +0.24, \phi_2\ \phi_3 = +0.22,$

$\phi_3\ \phi_4 = +0.20, \phi_4\ \phi_5 = +0.18, \phi_5\ \phi_1 = -0.16, \phi_1\ \phi_6 = -0.04, \phi_2\ \phi_7 = -0.12.\ \phi_3\ \phi_8 = -0.10.$ $\phi_4\ \phi_9 = -0.08, \phi_5\ \phi_{10} = -0.06, \phi_6\ \phi_8 = -0.01, \phi_8\ \phi_{10} = -0.02, \phi_{10}\ \phi_7 = -0.05, \phi_7\ \phi_9 = -0.03$, $\phi_9\ \phi_6 = -0.14$.The generalized Petersen graph attains afuzzy signed labeling graph and it also a balanced fuzzy signed labeling graph.

5. CONCLUSION

In this paper Fuzzy Signed Labeling of Cycle graph, Wheel graph and Petersen graph and some basic properties are examined.

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