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Some (r, 2, k)-regular graphs containing a given graph.

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Abstract

A graph G is called (r, 2 , k) - regular if each vertex of G is at a distance one from r vertices of G and each vertex of G is at a distance two from exactly k vertices of G. That is , if $d(v) = r$ and $d_2(v) = k$, for all v in G [9].This paper suggests a method to construct a $(m + n - 1, 2, (m n - 1))$- regular graph of order m n containing the given graph G of order $n \geq 2$ as an induced sub graph, for any $m \geq 1$, and this paper includes existence of some (r , 2 , k)-regular graphs and few examples of (2 , k) regular graphs.

Keywords: Distance degree regular, Induced subgraph, (d, k)-regular graphs, (2, k) - regular graphs, semi regular graphs.

MATHAMATICS SUBJECT CLASSIFICATION CODE (2010): 05C12.

1. Introduction.

By a graph we mean a finite, simple, undirected graph. We denote the vertex set and edge set of G by$V(G)$ and (G) respectively. The addition of two graphs G_1 and G_2 is a graph G_1+G_2 with $V(G_1+G_2) = V(G_1) \cup V(G_2)$ and $E(G_1+G_2) = E(G_1) \cup E(G_2) \cup \{ uv / u\in V(G_1), v\in V(G_2)\}$. The degree of a vertex v is the number of vertices adjacent to v and it is denoted by d (v). If all the vertices of a graph have the same degree r, we call that graph r-regular.

Two vertices *u* and *v* of *G* are said to be connected if there is a (u, v) – path in *G*. For a connected graph *G*, the distance *d* (u, *v)* between two vertices *u* and *v* is the length of a shortest (*u*, *v*) path in *G*. In any graph G, *d (u, v)* =1 if and only if u and v are adjacent. Therefore , the degree of a vertex v is the number of vertices at a distance one from v, and for d a positive integer and v a vertex of a graph G , the d th degree of v in G, denoted by $d_d(v)$ - is defined as the number of vertices at a distance d from v. Hence $d_1(v) = d(v)$.

A graph *G* is said to be distance *d* – regular if every vertex of *G* has the same number of vertices at a distance *d* from it [5]. Let us call a graph (dike)-regular if every vertex of *G* has exactly k number of vertices at a distance *d* from it .That is, a graph *G* is said to be **(*d, k*) - regular graph** if $d_d(v) = k$, for all *v* in *G* .The (1, k) – regular graphs and k-regular graphs are the same. A graph *G* is (2, *k*) regular if $d_2(v) = k$, for all *v* in *G.* A graph G is said to be (2, k) regular if $d_2(v) = k$, for all v in G, where $d_2(v)$ - means number of vertices at a distance 2 away from v. The concept of semi regular graph was introduced and studied by Alison Northup [2]. We observe that (2, k) - Regular and k –semi regular graphs are same.

An induced sub graph of *G* is a sub graph *H* of *G* such that E(*H*) consists of all edges of G whose end points belong to V(H).In 1936, Konig [8] proved that if G is any graph ,whose largest degree is r, then there is an r-regular graph H containing G as an induced sub graph.

The above results motivate us to suggest a method to construct , a (m + n - 1 , 2, (mn – 1))- regular graph H of order mn containing given graph G of order $n \geq 2$ as an induced sub graph, for any $m \geq 1$. Terms not defined here are used in the sense of **Harary** [6] and **J.A Bondy** and **U.S.R .Murty** [4].

2. (2, k) - regular graphs

Definition 2.1. A graph is said to be (2, k) - regular graph if each vertex of G is at a distance two away from exactly k vertices. That is, $d_2(v) = k$, for all vertex in G.Note that (2, k) - regular graph may be regular or non – regular.

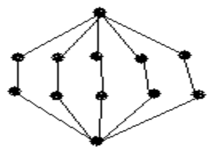
Examples 2.2.

Non-regular graph which are (2, k)-regular 2.3.

(i). Sunflower graph is the graph obtained by starting with an $n \geq 5$ cycles with consecutive vertices $v_1, v_2, v_3, v_4, \ldots\ldots v_n$ and creating new vertices $w_1, w_2, w_3, \ldots\ldots w_n$ with w_i connected with v_i and v_{i+1} (v_{n+1} is v_1) is (2, 4)- regular. We denote this graph by SF_n.

Proof: Let the vertex set $V(SF_n)=\{v_1,v_2,v_3,v_{4,\ldots\ldots}v_n\}U\{w_1,w_2,w_3,w_{4,\ldots\ldots}w_n\}$ and edge set $E(SF_n)=E(C)\ U\ \{VI_,\ wi/$ $(1\leq i\leq n).\}U\ \{v_{i+1}w_i/\ (1\leq i\leq n).\}U\ \{v_1w_n\}$. Here $d_2\ (v_i)=4$, $d_2\ (w_e)=4$, for $(1\leq i\leq n)\}$. Therefore, SF_n ($n \geq 5$) is (2, 4) -regular graph.

(ii) . For any $k \geq 1$, let G_k graph obtained from two disjoint copies of $K_{1,k}$ by adding a matching between two partite sets of size kiths graph G_k is (2, k)- regular graph order 2k +2. Graph G_5 in Figure 1 is (2,5) - regular graph



G_5

Figure 1

Regular graphs which are (2, k) – regular 2.4.

(i). Any complete m partite graph $K_{n1,\ n2,\ n3,\ n4,\ \cdot nm}$ is (2, k) - regular iff $_{n1 = n2 = n3 = n4, = nm.}$

(ii). Any positive integer n = m k, where m > 1and k ≥ 1 are positive integers. Then we construct complete 'm'partite graph which is $\left(2,\ \left(\frac{n}{m}\right)-1\right)$ regular.

We denote r-regular graphs which are (2, k)-regular by (r, 2, k)-regular graph.

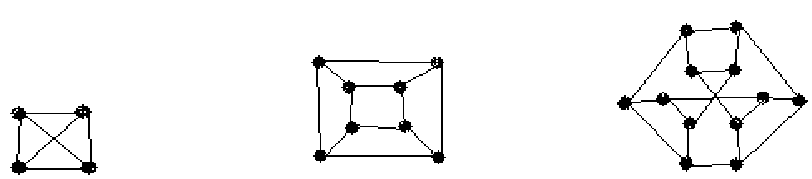
3. (r, 2, k)-regular graph.

Next, we will see some results related with (r, 2, k)-regular graph that we have already seen in [9], [10], [11], [12].

Definition 3.1

A graph G is called (r, 2, k)-regular if each vertex in graph G is at a distance one from exactly r-vertices and at a distance two from exactly k vertices. That is, d (v) = r and d_2 (v) = k, for all v in G.

Example 3.2.



(3, 2, 0)-regular graph (3, 2, 3)-regular graph (3, 2, 5)-regular graph

Figure 2.

The following facts can be verified easily:

Fact 1 [8] If G is (r, 2, k)-regular graph, then $0 \leq k \leq r\,(r-1)$.

Fact 2 [9] for any r > 1, a graph G is (r, 2, r(r-1)-regular if G is r-regular with girth at least five.

Fact 3 [9] for any $n \geq 5$, $(n \neq 6, 8)$ and any r > 1, there exists a (r, 2, r (r-1))-regular graph on $n \times 2^{r-2}$ vertices with girth five.

Fact 4 [10] for any odd $r \geq 3$, there is no (r, 2, 1)-regular graph

Fact 5 [10] Any (r, 2, k) - regular graph has at least k+r+1 vertices.

Fact 6 [10] If r and k are odd, then (r, 2, k)-regular graph has at least k+r+2 vertices.

Fact 7 [10] For any $r \geq 2$ and $k \geq 1$, G is a (r, 2, k)-regular graph of order r+k+1 if and only if dam (G) = 2.

Fact 8 [10] For any $r \geq 2$, there is a (r, 2, (r-1) (r-1))-regular graph on $4 \times 2^{r-2}$ vertices

Fact 9 [10] For r > 1, if G is a (r, 2, (r-1) (r-1))-regular graph, then G has girth four.

Fact 10[11] For any $r \geq 1$, there exist a (r, 2, r-1)-regular graph of order 2r.

Fact 11[11] For any $r \geq 1$, there exist a (r, 2, 2 (r-1)) - regular graph of order 4r-2.

Fact 12[11] For any $r \geq 2$, there is a (r , 2 , (r - 2) (r- 1))- regular graph on $3 \times 2^{r-2}$ vertices .

Fact 13[12] For any $r \geq 3$, there is a (r , 2 , (r - 3) (r- 1))- regular graph on $4 \times 2^{r-3}$ vertices .

Result 3.3.

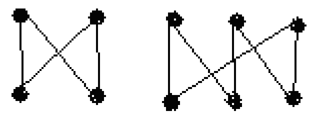
For any r > 0, there exists (r, 2, r+n-1) - regular bipartite graph of order 2(r+ n), for $(0 \leq n \leq r-1)$

Proof.

Let r>0, let G be a bipartite graph with the vertex set $V(G)=\{v_i, u_i / (1 \leq i \leq r+n)\}$ and edge set $E(G)= \{ v_i u_i, v_i u_{i+j} / (1 \leq i \leq r+n)$ and $1 \leq j \leq r-1)\}$.Subscripts are taken modulo r + n.This graph G is (r, 2, r+n-1) -regular bipartite graph of order 2(r+n).

Example 3.3. Figure 3 illustrates the result 3.2, for r = 2, 3.

When r=2.



n = 0 **n = 1.**

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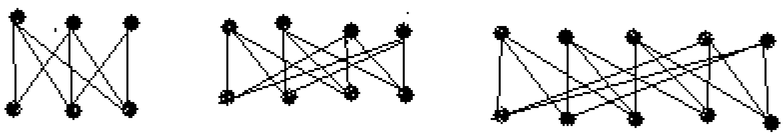
When r=3.



n=0 **n=1** **n=2**

Figure 3

4. The (r, 2, k) - regular containing given graph as an induced sub graph.

Konig [7] proved that if G is any graph, whose largest degree is r, then it is possible to add new points and to draw new lines joining either two new points or a new point an existing point, so that the resulting graph H is a regular graph containing G as an induced sub graph. We now suggests a method that may be considered an analogue to Konig's theorem for (r, 2, k)) - regular graphs.

Main theorem 4. 1

For any $m \geq 1$, every graph G of order $n \geq 2$ is an induced sub graph of ((n+m-1) ,2, (mn-1))- regular graph of order 2mn.

Proof.

Let G be a graph of order $n \geq 2$ with the vertex set V (G) = $\{v_1, v_2, v_3, \ldots\ldots v_n\}$. Let G_t denote a copy of G with the vertex set $V(G_t) = \{v_1^t, v_2^t, v_3^t, \ldots\ldots v_n^t\}$,for t =1,2,3,… m . Let G_{r+m} denote a copy of G with the vertex set $V(G_{r+m}) = \{u_1^r, u_2^r, u_3^r, \ldots\ldots u_n^r\}$,for r =1,2,3,…m) .The desired graph H has the vertex set $V(H) = \bigcup_{t=1}^{2m} V(G_t)$ and edge set E(H) = $\bigcup_{t=1}^{2m} E(G_t) \bigcup_{t=1}^{m} \{v_j^t u_i^t , u_j^t v_i^t / v_j^1 v_i^1 \notin V(G_1), (1 \leq j \leq n), (j+1 \leq i \leq n)\} \bigcup_{k=1}^{n} \{v_k^i u_k^{i+j} / (1 \leq i \leq m) \text{ and } (0 \leq j \leq m-1\}$. (Super scripts are taken modulo m).The resulting graph H contains G as an induced sub graph .

In H , for t =1,2,3,…m. $d(v_i^t) = d(u_i^t) = m + n - 1$ and $d_2(v_i^t) = d_2(u_i^t) = m n - 1$, for i= 1, 2, 3,…n. That is, H is ((m + n -1), 2, mn-**1)** **-** regular graph H of order 2mn containing any graph G of order n $\geq$ 2. For any graph of order n $\geq$ 2, there exist ((m+ n -1), 2, mn-**1)** **-** regular graph H of order 2mn containing given graph of order n $\geq$ 2 as an induced sub graph. Therefore, there are at least as many (m + n - 1), 2, m n **- 1)** **-** regular graph of order 2 m n as there are graph of order n $\geq$ 2.

Example 4.2. Figure 4 illustrates theorem 4.1, for m = 2 and n = 3.

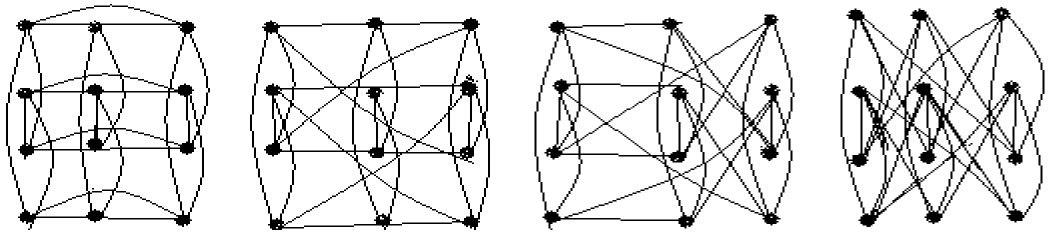


Figure 4

Corollary 4.3.

Every graph G of order n $\geq$ 2, is an induced sub graph of (n, 2, n-1) - regular graph of order 2n.

Proof.

This result is the particular case of theorem 4.1, for m = 1.

Let G be a graph of order n $\geq$ 2 with vertex set V (G) = $\{v_1, v_2, v_3, \ldots v_n\}$. Let G_1 denote a copy of G with the vertex set $V(G_1)=\{v_1^1, v_2^1, v_3^1, \ldots\ldots v_n^1\}$.Let G_2 denote a copy of G with the vertex set $V(G_2)=\{u_1^1, u_2^1, u_3^1, \ldots\ldots u_n^1\}$ The desired graph H has the vertex set V(H) = $\bigcup_{t=1}^{2} V(G_t)$ and edge set E(H)= $\bigcup_{t=1}^{2} E(G_t) \bigcup$ $\{v_j^1 u_i^1, u_j^1 v_i^1 / v_j^1 v_i^1 \notin V(G_1), (1 \leq j \leq n), (j+1 \leq i \leq n)\} \bigcup_{k=1}^{n} \{v_k^1 u_k^1\}$. The resulting graph H contains G as an induced sub graph.

In H , $d(v_i^1) = d(u_i^1) = n$ and $d_2(v_i^1) = d_2(u_i^1) = n-1$, for i =1,2,3,…n. That is, H is (n, 2, n-1**)** **-** regular graph of order 2n containing given graph G of order n $\geq$ 2.For any graph of order n $\geq$ 2, there exist (n, 2, (n - 1)) - regular graph H of order 2n containing given graph of order n $\geq$ 2, as an induced sub graph. Therefore, there are at least as many (n, 2, (n-1)) - regular graphs of order 2n as there are graphs of order n $\geq$ 2.

Example 4.4. Graphs in Figure 5 illustrates corollary 4.3, for n=3.

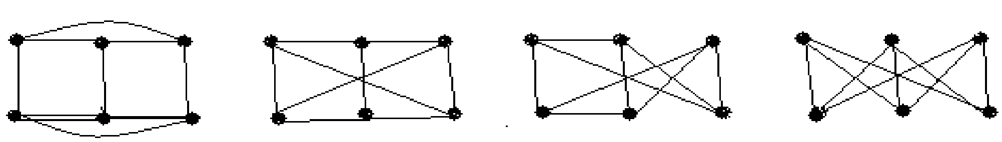


Figure 5

Corollary 4.5. Every graph G of order $n \geq 2$, is an induced sub graph of (n+1), 2, 2n-1) -regular graph of order 4n.

Corollary 4.6. Every graph G of order $n \geq 2$, is an induced sub graph of (n+2), 2, 3n-1) -regular graph of order 6n

Summary 4.7.

Therefore, there are at least as many ((n+m-1), 2, (m n-1)) - regular graphs of order 2mn as there are graphs of order n ≥ 2. If m =1, 2 , 3 , 4 ,5……n…, then we get (n, 2, n-1), (n+1, 2, 2n-1) , (n+2, 2, 3n-1) , (n+3, 2, 4n-1), (n+4, 2, 5n-1),…(2n, 2 , n^2-1)- regular graphs of order 2n, 4n, 6n, 8n, 10n……$2n^2$….containing given graph G of order $n \geq 2$ as an induced sub graphs.

Theorem 4.8.

Every graph G of order $n \geq 2$ is an induced sub graph of (n+1, 2, 2n)-regular graph of order 5n.

Proof.

Let G be a graph of order $n \geq 2$ with the vertex set V (G) = $\{v_1, v_2, v_3, \ldots\ldots v_n\}$. Let G_t denote five copies of G with the vertex set $V(G_t) = \{v_1^t, v_2^t, v_3^t, \ldots\ldots v_n^t\}$,for t =1,2,3,4,5. The desired graph H has the vertex set $V(H)=\bigcup_{t=1}^{5} V(G_t)$,and

edgeset $E(H)=\bigcup_{t=1}^{5} E(G_t)\bigcup_{t=1}^{4} \{v_j^t v_i^{t+1}, v_j^5 v_i^1 / v_j^1 v_i^1 \notin E(G_1) \quad (1\leq j\leq n),(j+1\leq i\leq n)\}\bigcup_{k=1}^{n} \{v_k^i v_k^{i+1}, v_k^5 v_k^1 / (1\leq i\leq 4)\}$.

The resulting graph H contains G as an induced sub graph. Moreover, for $(1\leq t\leq 5)$, In H , $d(v_i^t) = n+1$, $d_2(v_i^t)= 2n$, for $(1\leq i\leq n)$.That is, H is (n+1, 2, 2n) - regular graph with 5n vertices. For any graph G of order $n \geq 2$, there exists a (n+1, 2, 2n)-regular graph H of order 5n containing given graph G as an induced sub graph. Therefore, there are at least as many (n+1, 2, 2n) - regular graph of order 5n as there are graph G of order $n \geq 2$.

Example 4.9. Figure 6 illustrates the Theorem 4.8, for n = 4 (here we take only four graphs of order 4).

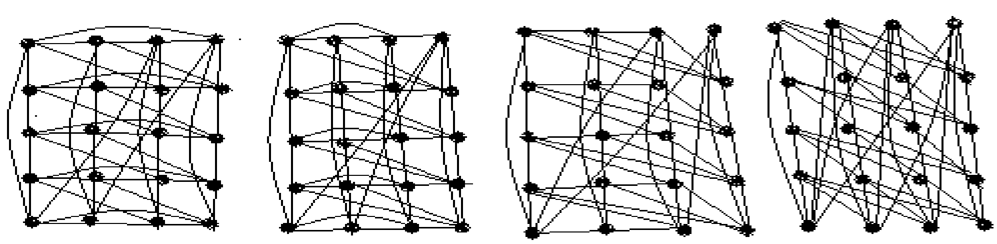


Figure 6

Corollary 4.10 . Every graph G of order $n \geq 2$ is an induced sub graph of (n+1, 2, 2n-2) - regular graph of order 3n.

For, if we take 3 copies of G instead of taking 5 copies of G in Theorem 4.8,then there is (n+1, 2, 2n-2) - regular graph containing every graph G of order $n \geq 2$.

Example 4.11. Figure 7 illustrates the Corollary 4.10, for n = 3.

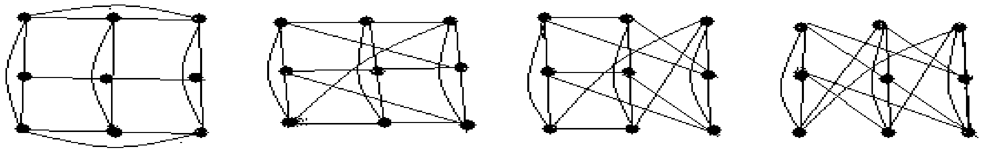


Figure 7

Corollary 4.12. Every graph G of order $n \geq 2$ is an induced sub graph of (n+1, 2, 2n-1)-regular graph of order 4n.

For, if we take 4 copies of G instead of taking 5 copies of G in Theorem 4.8,then there is (n+1, 2, 2n-1) - regular graph containing every graph G of order $n \geq 2$.

Example 4.13. Figure 8 illustrates the Corollary 4.12, for n = 3.

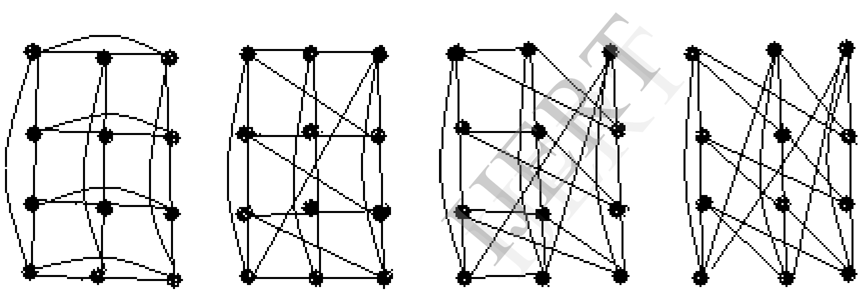


Figure 8

Remark 4.14 If we take 6,7,8,…. copies of G instead of taking 5 copies of G in Theorem 4.8, then there is only (n+1, 2, 2n) - regular graph of order 6n, 7n, 8n……

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