Some New Functions in Soft Topological Space

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***Abstract*— The purpose of this paper is to form some new functions like soft biclop.na-continuous and somewhat soft nearly biclop.na-continuous and also by using these concept, some theorems are analyzed.**

***Keywords*— Soft Feebly Open, Soft Feebly Closed, Soft δ-Open, Soft δ-Closed.**

I. PRELIMINARIES

Definition 1.1 [4]: Let X be an initial universe set and let E be the set of all possible parameters with respect to X. Let P(X) denote the power set of X. Let A be a nonempty subset of E. A pair (F,A) is called soft set over X, where F is a mapping given by F:A→P(X). A soft set (F,A) on the universe X is defined by the set of ordered pairs $(F,A)=\{(x,f_A(x)):x\tilde{\in}E,f_A(x)\tilde{\in}P(X)\}$ where f_A: E→P(X) such that $f_A(x)=\varphi$ if $x\notin A$. Here f_A is called an approximate function of the soft set (F,A). The collection of soft set (F,A) over a universe X and the parameter set A is a family of soft sets denoted by $SS(x)_A$.

Definition 1.2[3]: A set set (F,A) over X is said to be null soft set denoted by φ if for all $e\tilde{\in}A$, F(e) = φ. A soft set (F,A) over X is said to be an absolute soft set denoted by A if all $e\tilde{\in}A$, F(e)=X.

Definition 1.3[5]: Let Y be a nonempty subset of X, then Y denotes the soft set (Y,E) over X for which Y(e)=Y, for all $e\tilde{\in}E$. In particular, (X,E) will be denoted by X.

Definition 1.4 [5]: Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if (i)$\varphi,X\tilde{\in}\tau$(ii)If (F,E),(G,E) $\tilde{\in}\tau$ then $(F,E)\tilde{\cap}(G,E)\tilde{\in}\tau$(iii) If $\{(F_i,E)\}_{i\in I}\tilde{\in}\tau$ then $\tilde{\bigcup}_{i\in I}(F_i,E)\tilde{\in}\tau$. The pair (X,τ,E) is called a soft topological space. Every member of τ is called a soft open set. A soft set (F,E) is called soft closed in X if $(F,E)^c\tilde{\in}\tau$.

Definition 1.5: Let (X,τ,E) be a soft topological space over X and let (A,E) be a soft set over X
(i) the soft interior [7] *of* (A,E) is the soft set $\widetilde{int}(A,E)=\tilde{\cup}\{(O,E):(O,E)$ which is soft open and$(O,E)\tilde{\subseteq}(A,E)\}$
(ii) the soft closure [5] *of* (A,E) is the soft set $\widetilde{cl}(A,E)=\tilde{\cap}\{(F,E):(F,E)$ which is soft closed and $(A,E)\tilde{\subseteq}(F,E)\}$. Clearly $\widetilde{cl}(A,E)$ is the smallest soft closed set over X which contains (A,E) and $\widetilde{int}(A,E)$ is the largest soft open set over X which is contained in (A,E).

Definition 1.6 [2]: In a soft topological space (X,τ,E), a soft set (i) (A,E) is said to be soft feebly-open set if $(A,E)\tilde{\subseteq}s\widetilde{cl}(\widetilde{int}(A,E))$.
(ii) (A,E) is said to be soft feebly-closed set if $s\,\widetilde{int}(\widetilde{cl}(A,E))\tilde{\subseteq}(A,E)$.
It is said to be soft feebly-clopen if it is both soft feebly-open and soft feebly-closed.

Definition 1.7 [2]: Let (X,τ,E) be a soft topological spaces and let (A,E) be a soft set over X.
(i) Soft feebly-closure of a soft set (A,E) in X is denoted by $f\widetilde{cl}(A,E)=\tilde{\cap}\{(F,E)$: (F,E) which is a soft feebly-closed set and $(A,E)\tilde{\subseteq}(F,E)\}$.
(ii) Soft feebly-interior of a soft set (A,E) in X is denoted by $f\widetilde{int}(A,E)=\tilde{\cup}\{(O,E):(O,E)$ which is a soft feebly-open set and $(O,E)\tilde{\subseteq}(A,E)\}$. Clearly $f\widetilde{cl}(A,E)$ is the smallest soft feebly-closed set over X which contains (A,E) and $f\widetilde{int}(A,E)$ is the largest soft feebly-open set over X which is contained in (A,E).

Definition 1.8 ([4],[5],[1],[6]) : For a soft (F,E) over the universe U, the relative complement of (F,E) is denoted by (F,E)′ and is defined by (F,E)′ = (F′,E), where (F′,E), where F′ : E→P(U) is a mapping defined by F′ (e) = U – F(e) for all $e\tilde{\in}E$.

II. SOME NEWLY TWO MAPPINGS IN SOFT TOPOLOGICAL

*Definition 2.1:*Let(A,E) be a subset of soft topological space (X,τ,E). It is said to be soft δ-open if for each $x\tilde{\in}(A,E)$, there exists an soft open set (G,E) such that $x\tilde{\in}(G,E)\tilde{\subseteq}\widetilde{int}(\widetilde{cl}(G,E)\tilde{\subseteq}(A,E)$. On the other hand soft δ-open if for each $x\tilde{\in}(A,E)$, there exists a soft regular open set (U,E) of (X,τ,E) such that $x\tilde{\in}(U,E)\tilde{\subseteq}(A,E)$.

Definition 2.2: Let (A,E) be a subset of soft topological space (X,τ,E). It is said to be soft δ-clopen if it is both soft δ-open and soft δ-closed.

Remark 2.3: Union of two soft δ-clopen set is soft δ-clopen set .

*Definition 2.4:*A function f : (X,τ,E)→ (Y,τ,E) is said to be soft biclop.na-continuous if the inverse image of every soft δ-clopen set (V,E) of Y is soft feebly–clopen in X.

Definition 2.5: Let (A,E) be subset of soft topological space (X,τ,E). (A,E) is said to be somewhat soft nearly clopen if $\tilde{cl}(\tilde{int}(\tilde{cl}$(A,E))) = φ.

Definition 2.6: A function f : (X,τ,E)→ (Y,τ,E) is somewhat soft nearly biclop.na-continuous if f^{-1}(V,E) is somewhat soft nearly clopen for every soft feebly- clopen set (V,E) in Y such that f^{-1} (V,E) = φ.

Theorem 2.7: For a function f : (X,τ,E)→ (Y,τ,E), the following statements are equivalent.
(a) f is soft biclop.na-continuous
(b) A function f : (X,τ,E)→ (Y,τ,E) is soft biclop.na-continuous if for every soft feebly- clopen set (V,E) of Y containing f(x) there exist soft δ-clopen set (U,E) containing x such that f(U,E) $\tilde{\subseteq}$(V,E).
Proof :(a)⇒(b) : Let x$\tilde{\in}$X and let (V,E) be a soft feebly-clopen set in Y containing f(x). Then, by (b), f^{-1}(V,E) is soft δ-clopen in X containing x. Let (U,E) = f^{-1}(V,E). Then, f(U,E) $\tilde{\subseteq}$(V,E).
(b)⇒(a) : Let (V,E) be a soft feebly-clopen set of Y, and let x$\tilde{\in}f^{-1}$(V,E). Since f(x) $\tilde{\subseteq}$ (V,E), there exists (U,E) containing x such that f(U,E)$\tilde{\subseteq}$ (V,E). then follows that x$\tilde{\in}$(U,E) $\tilde{\subseteq}f^{-1}$(V,E). Hence f^{-1}(V,E) is soft δ-clopen.

Theorem 2.8: If f : (X,τ,E)→ (Y,τ,E) and X = $X_1 \tilde{\cup} X_2$ where X_1 and X_2 and soft δ-copen set and f/X_1 and f/X_2 are soft biclop.na-continuous, then f is soft biclop.na-continuous.
Proof: Let (A,E) be a soft feebly- clopen subset of Y. Then, since (f/X_1) and (f/X_2) are both soft biclop.na-continuous, therefore $(f/X_1)^{-1}$(A,E) and $(f/X_2)^{-1}$(A,E) are both soft δ-clopen set in X_1 and X_2 respectively. Since X_1 and X_2 are soft δ-clopen subsets of X, therefore $(f/X_1)^{-1}$(A,E) and $(f/X_2)^{-1}$(A,E) are both soft δ-clopen subsets of X. Also, f^{-1}(A,E) = $(f/X_1)^{-1}$(A,E) $\tilde{\cup}$ $(f/X_2)^{-1}$(A,E). Thus f^{-1}(A,E) is the union of two soft δ-clopen sets and is therefore soft δ-clopen. Hence f is soft biclop.na-continuous.

Theorem 2.9: If f : (X,τ,E)→ (Y,τ,E)and X = $X_1 \tilde{\cup} X_2$ and if (f/X_1) and (f/X_2) are both soft biclop.na-continuous at a point x belongs to $X_1 \tilde{\cap} X_2$, then f is soft biclop.na-continuous at x.
Proof: Let (U,E) be any soft feebly- clopen set containing f(x). Since x $\tilde{\in}$ $X_1 \tilde{\cap} X_2$ and (f/X_1), (f/X_2) are both soft biclop.na-continuous at x, therefore there exist soft δ-clopen sets (V_1,E) and (V_2,E) such that x$\tilde{\in}X_1 \tilde{\cap}$ (V_1,E) and f($X_1 \tilde{\cap}$ (V_1,E)) $\tilde{\subseteq}$ (U,E), and x$\tilde{\in}$ ($X_2 \tilde{\cap}$ (V_2,E)) and ($X_2 \tilde{\cap}$ (V_2,E)) $\tilde{\subseteq}$ U,E). Now since X = $X_1 \tilde{\cup} X_2$, therefore f ((V_1,E) $\tilde{\cap}$ (V_2,E)) =f($X_1 \tilde{\cap}$ (V_1,E) $\tilde{\cap}$ (V_2,E)) $\tilde{\cup}$f($X_2 \tilde{\cap}$ (V_1,E) $\tilde{\cap}$ (V_2,E)) $\tilde{\subseteq}$(f($X_1 \tilde{\cap}$ (V_1,E)) $\tilde{\cup}$f($X_2 \tilde{\cap}$ (V_2,E)) $\tilde{\subseteq}$ (U,E). Thus, (V_1,E) $\tilde{\cap}$ (V_2,E)=(V,E) is a soft δ-clopen set containing x such that f(V,E)) $\tilde{\subseteq}$(U,E) and hence f is soft biclop.na-continuous at x.

Theorem 2.10: Every restriction of a soft biclop.na-continuous mapping is soft biclop.na-continuous.
Proof: Let f be a soft biclop.na-continuous mapping of (X,τ,E) into (Y,τ,E) and t (A,E) be any soft subset of X. For any soft feebly-clopen subset (S,E) of Y, $(f/(A,E))^{-1}$(V,E) = (A,E) $\tilde{\cap}$ f^{-1}(V,E). But f is being soft biclop.na-continuous, f^{-1}(S,E) is soft δ-clopen and hence (A,E) $\tilde{\cap}f^{-1}$(V,E) is a relatively soft δ-clopen subset of (A,E), that is $(f/(A,E))^{-1}$(V,E) is a soft δ-clopen subset of (A,E). Hence f/(A,E) is soft biclop.na-continuous.

Theorem 2.11: Let f map (X,τ,E) into (Y,τ,E) and let x be a point of X. If there exist a soft δ-clopen set (N,E) of x such that the restriction of f to (N,E) is soft biclop.na-continuous at x, then f is soft biclop.na-continuous at x.
Proof: Let (U,E) be any soft feebly-clopen set containing f(x). Since f/(N,E) is soft biclop.na-continuous at x, therefore there is an soft δ-clopen set (V_1,E) such that x$\tilde{\in}$ (N,E) $\tilde{\cap}$ (V_1,E) and f((N,E) $\tilde{\cap}$ (V_1,E)) $\tilde{\subseteq}$ (U,E). Thus (N,E) $\tilde{\cap}$ (V_1,E) is soft δ-clopen set of x.

Theorem 2.12: Let X = (R_1,E) $\tilde{\cup}$ (R_2,E), where (R_1,E), (R_2,E) are soft δ-clopen sets in X. Let f : (R_1,E)→(Y,τ,E) and g : (R_2,E)→(Y,τ,E) be soft biclop.na-continuous.
If f(x) = g(x) for each x$\tilde{\in}$(R_1,E) $\tilde{\cap}$ (R_2,E). Then h : (R_1,E) $\tilde{\cap}$ (R_2,E)→(Y,τ,E) such that h(x) = f(x) for x$\tilde{\in}$ (R_1,E) and h(x) = g(x) for x$\tilde{\in}$ (R_2,E) is soft biclop.na-continuous.
Proof: Let (U,E) be a soft feebly- clopen set of Y. Now h^{-1}(U,E) = f^{-1}(U,E) $\tilde{\cup}$ g^{-1}(U,E). Since f and g are soft biclop.na-continuous, f^{-1}(U,E) and g^{-1}(U,E) are soft δ-clopen set in (R_1,E) and (R_2,E) respectively. But (R_1,E) and (R_2,E) are both soft δ-clopen sets in X. Since union of two soft δ-clopen sets is soft δ-clopen, so h^{-1}(U,E) is a soft δ-clopen set in X. Hence h is soft biclop.na-continuous.

Theorem 2.13: Let f : (X,τ,E)→ (Y,τ,E) be soft biclop.na-continuous surjection and (A,E) be soft δ-clopen subset of X. If f is soft feebly- clopen function, then the function g : (A,E) → f(A,E), defined by g(x) = f(x) for each x$\tilde{\in}$(A,E) is soft biclo.na-continuous.
Proof: Suppose that H = f(A,E). Let x$\tilde{\in}$ (A,E) and (V,E) be any soft feebly- clopen set in (H,E) containing g(x). Since (H,E) is soft feebly clopen set in Y and (V,E) is soft feebly-clopen in (H,E). Since f is soft biclop.na-continuous, hence there exist a soft δ-clopen set (U,E) in X containing x. Taking (W,E) = (U,E) $\tilde{\cap}$ (A,E), since (A,E) is soft δ-open and soft δ-clopen set in (A,E) containing x. Thus g is soft biclop.na-continuous.

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