Some Mapping on αc^*g-Open & Closed Maps in Topological Spaces

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Abstract

In this paper we have introduced the concept of Closed maps ,Open maps , Irresolute and Homeomorphism on the αc^*g-closed set and study some properties on them.

1. Introduction

Malghan [1] introduced and investigated some properties of generalized closed maps in topological spaces. The concept of generalized open map was introduced by Sundaram[2]. In this paper we introduced the concepts of αc^*g-closed maps and αc^*g-open maps in topological spaces.

2. Premilinaries

Definition: 2.1: A subset A of a topological space (X,τ) is called

(i) Generalized closed set (g-closed)[3] if cl(A) $\subseteq$U whenever A $\subseteq$U, and U is open in X.

(ii) α-generalized closed set αg-closed[4] if αcl(A) $\subseteq$U whenever A $\subseteq$U, and U is open in X.

(iii) αcg- closed set[5] if αcl(A) $\subseteq$U whenever A $\subseteq$U and U is C-set. The complement of αcg- closed set is αcg- open set[5].

(iv) αc*g-closed set[5] if αcl(A) $\subseteq$U whenever A $\subseteq$U and U is C*-set. The complement of αc*g - closed set is αc*g - open set[5].

(v) αc(s)g- closed set[5] if αcl(A) $\subseteq$U whenever A $\subseteq$U and U is C(s) set. The complement of αc(s)g- closed set is αc(s)g- open set[5].

Definition: 2.3: For a subset A of X is called

(i) a C-set(Due to Sundaram)[2] if A= G∩F where G is g-open and F is a t-set in X.

(ii) a C-set (Due to Hatir, Noiri and Yuksel)[9] if A = G∩F where G is open and F is an α*-set in X.

(iii) a C*set[11] if A= G∩F where G is g-open and F is an α*-set in X.

Definition 2.4: A function $f: X \to Y$ is said to be

(i) g-closed[3] in X for each closed set F in Y.

(ii) α-generalized continuous (αg-continuous)[15] if $f^{-1}(F)$ is αg-closed in X for each closed set F in Y.

(iii) closed map[1] if for each closed set F in X, $f(F)$ is closed in Y.

(iv) open map[1] if for each open set F in X, $f(F)$ is open in Y.

3. αc^*g-Closed maps & αc^*g-Open maps in topological spaces

Definition3.1: A map $f: X \to Y$ from a topological space X into a topological space Y is called αc^*g-closed map if for each closed set F in X, $f(F)$ is a αc^*g-closed set in Y.

Theorem 3.2: If a map $f: X \to Y$ is closed map then it is αc^*g-closed map but not conversely.

Proof: Since every closed set is αc^*g-closed set then it is αc^*g-closed map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.3: Let $X = Y = \{a, b, c\}$. Let f be a identity map such that

$f: (X, \tau_1) \to (Y, \tau_2)$. $\tau_1 = \{\varphi, Y, \{b, c\}\}$,
$\tau_2 = \{\varphi, X, \{a\}, \{a, c\}, \{a, b\}\}$.

Here

$C(Y, \tau_1) = \{\phi, Y, \{a\}\}$, $C(X, \tau_2) = \{\phi, X, \{b, c\}, \{b\}, \{c\}\}$.

$\alpha c^* g\_C(Y, \tau_2) = \{\phi, Y, \{b, c\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$

Then f is αc^*g-closed map but not closed map. Since for the closed set $\{a\}$ in (X, τ_1), $f(\{a\}) = \{a\}$ is not closed in Y.

Theorem 3.4: If a map $f: X \to Y$ is g-closed map then it is αc*g-closed map but not conversely.

Proof: Let $f: X \to Y$ be a g-closed map. Then for each closed set F in X, $f(F)$ is g-closed set in Y. Since every g-closed set is αc*g-closed set. Therefore $f(F)$ is αc*g-closed set. Hence f is αc*g-closed map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.5: Let $X = Y = \{a, b, c\}$. Let f be a identity map such that $f: (X, \tau_1) \to (Y, \tau_2)$. $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}, \{a, b\}, \{a, c\}\}$. Then f is αc*g-closed but not g-closed because for the closed set $\{a, c\}$ in X, $f(\{a, c\}) = \{a, c\}$ is not g-closed in Y. Therefore f is not g-closed map.

Theorem 3.6: If a map $f: X \to Y$ is α-closed map then it is αc*g-closed map but not conversely.

Proof: Let $f: X \to Y$ be a α-closed map. Then for each closed set F in X, $f(F)$ is α-closed set in Y. Since every α-closed set is αc*g-closed set. Therefore $f(F)$ is αc*g-closed set. Hence f is αc*g-closed map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.7: Let $X = Y = \{a, b, c\}$. Let f be a identity map such that $f: (X, \tau_1) \to (Y, \tau_2)$. $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}, \{a, b\}, \{a, c\}\}$. Then f is αc*g-closed but not α-closed because for the closed set $\{a, c\}$ in X, $f(\{a, c\}) = \{a, c\}$ is not α-closed in Y. Therefore f is not α-closed map.

Theorem 3.8: If a map $f: X \to Y$ is αg-closed map then it is αc*g-closed map but not conversely.

Proof: Let $f: X \to Y$ be a αg-closed map. Then for each closed set F in X, $f(F)$ is αg-closed set in Y. Since every αg-closed set is αc*g-closed set. Therefore $f(F)$ is αc*g-closed set. Hence f is αc*g-closed map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.9: Let $X = Y = \{a, b, c\}$. Let f be a identity map such that $f: (X, \tau_1) \to (Y, \tau_2)$. $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}, \{a, b\}, \{a, c\}\}$. Then f is αc*g-closed but not αg-closed because for the closed set $\{a, c\}$ in X, $f(\{a, c\}) = \{a, c\}$ is not αg-closed in Y. Therefore f is not αg-closed map.

Theorem 3.10: If a map $f: X \to Y$ is gs-closed map then it is αc*g-closed map but not conversely.

Proof: Let $f: X \to Y$ be a gs-closed map. Then for each closed set F in X, $f(F)$ is gs-closed set in Y. Since every gs-closed set is αc*g-closed set. Therefore $f(F)$ is αc*g-closed set. Hence f is αc*g-closed map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.11: Let $X = Y = \{a, b, c\}$. Let f be a identity map such that $f: (X, \tau_1) \to (Y, \tau_2)$. $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}, \{a, b\}, \{a, c\}\}$. Then f is αc*g-closed but not gs-closed because for the closed set $\{a, c\}$ in X, $f(\{a, c\}) = \{a, c\}$ is not gs-closed in Y. Therefore f is not gs-closed map.

Definition3.12: A map $f: X \to Y$ from a topological space X into a topological space Y is called αc*g-open map if $f(F)$ is a αc*g-open set in Y for every open set F in X.

Theorem 3.13: If a map $f: X \to Y$ is open map then it is αc*g-open map but not conversely.

Proof: Let $f: X \to Y$ be a open map. Let F be any open set in X, $f(F)$ is open set in Y. Then $f(F)$ is αc*g-open set. Since every open set is αc*g-open set. Hence f is αc*g-open map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.14: Let $X = Y = \{a, b, c\}$. Let f be a identity map such that $f: (X, \tau_1) \to (Y, \tau_2)$. $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}, \{a, b\}, \{a, c\}\}$.Then f is αc^*g-open map but not open map because for the open set $\{b\}$ in X, $f(\{b\}) = \{b\}$ is not open in Y. Therefore f is not open map.

Theorem 3.15: If a map $f: X \to Y$ is g-open map then it is αc^*g-open map but not conversely.

Proof: Let $f: X \to Y$ be a g-open map. Let F be any open set in X, $f(F)$ is g-open set in Y. Since every g-open set is αc^*g-open set. Then $f(F)$ is αc^*g-open set. Hence f is αc^*g-open map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.16: Let $X = Y = \{a, b, c\}$. Let f be a identity map such that $f: (X, \tau_1) \to (Y, \tau_2)$. $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}, \{a, b\}, \{a, c\}\}$.Then f is αc^*g-open map but not g-open map because for the open set $\{b\}$ in X, $f(\{b\}) = \{b\}$ is not g-open in Y. Therefore f is not g-open map.

Theorem 3.17: If a map $f: X \to Y$ is αg-open map then it is αc^*g-open map but not conversely.

Proof: Let $f: X \to Y$ be a αg-open map. Let F be any open set in X, $f(F)$ is αg-open set in Y. Since every αg-open set is αc^*g-open set. Then $f(F)$ is αc^*g-open set. Hence f is αc^*g-open map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.18: Let $X = Y = \{a, b, c\}$. Let f be a identity map such that $f: (X, \tau_1) \to (Y, \tau_2)$. $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}, \{a, b\}, \{a, c\}\}$.Then f is αc^*g-open map but not αg-open map because for the open set $\{b\}$ in X, $f(\{b\}) = \{b\}$ is not αg-open in Y. Therefore f is not αg-open map.

Theorem 3.19: If a map $f: X \to Y$ is α-open map then it is αc^*g-open map but not conversely.

Proof: Let $f: X \to Y$ be a α-open map. Let F be any open set in X, $f(F)$ is α-open set in Y. Since every α-open set is αc^*g-open set. Then $f(F)$ is αc^*g-open set. Hence f is αc^*g-open map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.20: Let $X = Y = \{a, b, c\}$. Let f be a identity map such that $f: (X, \tau_1) \to (Y, \tau_2)$. $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}, \{a, b\}, \{a, c\}\}$.Then f is αc^*g-open map but not α-open map because for the open set $\{b\}$ in X, $f(\{b\}) = \{b\}$ is not α-open in Y. Therefore f is not α-open map.

Theorem 3.21: If a map $f: X \to Y$ is gs-open map then it is αc^*g-open map but not conversely.

Proof: Let $f: X \to Y$ be a gs-open map. Let F be any open set in X, $f(F)$ is gs-open set in Y. Since every gs-open set is αc^*g-open set. Then $f(F)$ is αc^*g-open set. Hence f is αc^*g-open map.

The converse of the above theorem need not be true as seen from the following example.

Example 3.22: Let $X = Y = \{a, b, c\}$. Let f be a identity map such that $f: (X, \tau_1) \to (Y, \tau_2)$. $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}, \{a, b\}, \{a, c\}\}$.Then f is αc^*g-open map but not gs-open map because for the open set $\{b\}$ in X, $f(\{b\}) = \{b\}$ is not gs-open in Y. Therefore f is not gs-open map.

Theorem 3.23: If $f: X \to Y$ is αc*g-continuous and αc*g-closed and A is a αc*g-closed set of X, then $f(A)$ is αc*g-closed in Y.

Proof: Let $f(A) \subseteq O$, where O is c*-set of Y, Since f is αc*g-continuous, $f^{-1}(O)$ is c*-set containing A. Hence $cl(A) \subseteq f^{-1}(O)$ as A is αc*g-closed. Since f is αc*g-closed, $f(cl(A))$ is αc*g-closed set contained in c*-set O, which implies that $cl[f(cl(A))] \subseteq O$ and hence $cl(f(A)) \subseteq O$. So $f(A)$ is αc*g-closed in Y.

Corollary 3.24: If $f: X \to Y$ is continuous and closed map and if A is αc*g-closed set in X, then $f(A)$ is αc*g-closed in Y.

Proof: Since every continuous map is αc*g-continuous and every closed map is αc*g-closed, by the above theorem the result follows.

Theorem 3.25: If $f: X \to Y$ is closed and $h: Y \to Z$ is αc*g-closed then $h \circ f: X \to Z$ is αc*g-closed.

Proof: Let $f: X \to Y$ is a closed map and $h: Y \to Z$ is αc*g-closed map. Let V be any closed set in X. Since $f: X \to Y$ is closed, $f(V)$ is closed in Y and since $h: Y \to Z$ is αc*g-closed ,$h(f(V))$ is αc*g-closed set in Z. Therefore $h \circ f: X \to Z$ is αc*g-closed map.

Theorem 3.26: If $f: X \to Y$ is αc*g-closed and A is closed set in X. Then $f_A: A \to Y$ is αc*g-closed.

Proof: Let V be closed set in A. Then V is closed in X. Therefore f is αc*g-closed set in Y. By theorem 1.24 $f(V)$ is αc*g-closed. That is $f_A(V) = f(V)$ is αc*g-closed set in Y. Therefore $f_A: A \to Y$ is αc*g-closed.

4. αc*g -irresolute map in Topological Spaces

Crossely and Hildebrand[9] introduced and investigated the concept of irresolute function in topological spaces. Sundaram[2] , Maheshwari and Prasad[10], Jankovic[11] have defined gc-irresolute maps, α-irresolute maps and p-open maps in topological spaces.

In this section, we have introduced a new class of map called αc*g -irresolute map and study some of their properties.

Definition 4.1: A map $f: X \to Y$ from topological space X into a topological space Y is called αc*g -irresolute map in the inverse of every αc*g -closed(αc*g -open) set in Y is αc*g -closed (αc*g -open) in X.

Theorem 4.2: If a map $f: X \to Y$ is αc*g -irresolute, then it is αc*g -continuous, but not conversely.

Proof: Assume that f is αc*g -irresolute. Let F be any closed set in Y. Since every closed set is αc*g -closed, F is αc*g -closed in Y. Since f is αc*g -irresolute, irresolute, $f^{-1}(F)$ is αc*g -closed in X. Therefore f is αc*g -continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 4.3: Consider the topological space $X = Y = \{a, b, c\}$ with topology $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}\}$ Let $f(X, \tau_1) \to (Y, \tau_2)$ be the identity map then f is αc*g -continuous , because for the inverse image of every closed in Y is αc*g -closed in X, but not αc*g -irresolute. Because for the inverse image of every αc*g -closed in Y is not αc*g -closed in X. (ie) for the αc*g -closed set {b} in Y the inverse image $f^{-1}(\{b\}) = \{b\}$ is not αc*g -closed in X.

Theorem 4.4: Let X,Y,and Z be any topological spaces.For any αc*g -irresolute map $f: X \to Y$ and any αc*g -continuous map $g: Y \to Z$ the composition $g \cdot f: X \to Z$ is αc*g -continuous.

Proof: Let F be any closed set in Z. Since g is αc*g -continuous, $g^{-1}(F)$ is αc*g -closed in Y. Since f is αc*g -irresolute $f^{-1}(g^{-1}(F))$ is αc*g -closed $f^{-1}(g^{-1}(F)) = (g \cdot f)^{-1}(F)$. Therefore $g \cdot f$ is αc*g -continuous.

Theorem 4.5: If $f: X \to Y$ from topological space X into a topological space Y is bijective, αc*g -open set and αc*g -continuous then f is αc*g -irresolute.

Proof: Let A be a αc^*g -closd set in Y. Let $f^{-1}(A) \subseteq O$,Where O is C^*-set in X. Therefore $A \subseteq f(O)$holds. Since$f(O)$ is αc^*g –open set and A is αc^*g -closed in Y, $\alpha cl(A) \subseteq f(O), f^{-1}(\alpha cl(A)) \subseteq f(O)$

Since f is αc^*g -continuous and αcl(A) is closed in Y. $\alpha cl(f^{-1}(\alpha cl(A)) \subseteq O$

and so $\alpha cl(f^{-1}(A)) \subseteq O$ .Therefore $f^{-1}(A)$ is

αc^*g -closed in X. Hence f is αc^*g -irresolute.

The following examples show that no assumption of the above theorem can be removed.

Example 4.6: Consider the topological space $X = Y = \{a, b, c\}$ with topology $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}\}$. Then the defined identity map$f(X, \tau_1) \rightarrow (Y, \tau_2)$ is αc^*g -continuous, bijective and not αcg-open. So f is not αcg-irresolute. Since for the αc^*g -closed set {a} in Y the inverse image $f^{-1}(\{a\}) = \{a\}$ is not αc^*g -closed in X.

Example 4.7: Consider the topological space $X = Y = \{a, b, c\}$ with topology $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}\}$. Then the map $f(X, \tau_1) \rightarrow (Y, \tau_2)$be defined by $f(a) = a, f(b) = b, f(c) = a$.Then f is αc^*g -continuous, αc^*g -open and not bijective. So f is not αc^*g -irresolute. Since for the αc^*g -closed set {b} in Y the inverse image $f^{-1}(\{b\}) = \{b\}$ is not αc^*g -closed in X.

Example 4.8: Consider the topological space $X = Y = \{a, b, c\}$ with topology $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}, \{a, b\}, \{a, c\}\}$ Then the defined identity map $f(X, \tau_1) \rightarrow (Y, \tau_2)$ is bijective, αc^*g –open and not αc^*g -continuous,. So f is not αc^*g -irresolute. Since for the αc^*g -closed set {b} in Y the inverse image $f^{-1}(\{b\}) = \{b\}$ is not αc^*g -closed in X.

Remark 4.9: The following two examples show that the concepts of irresolute maps and αc^*g -irresolute maps are independent of each other.

Example 4.10: Consider the topological space $X = Y = \{a, b, c\}$ with topology $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}, \{a, b\}\}$ Then the defined identity map $f(X, \tau_1) \rightarrow (Y, \tau_2)$ is irresolute but not αc^*g -irresolute. Since {b} is αc^*g -closed set in Y has its inverse image $f^{-1}(\{b\}) = \{b\}$ is not αc^*g -closed in X.

Example 4.11: Consider the topological space $X = Y = \{a, b, c\}$ with topology $\tau_1 = \{\varphi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}, \{b\}, \{a, b\}\}$. Then the defined identity map $f(X, \tau_1) \rightarrow (Y, \tau_2)$ is αc^*g -irresolute but not irresolute. Since the closed set {a,c} in Y has its inverse image $f^{-1}(\{a, c\}) = \{a, c\}$ is not closed in X.

Remark 4.12: From the following diagram we can conclude that αc^*g -irresolute map is independent with irresolute map.

αc^*g -irresolute map ←|→ irresolute map

5. αc^*g -homeomorphism maps in Topological Spaces

Several mathematicians have generalized homeomorphism in topological spaces. Biswas[14],Crossely and Hildebrand[9], Gentry and Hoyle[13] and Umehara and Maki[12] have introduced and investigated semi-homeomorphism, which also a generalization of homeomorphism. Sundaram[2] introduced g-homeomorphism and gc-homeomorphism is topological spaces.

In this section we introduce the concept of αc^*g -homeomorphism and study some of their properties.

Definition 5.1: A bijection $f(X, \tau_1) \rightarrow (Y, \tau_2)$ is called αc^*g - homeomorphism if f is both αc^*g -open and αc^*g -continuous.

Theorem 5.2: Every homeomorphism is a αc^*g -homeomorphism but not conversely.
Proof: Since every continuous function is αc^*g -continuous and every open map is αc^*g -open the proof follows.

The converse of the above theorem need not be true as seen from the following example.

Example 5.3: Let $X = Y = \{a, b, c\}$with $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, X, \{a, b\}\}$ then $f(X, \tau_1) \rightarrow (Y, \tau_2)$ is αc^*g -homeomorphism but not homeomorphism.

Theorem 5.4: For any bijection $f: X \rightarrow Y$ the following statements are equivalent.

i) $f^{-1}: Y \rightarrow X$ is αc^*g -continuous.

ii) f is a αc^*g -open map.

iii) f is a αc^*g -closed map.

Proof: $(i) \Rightarrow (ii)$Let G be any open set in X.Since f^{-1} is αc^*g -continuous, the inverse image of G under f^{-1} namely $f(G)$ is αc^*g -open in Y.So f is αc^*g -open map.

$(ii) \Rightarrow (iii)$ Let F be any closed set in X.Then F^c is open in X. Since f is αc^*g -open map $f(F^c)$ is αc^*g -open map in Y.But $f(F^c) = Y - f(F)$ and so $f(F)$ is αc^*g -open map in Y.Therefore f is a αc^*g -closed map.

$(iii) \Rightarrow (i)$ Let F be any closed set in X. Then $(f^{-1})^{-1}F = f(F)$ is αc^*g -closed map in Y.Therefore $f^{-1}: Y \rightarrow X$ is αc^*g -continuous.

Theorem 5.5: Let $f(X, \tau) \rightarrow (Y, \sigma)$ be a bijective and αc^*g -continuous map the following statement are equivalent.

i) f is a αc^*g -open map.

ii) f is a αc^*g -homeomorphism.

iii) f is a αc^*g -closed map.

Proof: The proof easily follows from definitions and assumptions.

The following examples shows that the composition of two αc^*g -homeomorphism need not be αc^*g -homeomorphism.

Example 5.6: Let $X = Y = Z = \{a, b, c\}$ with topologies $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$,

$$\tau_2 = \{\varphi, Y, \{a\}, \{a, b\}\}, \tau_3 = \{\varphi, Z, \{a, b\}\}$$

Let f and g be identity maps such that $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ then f and g are αc^*g -homeomorphism, but their composition $g \cdot f: X \rightarrow Z$ is not αc^*g -homeomorphism.

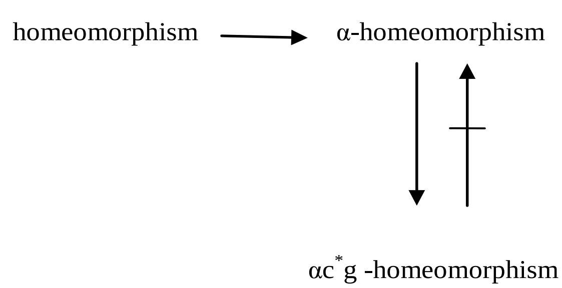
Theorem 5.7: Every α-homeomorphism is a αc^*g- homeomorphism.

Proof: Let $f: X \rightarrow Y$ be a α-homeomorphism then f is α-continuous and α-closed. Since every α-continuous is αc^*g -continuous and every α -closed is αc^*g -closed, f is αc^*g -continuous and αc^*g -closed. Therefore f is αc^*g -homeomorphism.

The converse of the above theorem need not to be true as seen from the following example.

Example 5.8:
Consider the topological space $X = Y = \{a, b, c\}$ with topology $\tau_1 = \{\varphi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\varphi, Y, \{a\}, \{b\}, \{a, c\}\}$. Then the defined identity map $f(X, \tau_1) \rightarrow (Y, \tau_2)$ is αc^*g -homeomorphism but not α-homeomorphism.Since for the open set {a} in X the inverse image $f^{-1}(\{a\}) = \{a\}$ is not α-open in Y.

From the above observations we get the following diagram:



Definition 5.9 : A bijection $f(X, \tau) \rightarrow (Y, \sigma)$ is said to be $(\alpha c^*g)^*$ homeomorphism if f and its inverse f^{-1} are αc^*g -irresolute map.

Notation 5.10: Let the family of all $(\alpha c^*g)^*$-homeomorphism from (X, τ) onto itself be denoted by $(\alpha c^*g)^*h(X, \tau)$ and the family of all αc^*g -homeomorphism from (X, τ) onto itself be denoted by $(\alpha cg)h(X, \tau)$. The family of all homeomorphism from (X,τ) onto itself be denoted by $h(X,\tau)$.

Theorem 5.11: Let X be a topological space. Then i) The set $(\alpha c^* g)^* \, h(X)$ is group under composition of maps. (ii) h(x) is a subgroup of $(\alpha c^* g)^* \, h(X)$
(iii) $(\alpha c^* g)^* \, h(X) \subset (\alpha c^* g) h(X)$.

Proof for (i): Let $f, g \in (\alpha c^* g)^* \, h(X)$, then $g \cdot f \in (\alpha c^* g)^* \, h(X)$ and so $(\alpha c^* g)^* \, h(X)$ is closed under the composition of maps.The composition of maps is associative. The identity map I:X→X is a $(\alpha c^* g)^*$-homeomorphism and so $I \in (\alpha c^* g)^* \, h(X)$. Also $f \cdot I = I \cdot f = f$ for every
$f \in (\alpha c^* g)^* \, h(X)$. If $\in (\alpha c^* g)^* \, h(X)$, then $f^{-1} \in (\alpha c^* g)^* \, h(X)$ and
$f \cdot f^{-1} = f^{-1} \cdot f = I$.Hence $(\alpha c^* g)^* \, h(X)$ is a group under the composition of maps.

Proof for (ii): Let $f(X,\tau) \to (Y,\sigma)$ be a homeomorphism. Then by theorem 4.5.Both of f and f^{-1} are $(\alpha c^* g)^*$- irresolute and so f is a $(\alpha c^* g)^*$-homeomorphism.Therefore every homeomorphism is a $(\alpha c^* g)^*$-homeomorphism and so $h(x)$ is a subset of $(\alpha c^* g)^* \, h(X)$.
Also $h(x)$ is a group under composition of maps.Therefore $h(x)$ is a subgroup of group $(\alpha c^* g)^* \, h(X)$.

Proof for (iii): Since every $(\alpha c^* g)^*$-irresolute map is $\alpha c^* g$-continuous, $(\alpha c^* g)^* \, h(X)$ is a subset of $(\alpha c^* g)^* \, h(X)$.

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