# SOME LRS BIANCHI TYPE-I COSMOLOGICAL MODELS WITH ZERO-MASS SCALAR FIELD

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**ABSTRACT**

In this paper, we have considered some LRS Bianchi I cosmological models in the presence of zero-mass scalar fields associated with a perfect fluid distribution on it. We have also discussed various physical and geometrical features of the models.



**Keywords :** LRS Bianchi I models, Zero-mass scalar fields, Conservation equation, Four vector velocity, Energy momentum tensor.

## 1. INTRODUCTION

The study of scalar meson fields has attracted the attention of many workers. Brahamachary [2] considered the massive, whereas Bergmann and Leipnik [1] considered the massless scalar field coupled to spherically symmetric gravitational fields. Janis et. al. [13] have further considered the problem from the point of view of singularities and Gautreau [9 ] and Singh [32] have extended the study to the case of non-spherical Weyl and plane symmetric fields respectively. Later on, the workers in the field, with a few exceptions (Stephenson [35] have directed their efforts to the study of the massless scalar fields coupled to gravitational and electromagnetic fields (Mishra and Pandey [18]; Rao et. al. [24], [25]; Reddy [27], Singh [33]. The generalization of the Reissner-Nordstrom solution in the presence of a massless scalar field was done by Penny [20]. Janis et. al. [14] obtained the solutions of the Einstein-scalar and Brans-Dicke [3] field equations for static space time and also gave a procedure to generate static solutions of the coupled Einstein-Maxwell-scalar field equations. The solutions of axially symmetric Einstein-Maxwell-scalar field equations have been given by Eris and Gurses [8].



Singh et. al. [34] have found a method to obtain solutions to the cylindrically symmetric gravitational field coupled to massless scalar and non-null Maxwell fields. They have shown that starting from any solution to the electrovacuum field equations it is possible to generate a whole class of solutions to the coupled Einstein-Maxwell-scalar field equations by a suitable redefinition of one of the space-time metric coefficients. They have further applied the technique to the solution due to Chitre et. a1. [7] and have also obtained the dual solution by an extension of Bonnor's theorem [4].

As a matter of fact following the development of inflationary models, the importance of scalar fields (mesons) in cosmology has become well known [15]. The study of interacting fields, one of the fields being a zero-mass scalar field is basically an attempt to look into the yet unsolved problem of the unification of gravitational and quantum theories [29, 30]. Considerable interest has been focused on a set of field equations representing zero-mass scalar-fields coupled with the gravitational field for the last three decades. Bergamann and Leipnik [1] and Brahmachary [2] have investigated the spherically symmetric fields associated with zero-rest-mass. The static solutions for axially symmetric fields have been investigated by Buchdahl [5]. Janis et. al. [13-14], in an attempt to present an extension of Israel's idea of a singular even horizons [12] have considered the spherically symmetric solutions of the fie-ld equations of general relativity containing zero-rest-mass meson fields. Penny [21] and Gautreau [9] have extended the study of the case of axially symmetric fields and have found that the scalar fields obey a flat space Laplace equation and a large class of solution exist. Singh [32], Patel [19] and Reddy [27] have investigated plane symmetric solutions of the field equations corresponding to zero-mass scalar fields. Stephenson [35], Rao et. al. [24], Chatterjee and Roy [6], Reddy and Rao [26], Verma [36], Shanthi and Rao [31], Pradhan et. al. [22] are some of the authors who have studied various aspects of interacting fields in the framework of general relativity. At the present state of evolution, the universe is spherically symmetric with isotropic and homogeneous matter distribution. But in its early stages of evolution, it could have not had a smoothed out picture. Close to the big bang singularity, neither the assumption of spherically symmetric nor of isotropy can be strictly valid. So we, consider plane symmetry, which is less restrictive than spherical symmetry and provides an avenue to study in homogeneities. For simplification and description of the large scale behaviour of the actual universe, locally rotationally symmetric (LRS**)** Bianchi I space-time has been widely studied. Mazumdar [16] has obtained solutions of an LRS Bianchi I space-time filled with a perfect fluid. Hajj-Boutros and Sfeila [10] and Sri Ram [28] have also obtained some solutions for the same field equations by using their solution -generating techniques. Pradhan et. al. [23] have studied LRS Bianchi I space-time with zero-mass scalar field. In fact cosmological models based on scalar fields of various kinds have had enormous success in solving cosmological problems among which are the causality, entropy, initial singularity and cosmological constant problem.

Here in this paper, we have considered some LRS Bianchi I cosmological models in the presence of zero-mass scalar fields associated with a perfect fluid distribution in it. We have also discussed various physical and geometrical features of the models.

## 2. THE FIELD EQUATIONS:

The metric for the LRS Bianchi I space-time is of the form [17].

(2.1) $$ds^2 = -dt^2 + \lambda^2 dx^2 + \mu^2(dy^2 + dz^2)$$

Where λ and μ are functions of the cosmic time t. The energy momentum tensor of a perfect fluid together with a zero - mass scalar field is given by

(2.2) $$T_{ij}^{(m)} \; T_{(ij)}^{(s)}$$

Where

(2.3) $$T_{ij}^{(m)} = (\rho + P)u_j \; u_j + pg_{ij}$$

is the energy momentum tensor corresponding to perfect fluid distribution with the four vector velocity $u^i$ satisfying $u_i \; u^i = -1$, p the pressure and ρ the mass - energy density. The energy momentum tensor $T_{ij}^{(s)}$ corresponds to zero - mass scalar fields ϕ and is

(2.4) $$T_{ij}^{(s)} = \phi_{ij} \; \phi_j - \frac{1}{2} g_{ij} \; g^{ab} \; \phi, a^{\phi}, b,$$

where ϕ(t) (a function of t only) is the zero - mass scalar field which satisfies the wave equation

(2.5) $$g^{ij} \phi_{;ij} = 0$$

The scalar field $\phi$ is not directly coupled to matter. It interacts with matter indirectly through gravity. The Einstein's field equations

(2.6) $$R_{ij} - \frac{1}{2} R g_{ij} = k T_{ij}$$

together with energy momentum tensor defined by equation (2.2) give the following equations

(2.7) $$-Kp + \phi^2 = \frac{2\ddot{\mu}}{\mu} + \frac{\dot{\mu}^2}{\mu^2}$$

(2.8) $$-Kp + \phi^2 = \frac{\ddot{\mu}}{\mu} + \frac{\dot{\lambda}\dot{\mu}}{\lambda\mu} + \frac{\ddot{\lambda}}{\lambda}$$

and

(2.9) $$K_\rho - \phi^2 = \frac{2\dot{\lambda}\dot{\mu}}{\lambda\mu} + \frac{\dot{\mu}^2}{\mu^2}$$



where k =8πG, G the gravitational constant. The overdot indicates a derivative with respect to time t. The wave equation (2.5) yields

(2.10) $$\left(\frac{\dot{\lambda}}{\lambda} + \frac{2\dot{\mu}}{\mu}\right)\dot{\phi} + \ddot{\phi} = 0$$

and the energy conservation equation for the matter $T^{(m)}_{ij,i} = 0$ leads to

(2.11) $$\dot{\rho} + \left(\frac{\dot{\lambda}}{\lambda} + \frac{2\dot{\mu}}{\mu}\right)(\rho + p) = 0$$

## 3. SOLUTIONS OF THE FIELD EQUATIONS

From equations (2.7) and (2.8) we obtain

(3.1) $$\frac{\ddot{\mu}}{\mu}+\frac{\dot{\mu}^2}{\mu^2}-\frac{\ddot{\lambda}}{\lambda}-\frac{\dot{\lambda}\dot{\mu}}{\lambda\mu}=0$$

Which has first integral

(3.2) $$\mu^2\dot{\lambda}-\lambda\mu\dot{\mu}=A$$

Where A is an integrating constant

Equation (3.2) is a linear differential equation in $\lambda(t)$ and has an exact solution

(3.3) $$\lambda=C_1\mu+A\mu\int\frac{df}{\mu^3(t)}$$

Similarly equation (3.2) is also a linear differential equation in $\mu(t)$, which has an exact solution,

(3.4) $$\mu^2=C_2\lambda^2-2A\lambda^2\int\frac{df}{\lambda^3(t)}$$

On integration, equation (2.10) yields

(3.5) $$\Phi=C_4+\int\frac{C_3dt}{\lambda(t)\mu^2(t)}$$

Where $C_1$,$C_2$,$C_3$ and $C_4$ are integration constants.

Thus, for any arbitrary $\mu(t)$, equation (3.3) gives $\lambda(t)$ and then $\Phi$ is known from equation (3.5). Similarly for an arbitrary $\lambda(t)$ one can calculate $\mu(t)$ and $\Phi$ from equation (3.4) and (3.5). Then from equations

(2.7) and (2.9), p and ρ can be obtained and hence the solution of the field equations is completely known

To illustrate our problem, we choose $\mu = t^{\frac{1}{2}(1-3f)}$. From eqns. (3.3) and (3.5) we obtain

(3.6) $$\lambda = C_1 t^{k(1-3f)} + \frac{2A}{9f-1} t^{3f}$$

and

(3.7) $$\Phi^2 = C_3^2 \left[ C_1 t^{k(1-3f)} + \frac{2A}{(9f-1)} t^{3f(1-3f)} \right]$$

where k and f (≠1) are real constants. So, in this case, the geometry of our universe is given by metric.

(3.8) $$ds^2 = -dt^2 + \left[ C_1 t^{k(1-3f)} + \frac{2A}{(9f-1)} t^{3f} \right] dx^2 + t^{2k(1-3f)}\left(dy^2 + dz^2\right)$$



For the metric (3.8) from the equations (2.7) – (2.9) find the expression for p and ρ

(3.9) $$Kp = C_3^2 \left[ C_1 t^{k(1-3f)^2} + \frac{2A}{(9f-1)} t^{3f(1-3f)} \right]^{-2} + \frac{(1-3f(1+9f)}{4t^2}$$

(3.10) $$K_\rho = C_3^2 \left[ C_1 t^{k(1-3f)^2} + \frac{2A}{(9f-1)} t^{3f(1-3f)} \right]^{-2}$$

$$+\frac{2A(1-3f)(1+9f)t^{k(9f-1)}+3C_1(1-3f)^2(9f-1)}{4t^2\left[2A\,t^{\frac{1}{2}(9f-1)}+k_1(9f-1)\right]}$$

When $k=\frac{1}{2}$ we get solution due to Pradhan et. al. [23] by suitable adjustment of constants. However, when $k=\frac{1}{2}$ we get

(3.11) $$ds^2=-dt^2+\left[C_1t^{\frac{1}{2}(1-3f)}+\frac{2A}{(9f-1)}t^{3f}\right]dx^2$$

$$+t^{(1-3f)}\left(dy^2+dz^2\right)$$

Also p and ρ are given by

(3.12) $$Kp=C_3^2\left[C_1t^{\frac{1}{2}(1-3f)^2}+\frac{2A}{(9f-1)}t^{3f(1-3f)}\right]^{-2}$$

$$+\frac{(1-3f)(1+9f)}{4t^2}$$

(3.13) $$K_\rho=C_3^2\left[C_1t^{\frac{1}{2}(1-3f)^2}+\frac{2A}{(9f-1)}t^{3f(1-3f)}\right]^{-2}$$

$$+\frac{2A(1-3f)(1+9f)t^{\frac{1}{2}(9f-1)}+3C_1(1-3f)(9f-1)}{4t^2\left[2A\,t^{\frac{1}{2}(9f-1)}+C_1(9f-1)\right]}$$

When $f=\frac{1}{3}$, p = ρ= constant, whereas in the absence of scalar field we get p=ρ=0 [16].

The energy conditions [8(a)]

(i) $(\rho+p) > 0$

(ii) $(\rho+3p)>0$ and

(iii) $\rho>0$

are satisfied when $C_1 > 0, A > 0,$ and $\frac{1}{9} < f < \frac{1}{3}$ and the dominant energy conditions [11].

(i) $(\rho - p) \geq 0$ and

(ii) $(\rho+p) \geq 0$

when $A > 0, C_1 > 0$ and $\frac{1}{9} < f < \frac{2}{9}$



The expansion scalar θ, the shear tensor $\sigma_{\alpha\gamma}$, the rotation $\omega_{\alpha\gamma}$ and the acceleration vector $a_\alpha$ for the velocity field $u_\alpha$ are defined by

(3.14) $\theta = u^{\alpha}_{;\alpha}$

(3.15) $\sigma_{\alpha\gamma} = \frac{1}{2}\left(u_{\alpha;\gamma} + u_{\gamma;\alpha}\right) - \frac{1}{2}\left(u_\alpha a_k + u_\gamma a_\alpha\right) - \frac{1}{3}\theta\left(g_{\alpha\gamma} + u_\alpha u_\gamma\right),$

(3.16) $\omega_{\alpha\gamma} = u_{\alpha;\gamma} - \sigma_{\alpha\gamma} - \frac{1}{3}\theta\left(g_{\alpha\gamma} + u_\alpha u_\gamma\right) - u_{\alpha;\beta}u^\beta u_\gamma$

and

(3.17) $$a_\alpha = u^\gamma u_{\alpha;\gamma}$$

Here the semicolon indicates covariant differentiation. The spatial volume is given by

$$V = \lambda\mu^2$$

For the velocity field $u_\alpha$ these kinematical parameters are found to have the following expressions :

(3.18) $$V = \frac{t\left(C_1(9f-1) + 2At^{\frac{1}{2}(9f-1)}\right)}{(9f-1)t^{\frac{1}{2}(9f-1)}}$$

(3.19) $$\theta = \frac{4At^{\frac{1}{2}(9f-1)} + 3C_1(1-f)(9f-1)}{2t\left[2A\,t^{\frac{1}{2}(9f-1)} + C_1(9f-1)\right]}$$

(3.20) $$\sigma = \frac{1}{\sqrt{6}}\left[\frac{2At(9f-1)t^{\frac{1}{2}(9f-1)}}{t\left[2A\,t^{\frac{1}{2}(9f-1)} + C_1(9f-1)\right]}\right]$$

and

(3.21) $$\omega = 0$$

(3.22) $$a_\alpha = [0,\ 0,\ 0,\ 0].$$

**4. <u>DISCUSSION</u>**

From above equations [3.19 - 3.22] we see that our model is expanding, shearing and non-rotating. The acceleration vector $a_\alpha$ is zero and consequently the stream links of the perfect fluid are geodetic As the shear tensor is not zero, the model is clearly anisotropic.

For $f = \frac{1}{3}$, the metric (3.11) represents a non-static cosmological model filled with a stiff fluid, the pressure and density of which are given by

(4.1) $$Kp = K\rho = \frac{C_3^2}{(C_1 + A)^2}$$

The models with ρ=p are important in relativistic cosmology for the description of very early stages of the universe



Choosing $\lambda = h_1 t^{k(1-3f)} + h_2 t^{3f}$ and A=0 in equation [3.3], we find

(4.2) $$\mu^2 = g_1 t^{2k(1-3f)} + g_2 t^{k+3f(1-k)} + g_3 t^{6f}$$

Where $g_1 = C_2 h_1^2, g_2 = 2C_2 h_1 h_2, g_3 = C_2 h_2^2$

Hence, in this case, the geometry of our universe is given by metric

(4.3) $$ds^2 = -dt^2 + \left(h_1 t^{k(1-3f)} + h_2 t^{3f}\right)^2 dx^2$$
$$+ \left[g_1 t^{2k(1-3f)} + g_2 t^{k+3f(1-k)} + g_3 t^{6f}\right]\left(dy^2 + dz^2\right)$$

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