Some Investigation LRS Bianchi Type- I Model in General Relativity

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***Abstract*-Locally Rotationally symmetric (LRS) Bianchi type-I cosmological model is studied in the context of general theory of relativity with the matter cosmic cloud string and electromagnetic field respectively. Further some physical and kinematical properties are discussed.**

***Keyword*- *LRS Bianchi Model, cosmic cloud string, electromagnetic field, general relativity*.**

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1. INTRODUCTION

Space-times admitting a three parameter group of automorphisms are important in discussing the cosmological models. The case where the group is simply transitive over the 3-dimensional, constant time subspace is particularly useful for two reasons. First, Bianchi has shown that there are only nine distinct sets of structure constants for groups of this type. Therefore, we can use algebra to classify the homogeneous Space -times. The second reason for the importance of Bianchi type Space - times is the simplicity of the field equations.

When we study the Bianchi type models, we observe that the models contain isotropic special cases and they permit arbitrarily small anisotropic levels at some instant of cosmic times.

Bianchi type cosmological models are important in the sense that these models are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view, anisotropic universe has a greater generality than isotropic models. The simplicity of the field equations made Bianchi space time useful in constructing models of spatially homogeneous and anisotropic cosmologies.

Hence, these models are to be known as suitable models of our universe. Therefore, study of Bianchi type models create much more interest.

Now, we discuss LRS Bianchi type - I cosmological space - times in general relativity. The present day observations indicates that the universe at large scale is homogeneous and isotropic, and we have accelerating phase of universe

It is well known that the exact solutions of general theory of relativity for homogeneous space times belong to either Bianchi types or Kantowski–Sachs. For simplification and description of the large scale behavior of the actual universe, locally rotationally symmetric (Henceforth referred as to LRS) Bianchi-I space-time has been widely studied. In order to study problems like the formation of galaxies of the process of homogenization and isotropization of the universe, it is necessary to study problems relating to inhomogeneous and anisotropic space-time.Mazumder [1] has obtained cosmological solutions for LRS Bianchi-I space-time filled with a perfect fluid with arbitrary cosmic scale functions and studied kinematical properties of the particular form of the solution. Mohanti [2,3] also obtained some solutions for the same field equations by using solution generation technique with the matter perfect fluid. Banerjee [4] studied Bianchi type I cosmologies with cosmic strings. Numbers of researches [5-17] have studied Bianchi Type- I cosmological models in general relativity.

2. Metric and Field Equation

The LRS Bianchi type I line element is given by

$$ds^2 = -dt^2 + A^2 dt^2 + B^2(dy^2 + dz^2) \qquad (2.1)$$

Where, A and B are the scale factor (metric potential) and function of the cosmic time t only (non-static case).

The Einstein field equation in the general relativity is given by

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi k T_i^j \qquad (2.2)$$

Where, R_i^j is known as Ricci Tensor

And $R = g^{ij} R_{ij}$ is the Ricci Scalar

T_i^j is energy momentum tensor for the matter.

Einstein’s field equation (2.2) for line element (2.1) lead to

$$R_{11} = -A\left[\ddot{A} + 2\frac{\dot{A}\dot{B}}{B}\right] \quad (2.3)$$

$$R_{22} = -B\left[\ddot{B} + \frac{\dot{A}\dot{B}}{A} + \frac{\dot{B}^2}{B}\right] \quad (2.4)$$

$$R_{33} = -B\left[\ddot{B} + \frac{\dot{A}\dot{B}}{A} + \frac{\dot{B}^2}{B}\right] \quad (2.5)$$

$$R_{44} = \frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} \quad (2.6)$$

Case I: - For Massive cloud string

Here we consider the energy momentum tensor for a cloud massive string is given by

$$T_i^j = \rho v_i v^j - \lambda x_i x^j \quad (2.7)$$

Where, ρ is the rest energy density for a cloud of string with particles attached along the extension.

Thus, $\rho = \rho_p + \lambda$ (2.8)

Where, ρ_p is particle energy and λ is the tension density of the string.

v^i is the four vector representing the velocity of cloud of particles and x^i is the four vector representing the direction anisotropy, i.e. z- direction.

Where v_i and x_i satisfy condition

$$v_i v^j = -1,\ x_i x^j = 1 \text{ and } v_i x^i = 0 \quad (2.9)$$

The field equation (2.2) together with (2.7) for the line element (2.1) subsequently lead to the following system of equation

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\ddot{B}}{B} = 0 \quad (2.10)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = 0 \quad (2.11)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = 8\pi k\lambda \quad (2.12)$$

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} = 8\pi k\rho \quad (2.13)$$

Where, $\dot{A} = \frac{\partial A}{\partial t}$, $\ddot{A} = \frac{\partial^2 A}{\partial t^2}$ etc.

From equation (2.11) and (2.12) we get

$$\lambda = 0 \quad (2.14)$$

Thus, cosmic cloud string does not exist.

Hence Vacuum solution are

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\ddot{B}}{B} = 0 \quad (2.15)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = 0 \quad (2.16)$$

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} = 0 \quad (2.17)$$

Solving the equation (2.15) to (2.17) we obtain

$$A = e^{(kt+k_1)} \quad (2.18)$$

$$B = \frac{e^{(kt+k_2)}}{k} + k_3 \quad (2.19)$$

Where, k_i 's are constants of integration.

The corresponding vacuum cosmological model can be written as

$$ds^2 = -dt^2 + e^{2(kt+k_1)}dx^2 + \left[\frac{e^{(kt+k_2)}}{k} + k_3\right]^2 (dy^2 + dz^2) \quad (2.20)$$

Case II: - For electromagnetic field

Here, the energy momentum tensor for electromagnetic field is defined as

$$E_i^j = -F_{ir}F^{jr} + \frac{1}{4}F_{ab}F^{ab}g_i^j \quad (2.21)$$

Where, E_i^j is electromagnetic energy tensor and F_i^j is electromagnetic field tensor.

In comoving coordinate system, the magnetic field is taken along Z-axis so that the non-vanishing components of electromagnetic field tensor F_{ij} is F_{12}.

The first set of Maxwell's equation

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \quad (2.22)$$

Lead to

$F_{12} = \omega =$ constant [Here $F_{14} = 0 = F_{24} = F_{34}$]

(2.23)

Now from equation (2.21) we obtain the components of the electromagnetic field

$$E_1^1 = E_2^2 = -E_3^3 = -E_4^4 = -\frac{\omega^2}{2A^2B^2} \quad (2.24)$$

Using comoving coordinate system, the field equations for the matric (2.1) can be written as

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\ddot{B}}{B} = 4\pi k \frac{\omega^2}{A^2B^2} \quad (2.25)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = 4\pi k \frac{\omega^2}{A^2B^2} \quad (2.26)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -4\pi k \frac{\omega^2}{A^2B^2} \quad (2.27)$$

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} = -4\pi k \frac{\omega^2}{A^2B^2} \quad (2.28)$$

From equation (2.26) and (2.27) we have

$$F_{12} = \omega = 0 \quad (2.29)$$

Here we observed that, in anisotropic bianchi type I cosmological model electromagnetic field does not exist. Thus we have a new set of vacuum field equations which are same as the case I equation (2.15) to (2.17). We obtain the same vacuum cosmological model defined as in equation (2.20).

3. Physical and Kinematical Properties

The spatial volume for the model (2.1) is given by

$$V^3 = AB^2 \quad (3.1)$$

Where, $V = (AB^2)^{\frac{1}{3}}$ as the average scale factor.

So that the Hubble parameter in anisotropic model can be defined as

$$\theta = \frac{\dot{V}}{V} = \frac{1}{3}\left[\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right] \quad (3.2)$$

The physical quantities of the expansion scalar θ is defined as

$$\theta = 3\frac{\dot{V}}{V} = \left[\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right] \quad (3.3)$$

The non-vanishing component of shear tensor σ_{ij} defined by

$\sigma_{ij} = u_{i;j} + u_{j;i} - \frac{\theta}{3}(g_{ij} + u_i u_j)$ are obtained as

$$\sigma_{11} = -A\dot{A} - \frac{\theta}{3}A^2$$

$$\sigma_{22} = \sigma_{33} = -B\dot{B} - \frac{\theta}{3}B^2$$

$$\sigma_{44} = 0$$

Thus shear scale σ is obtained as

$$\sigma^2 = -\frac{1}{2}\left[\frac{\dot{A}^2}{A^2} + 2\frac{\dot{B}^2}{B^2} + \theta^2\right] \quad (3.4)$$

4. CONCLUSION

We have investigated non-static LRS Bianchi type I Cosmological models with the matter cosmic cloud string and electromagnetic field respectively and further observed that in this model cosmic cloud string as well as electromagnetic field does not exist and vacuum solutions have been obtained.

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