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Solving Electrical Power Flow Problems using Intervals Arithmetic

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***Abstract*—The present article proposes an Improved Interval Fast Decoupled Power Flow (IIFDPF) algorithm to provide the load flow. The solution is achieved by the modelling of the load and generator bus data with consideration of uncertainties due to measurement error. Interval Arithmetic based techniques are adopted contextually to deal the bus data with the uncertainty in Interval Improved Fast De-coupled Power flow (IIFDPF) algorithm that leads to solve the Interval power flow method. The initial nonlinear model is solved by using Interval Newton method and two sets of linear Interval equations i.e., the decoupled P and Q equations. Based upon different strategies of updating the voltage angle and the bus voltage in each iteration, the strategic solution is obtained by saving computing time with faster convergence. The IIFDPF algorithm is validated with the available IEEE standard test data.**

Keywords—Uncertainty, Power flow, Interval arithmetic, Interval Improved Fast Decoupled, Interval Newton's method.

I. INTRODUCTION

Since recent years much efforts has been made by the researchers in the area of load flow solutions. The available conventional methodology does not answer the presence of uncertainties in the mathematical modeling of power systems. Common uncertainties can be categorized as environmental, regulatory and technological. The sources of uncertainty happen to be (i) The type of the assumed mathematical model, (ii) Representation of various physical components (iii) Values of the parameters may be error (iv) Introduction of noise at the inputs and (v) Numerical modeling using finite arithmetic. Qualitative and quantitative aspects govern the classification of uncertainties. Qualitative uncertainty is generally expressed verbally like 'near to', 'smaller to' etc., whereas the quantitative uncertainty is quantifiable in numerical terms following the Interval arithmetic. Probabilistic methods, Fuzzy logic and Interval arithmetic are the ways to determine the solutions to any systems with uncertainty.

Conventional methodologies available in the literature propose the use of probabilistic methods for these type of studies. Which accounts for the variability and stochastic nature of the input data. In particularly, Uncertainty propagation studies based on sample-based methods, such as Monte carlo's require several model runs that sample various combinations of inputs values. Since the number of required model runs may be rather large, the needed computation resources for these types of studies could be prohibitively expensive.

Probabilistic methods are useful tool, especially for planning studies. However, as discussed in [1-2],these present various shortcomings due mainly to non-normal probability distribution and the statistical dependence of the input data, as well as the problems associated with accurately identifying probability distribution for some input data, as in the case for example of power generated by wind or solar generators. These could lead to complex computation that may limits the use of these methods in practical applications, especially in the study of large networks.

The second family of the load flow algorithms incorporating uncertainty has been developed more recently and utilized fuzzy sets for its for modelling [3-7]. This is a qualitatively different way of expressing uncertainty. It represents impressive, or vargue, knowledge rather than uncertainty related to the frequency of occurrence. One inherent advantage of this approach is the ability to easily incorporate experts knowledge about the system under study. With this approach, inputs variables are represented as fuzzy numbers(FNs), which are special type of fuzzy sets , Although the calculation in fuzzy analysis are somewhat simpler than the in a probabilistic case (convolution is not needed), it is still far too complex to be applied directly to the full system model. Therefore, again a linearized model of the system is used and results obtained are approximate.

In recent years the uncertain variable in the load flows are being represented using Interval numbers. By considering Interval arithmetic the obtained solution for the load flow can be attributed to the every punctual or instant value of the problem with uniform validity. This attractive feature of the Interval arithmetic made many researchers to put their efforts in solving the uncertain electrical power flow problems using Interval arithmetic. The contributions made by Barboza, Zian Wang etc., [7-12] are significant in International literature.

In their contributions Zian Wang and F.L. Alvarado [7] suggested a method for solving the load flow using Interval arithmetic taking the uncertainty at the nodal values. It is stated in their article that the required solution to the of non-linear equations can be obtained by Interval Newton operator, Krawczyk operator or Hansen-Sengupta operator. In their series of research articles [7-13] Barboza and others presented their methodology for solving the uncertain power flow problems. Also in the literature Interval mathematics has been applied to the load flow analysis [9-12] by considered Krawczyk's method to solve the non-linear equations. It is mentioned that the existing problem of excessive conservatism in solving the Interval linear equations could be overcome by Krawczyk's method. In these methods the linearized power flow equations should be preconditioned by an M-matrix in order to guarantee convergence.

In another paper [8] the set of non-linear equations were solved by Gauss-Seidal method. Preconditioning is required and no guarantee of convergence if Interval input is too large, hence this method cannot give exact solution.

In the article [14] Fast Decoupled power flow using Interval arithmetic has been used to obtain the solution to the power flow with uncertainty. Linearization is done by Interval gauss elimination method. In particularly the use of the Interval Gauss elimination in the power flow process leads to realistic solution bounds only for certain special classes of matrices. This solution show excessive conservatism.

The present article proposes an Interval Improved Fast Decoupled Power Flow (IIFDPF) algorithm under data uncertainty, which introduces Interval arithmetic into traditional fast decoupled power flow used in power systems. In this proposed algorithm Interval numbers are used to express all classes of uncertainty. IIFDPF algorithm leads to solve the Interval power flow method and the resulting non linear model is solved by using Interval Newton's method and two sets of linear Interval equations i.e. the decoupled P and Q equations. Based upon different strategies of updating the voltage angle and the bus voltage in each iteration, the strategic solution is obtained to save computing time with faster convergence and better accuracy. The basic idea of the Interval Improved Fast Decoupled Power Flow using Interval arithmetic being proposed in this paper is as follows. Firstly, use of Interval numbers is made to express the uncertain variables in power system such as the uncertainty of all the load bus and generator bus. Then the mathematical model of the Improved Fast Decoupled Power Flow using Interval arithmetic is built. The proposed algorithm has been validated using standard IEEE 3,9,14 bus system data available in International literature.

II. INTERVAL IMPROVED FAST DECOUPLED POWER FLOW (IIFDPF) UNDER UNCERTAINTY

All load and generator bus data in the electric system are provided by measuring instruments which are probably inaccurate. Moreover the specified variables like real power at PVbuses also can be uncertain since their values are obtained via measurement equipment. This uncertainty in the input data can be enlarged due to both rounding and truncating processes that occur in numerical computation. As a consequence the actual error presented in the final results cannot be easily evaluated. This paper proposes to apply techniques of Interval Arithmetic for a more reliable load and generator modelling.

As the Fast Decoupled power flow method (FDPFM) is one of the improved methods, which is based on simplification of Newtons –Raphson method and reported by Stott and O.Alsac [15]. This method answers the complexity in its calculations, fast convergence and reliable results becomes widely used in power flow analysis in conversional methods.

The basic idea of the Interval Improved Fast Decoupled Power Flow using Interval arithmetic being proposed in this paper is as follows. Firstly, use Interval numbers to express the uncertain variables in power system such as the uncertainty of all the load bus (PQ bus i.e. P_l and Q_l) and generator bus (PV bus i.e. Pg and Qg). Then the mathematical model of the Improved Fast Decoupled Power Flow using Interval arithmetic is built.

Using the simplifications and assumption of NR method, the following expressions are formulated:

$$\frac{\Delta P}{|V|} = - B' \Delta\delta \qquad \text{…..(1)}$$

$$\frac{\Delta Q}{|V|} = - B'' \Delta|V| \qquad \text{….. (2)}$$

In the above two equations, the branch parameter matrices are Interval matrices and state variable vectors are Interval vectors, which mean that the elements of them are mostly Interval numbers.

In the operation of actual power system, the influence of parameter uncertainty of electric lines and transformers is often small enough to be neglected so the equations (1) and (2) can be simplified as below:

$$\Delta\delta = - [B'] \setminus \left[\frac{\Delta P}{|V|}\right] \qquad \text{….. (3)}$$

$$\Delta V = - [B''] \setminus \left[\frac{\Delta Q}{|V|}\right] \qquad \text{…..(4)}$$

IIFDPF algorithm leads to solve the Interval power flow method and the resulting nonlinear model is solved by using Interval Newton's method and two sets of linear Interval equations i.e. the decoupled P equations and Q equations (3) and (4). Based upon different strategies of updating the voltage angle and the bus voltage in each iteration are based on the weak coupling between ΔP and ΔV and between ΔQ and $\Delta\delta$. In Fast decupled method, instead of updating the voltage magnitude and the voltage angle once and simultaneously in each iteration, IIFDPF algorithm updated either the voltage angle on the voltage magnitude at each bus, to recalculate the real and reactive power and then updated the second variable based on what was updating first. Moreover for speed improvements and convergence reliability, the update of one of the two variables repeated several times, holding the other variable at its last calculated value, which reduces the number of floating point operations of the algorithm and thus lead to the faster convergence.

III. PROPOSED ALGORITHM

This section gives the procedural steps of the proposed algorithm.

Step 1: Create the admittance Y-bus according to the line data given by the IEEE standard bus test systems.

Step 2: Find the load and generator data in the form of Interval data, since they are specified considering the probable measurement error. Initial guesses are provided by punctual algorithm for the power flow problem, transformed into Interval that considers the maximum measurement relative error.

Step 3: Calculate midpoint of initial voltage (V) and phase angle (δ) i.e. $\tilde{V}$ and $\tilde{\delta}$.

Step 4: Detect the type of bus from the bus number according to the bus data given. Set all bus voltages to an initial value equal to midpoint value of Voltage and Phase angle (PQ bus as 0, slack bus as 1 and PV bus as 2).

Step 5: Create the matrices B' and B" .

Initialize the iteration counter iter = 0.

Step 6: If max (ΔP, ΔQ) $\leq$ accuracy, then go to step 8 else,

I. Calculation of the real (P_i) and reactive power (Q_i) at each bus using midpoint value of voltage ($\tilde{V}$) and phase angle ($\tilde{\delta}$), and checking if MVAR of generator buses are within the limits, otherwise update the voltage magnitude at these buses by ±5 %.

II. Calculation of the power residuals, ΔP and ΔQ.

III. Calculation of the bus voltage and voltage angle updates $\Delta V$and $\Delta\delta$ according to the mathematical model of the fast decoupled power flow using Interval arithmetic using,

$$\Delta\delta = - B' \backslash \Delta PV$$

$$\Delta V = - B'' \backslash \Delta QV$$

IV. Update of the voltage magnitude V and the voltage angle δ at each bus.

$$N(\tilde{\delta}^{(k)}, \delta^{(k)}) = \tilde{\delta}^{(k)} + \Delta\delta^{(k)}$$

$$\delta^{(k)+1)} = \delta^{(k)} \cap N(\tilde{\delta}^{(k)}, \delta^{(k)})$$

$$N(\tilde{V}^{(k)}, V^{(k)}) = \tilde{V}^{(k)} + \Delta V^{(k)}$$

$$V^{(k)+1)} = V^{(k)} \cap N(\tilde{V}^{(k)}, V^{(k)})$$

Where, N (.) is Newton operator.

This algorithm updates either the voltage angle or the voltage magnitude at each bus jumped to subroutine 1 step 6 to recalculate the real and reactive power and then updated the second variable based on what was updated first.

These combinations will be noted according to the number of loops of update of each variable. For instance, updating twice the voltage angle (δ) and then once the voltage magnitude (V) in the same iteration will be written as (2:1).

V. Increment of the iteration counter

Iter = Iter + 1

Step 7:check for convergence, whether obtained or not.

Step 8*:* Print out of the power flow solution, computation and display of the voltages (V) and phase angle (δ), line flow and Losses.

IV. NUMERICAL RESULTS

To demonstrate the performance of proposed method, an IEEE-14 test system is considered with an uncertainty of 5% in the load values at PQ buses and 2% in generation data at all PV buses.

The solution obtained by the algorithm is shown in Table I ,II.III. The results of Interval method encloses the results of traditional crisp method, when the parameters being obtained as the midpoint of given Interval, this demonstrates the validity of the proposed method. It satisfies the rule base of Interval arithmetic and the Interval Improved Fast Decoupled Power Flow method (IIFDPF) is validated. The performance of the IIFDPFL was tested on IEEE 3,9 and 14bus systems with convergence accuracy of 10^{-3}on a MVA base of 100 for both power residuals delta P and delta Q.This numerical analysis involved a speed comparison between interval FDPF and IIFDPF method based on the Elapsed time and Maximum power mismatch shown in table IV. In addition, as mentioned in the previous part, the algorithm of this paper updated the voltage angle several times before updating the voltage magnitude or vice versa which resulted in a different speed in convergence for each combination used for the same IEEE bus system. These combinations will be noted according to the number of loops of update of each variable. For instance, updating twice the voltage magnitude (V) and then once the voltage angle (δ) in the same iteration will be written as (2:1). Moreover, we studied different combinations of updating the voltages and phase angles effects on computing saving and thus higher speed of convergence under uncertainty.

V. CONCLUSIONS

This article presents an algorithm to perform the load flow analysis of a complex bus system that has inherent error in measurement and parametric uncertainties due to ageing, environmental effects etc., The proposed algorithm is fast converging and takes the consideration of retaining the mid-point of the load flow studies. This is a specific feature that ensures the convergence in accordance with the punctual load flow studies. Thus the proposed algorithm is effective and avoids un-necessary computation effort like preconditioning. The algorithm is tested successfully on standard IEEE 3.9, 14 bus data available in International literature.

Table I
Interval Voltage and phase angle - IEEE-3

BUS NO	Type of Bus	Traditional method results : Voltage in (pu)	Interval Improved FDPF Voltage in (pu)	Traditional method phase angle in(°)	Interval Improved FDPF Phase angle in (°)
1	SLACK	1.05	[0.9975, 1.1026]		
2	PQ	0.9844	[0.9829, 0.9856]	-9.5786	[-9.8020, -9.3578]
3	PQ	0.9388	[0.9373, 0.9405]	-14.4667	[-14.8125, -14.1167]

Table II
Interval Voltage and phase angle - IEEE-9

BUS NO	Type of Bus	Traditional method results : Voltage in (pu)	Interval Improved FDPF Voltage in (pu)	Traditional method phase angle in(°)	Interval Improved FDPF Phase angle in (°)
1	SLACK	1.04	[0.9880, 1.0921]		
2	PV	1.0253	[1.02530, 1.02530]	-14.637	[-15.0322, -14.2418]
3	PV	1.0253	[1.02530, 1.02530]	-15.3878	[-15.8005, -14.9752]
4	PQ	1.0496	[1.0493, 1.0499]	-5.9129	[-6.0681, -5.7578]
5	PQ	1.0284	[1.0276, 1.0291]	-12.0504	[-12.3656, -11.7353]
6	PQ	1.0315	[1.0311, 1.0318]	-15.3878	[-15.8005, -14.9752]
7	PQ	1.0197	[1.0190, 1.0203]	-17.1923	[-17.6486, -16.7359]
8	PQ	1.0326	[1.0322, 1.0329]	-14.637	[-15.0322, -14.2418]
9	PQ	1.0581	[1.0577, 1.0584]	-9	[-9.2401, -8.7600]

Table III
Interval Voltage - IEEE-14

BUS NO	Type of Bus	Traditional method results : Voltage in (pu)	Interval Improved FDPF Voltage in (pu)	Traditional method phase angle in(°)	Interval Improved FDPF Phase angle in (°)
1	Slack	1.06	[1.0060, 1.0060]		
2	PV	1.045	[1.0450, 1.0450]	-4.9874	[-5.1388, -4.8360]
3	PV	1.01	[1.0100. 1.0100]	-12.7484	[-13.0736, -12.4233]
4	PQ	1.0142	[1.0140, 1.0143]	-10.2664	[-10.5318, -10.0011]
5	PQ	1.0173	[1.0172, 1.0175]	-8.7501	[-8.9790, -8.5213]
6	PV	1.07	[1.0700, 1.0700]	-14.2358	[-14.5960, -13.8753]
7	PQ	1.059	[1.0586, 1.0593]	-13.318	[-13.6575, -12.9788]
8	PV	1.09	[1.0900, 1.0900]	-13.318	[-13.6575, -12.9788]
9	PQ	1.0524	[1.0517, 1.0530]	-14.8993	[-15.2747, -14.5244]
10	PQ	1.048	[1.0474, 1.0487]	-15.0669	[-15.4494, -14.6851]
11	PQ	1.0554	[1.0550, 1.0558]	-14.7806	[-15.1567, -14.4045]
12	PQ	1.0549	[1.0547, 1.0552]	-15.0941	[-15.4713, -14.7009]
13	PQ	1.0499	[1.0495, 1.0502]	-15.1609	[-15.5551, -14.7735]
14	PQ	1.0333	[1.0326, 1.0339]	-16.0174	[-16.4317, -15.6059]

Note: At PV bus voltage constant at specified value.

Table IV : Number of iteration, Power mismatch and Elapsed time - IEEE-14

	Interval improved FDPF Method 1:1	Interval Improved FDPF Interval Method 1:2	Interval Improved FDPF Method 2:1	Interval Improved FDPF Method 3:2	Interval Improved FDPF Method 2:3	Interval Improved FDPF Method 4:4
No iterations	02	02	1	2	1	1
Maximum power Mismatch	7.1377e-004	5.1891e-004	9.8058e-004	7.4920e-004	8.3364e-004	6.1229e-004
Elapsed time in sec	0.459178	0.765915	0.441406	0.659725	0.573006	0.601335

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