Solution to the Mode Mixing Problem in Two Harmonic Signals: Empirical Wavelet Transform

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***Abstract*—Empirical Wavelet Transform and Empirical Mode Decomposition are two powerful adaptive signal decomposition techniques. Although widely used, EMD method has some limitations in decomposing signals where amplitude-frequency ranges are too close to each other. The aim of present paper is to review existing knowledge base for mode mix problem of two harmonics decomposition and introduce its solution using EWT for this topic. In this paper, the EWT decomposition is compared with the conventional EMD by numerical examples and the results of EWT decomposition are compared to original signal parameters such as energy, phase and amplitude. Finally, it is concluded from the parameters that EWT efficiently detects the components of the signals where EMD suffers mode mixing.**

Keywords— Empirical Mode Decomposition, Empirical Wavelet Transform, Two harmonic decomposition, mode mixing .

I. INTRODUCTION

The adaptive data analysis method that has gained a lot of interest in signal processing since last decade is the algorithm called "Empirical Mode Decomposition" (EMD) proposed by Huang, et al. [1]. The purpose of this method is to decompose the signal into its principal "modes" (a mode corresponds to a signal which has a compactly supported Fourier spectrum). It has proven to be quite versatile in a broad range of applications for extracting signals from data generated in noisy nonlinear and non-stationary processes. These include study of wide variety of data including rainfall [2], earthquakes detection [3], Sunspot number variation, heart-rate variability, financial time series, and ocean waves [4], fault diagnosis [5], signal denoising, image processing [6], biomedical signal processing, speech signal analysis [7], pattern recognition [8]. In addition to successful applications of the EMD methods, a large number of publications have attempted to improve, or at least to modify, the original method. These include modifying definition of Intrinsic Mode Function [9, 10], replacing cubic spline interpolation by some other higher order interpolation, choosing the stoppage criteria [11]. Concomitantly, studies devoted to analyze the important shortcomings of the EMD and its limitations in comparison with other decomposition methods began to appear. One of the first precincts found was the method's rather low frequency resolution meaning that the EMD can resolve only distant spectral components differing by more than octave. Another was the problem of mode mixing i.e. it receives false artificial components not present in the initial composition. Also, the main issue of the EMD approach is its lack of mathematical theory. To overcome this and to propose a mathematical background for EMD , Flandrin [12] show that EMD behaves like an adaptive filter bank. Rilling and Flandrin [13] provided analytical basis of EMD using two harmonic signals. Their work was further continued by Feldman [14] who proposed a theoretical limiting amplitude-frequency resolution in EMD method and thus proposing conditions for mode mixing problem for two harmonics. The solution of mode mixing in EMD was provided by Shuen-De, et al. [9] .The author proposed two solutions, namely, to increase the number of EMD iterations and additional mathematical operations based on integration and differentiation both of them were computationally expensive and integration process introduces errors. However, the Ensemble Empirical Mode Decomposition (EEMD) [15] method largely overcomes the false mode mixing problem of the original EMD and provides physically unique decompositions but again is computationally expensive.

A new adaptive data analysis method having a similar goal like EMD is the Empirical Wavelet Transform (EWT), proposed by Gilles [16] which explicitly builds an adaptive wavelet filter bank to decompose a given signal into different modes. It is a new approach to construct adaptive wavelets competent of extracting Amplitude modulated-Frequency modulated components of a signal which have a compact support Fourier spectrum (i.e. the modes). Separating various modes corresponds to segmentation of the Fourier spectrum and then applying some filter related to each detected support. The EWT performs local maxima detection of the Fourier spectra of the signal, then performs spectrum segmentation based on detected maxima and, finally, constructs a corresponding wavelet filter bank. This paper shows that the mode mixing problem in two harmonics can be solved using EWT decomposition.

The paper is organized as follows. In Section II EMD & the problem of mode mixing is recalled. In Section III a review of EWT is presented. Section IV demonstrates results numerical examples of two harmonic decomposition using EWT and their comparison with conventional EMD. In addition, the results of EWT decomposition are compared to original signal parameters such as energy, phase and amplitude.

II. EMPIRICAL MODE DECOMPOSITION & PROBLEM OF MODE MIXING

In 1998, Huang, et al. [1] proposed an adaptive data analysis method called Empirical Mode Decomposition (EMD) which decomposes a signal into specific modes. The EMD works in

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temporal space directly, is intuitive, direct, and adaptive, with a posteriori defined basis derived from the data. The decomposition works on a supposition that, at all instants, the data exists in simple oscillatory modes of considerably different frequencies, one overlaying on the other.

EMD aims to decompose a signal f(t) as a (finite) sum of $N + 1$ Intrinsic Mode Functions (IMF) $f_k(t)$ such that

$$f(t) = \sum_{k=0}^{N} f_k(t) \quad (1)$$

Mode mixing is defined as a single IMF either consisting of signals of widely dissimilar scales or a signal of a similar scale existing in different IMF components. Feldman [14] described the problem of mode mixing in two harmonic signals. The more frequencies are spaced from each other; the smaller amplitude ratio of two harmonics is suitable for EMD separation. The logical result of the provided theoretical analysis as provided by author [14] is that the frequency (ω) and amplitude (A) ratios of the harmonics can be separated in to three different groups:

(i). Harmonics with very close frequencies and small amplitude

$$\frac{A_2}{A_1} \le \left(\frac{\omega_1}{\omega_2}\right)^2$$

are unsuitable for EMD decomposition;

(ii). Close frequency harmonics

$$\left(\frac{\omega_1}{\omega_2}\right)^2 \le \frac{A_2}{A_1} < 2.4 \left(\frac{\omega_1}{\omega_2}\right)^{1.75}$$

requiring several sifting iterations; Mode mixing occurs typically in this case.

(iii). Distant frequencies and large amplitude harmonics

$$\frac{A_2}{A_1} \ge 2.4 \left(\frac{\omega_1}{\omega_2}\right)^{1.75}$$

that are well separated for a single iteration.

An example of mode mixing is illustrated in figure below this signal is taken from Shuen-De, et al. [9]:

$$x(t) = \sin(2\pi t) + 0.1207 \sin(6.6\pi t) \quad (2)$$

Here, $\omega_1 = 2$, $\omega_2 = 6.6$, $A_1 = 1$ & $A_2 = 0.01207$.this signals satisfies condition (ii).

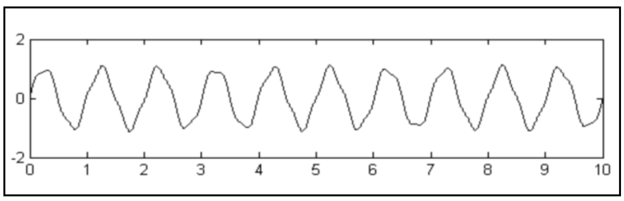

Figure 1:Signal x(t)- example of mode mixing [9]

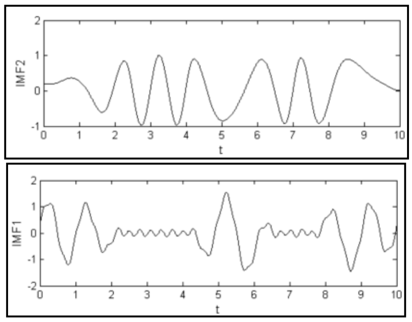

Figure 2 : EMD Decomposition showing mode mixing [9]

The IMF's obtained from EMD decomposition clearly depict the problem of mode mixing (overlapping of frequencies).The author [9] proposed the solutions for the mode mixing phenomenon but the solutions itself introduces some errors.

III. EMPIRICAL WAVELET TRANSFORM

In 2013, Jerome Gilles [16] introduced a new adaptive data analysis method called Empirical Wavelet Transform which explicitly builds an adaptive wavelet filter bank to decompose a given signal into different modes. EWT also aim's like the EMD, to extract AM-FM components from a signal. The EWT works in frequency space unlike EMD, is intuitive, direct, and adaptive algorithm supported by a strong mathematical background. EWT proposes a method to build a family of wavelets adapted to the processed signal.

In EWT, the Fourier support is divided into N contiguous segments, and then N-1 boundaries need to be extracted excluding 0 and π. To find the boundaries, the local maxima positions in the spectrum are detected and are sorted in decreasing order and boundaries are defined as average between the consecutive maxima's. If we denote ω_n to be the limits between each segments (where $\omega_0=0$ and $\omega_n=\pi$) and if each segment is denoted as, then $\bigcup_{n=0}^{N} \wedge_n = [0, \pi]$.

A transition phase T_n of width $2\tau_n$ (such that where $0 < \gamma < 1$) is defined around the centre of each $\wedge_n$. The empirical wavelets are defined as band pass filters on each $\wedge_n$. For this, the author, Gilles [16] has utilized the idea used in the construction of both Littlewood-Paley and Meyer's wavelets. The Fourier transform of empirical scaling function and the empirical wavelets are defined by Equations (3) and (4), respectively

$$\hat{\phi}_n(\omega) = \begin{cases} 1 & \text{if } |\omega| \le (1-\gamma)\omega_n \\ \cos\left[\frac{\pi}{2}\beta\left(\frac{1}{2\gamma\omega_n}(|\omega| - (1-\gamma)\omega_n)\right)\right] & \text{if } (1-\gamma)\omega_n \le |\omega| \le (1+\gamma)\omega_n \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

And

$$\hat{\psi}_n(\omega) = \begin{cases} 1 & \text{if } (1+\gamma)\omega_n \le |\omega| \le (1-\gamma)\omega_{n+1} \\ \cos\left[\frac{\pi}{2}\beta\left(\frac{1}{2\gamma\omega_{n+1}}(|\omega| - (1-\gamma)\omega_{n+1})\right)\right] & \text{if } (1-\gamma)\omega_{n+1} \le |\omega| \le (1+\gamma)\omega_{n+1} \\ \sin\left[\frac{\pi}{2}\beta\left(\frac{1}{2\gamma\omega_n}(|\omega| - (1-\gamma)\omega_n)\right)\right] & \text{if } (1-\gamma)\omega_n \le |\omega| \le (1+\gamma)\omega_n \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

where β(x) is an arbitrary C^k([0,1]) function such that

$$\beta(x) = \begin{cases} 0 & x \leq 0 \\ 1 & x \geq 0 \\ x^4(35 - 84x + 70x^2 - 20x^3) & x \in [0,1] \end{cases} \quad (5)$$

and

$$\beta(x) + \beta(1-x) = 1 \quad \forall\, x \in [0,1] \quad (6)$$

After defining the empirical wavelet and scaling function, the empirical wavelet transform, $W_f^{\epsilon}(n,t)$ of a signal f(t) is defined in a way similar to the classic wavelet transform. The detail coefficients are given by the inner products with the empirical wavelets.

$$W_f^{\epsilon}(s,t) = \langle f, \psi_n \rangle = \int f(\tau)\overline{\psi_n(\tau - t)}d\tau \quad (7)$$

And the approximation coefficients by the inner product with the scaling function

$$W_f^{\epsilon}(0,t) = \langle f, \emptyset_1 \rangle = \int f(\tau)\overline{\emptyset_{1n}(\tau - t)}d\tau \quad (8)$$

The results of two harmonic decomposition by EWT and their comparison with those obtained by [9] are described in the next section.

IV. NUMERICAL SIMULATIONS USING EWT & COMPARISON WITH EMD

Picking the example described above using equation 4, taken from [9],(represents condition (ii) as described by Feldman) the results obtained after EWT decomposition of the signal ae shown in figure 3. On comparison of results obtained from EMD & EWT decomposition (Figure 2 & Figure 3) respectively, it's obvious that EWT frequency to amplitude ratio for decomposing two harmonics is lower than obtainable via EMD.

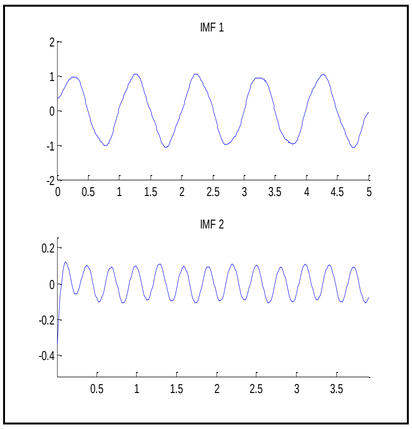

Figure 3: EWT Decomposition of signal x(t)

Also, as observed from figure 4(Magnitude spectrum), the EWT components obtained after decomposition have almost same frequency as in original signal.

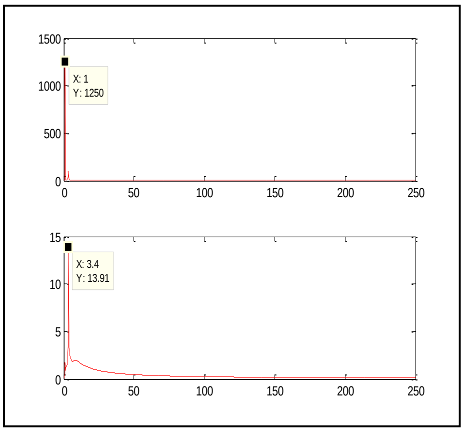

Figure 4: Magnitude spectrum of the components obtained in figure 3.

Another example from [9] where EMD fails, (it represents condition (i)) to separate signal into its components while EWT successfully does it is y(t) represented in equation (9) and shown in figure(5).

$$y(t) = \sin(2\pi t) + 0.02\sin(8\pi t) \quad (9)$$

Here, $\omega_1 = 2$, $\omega_2 = 8$, $A_1 = 1$ & $A_2 = 0.02$.
The EMD & EWT decomposition of y (t) is shown in figures 6 & 7 respectively.

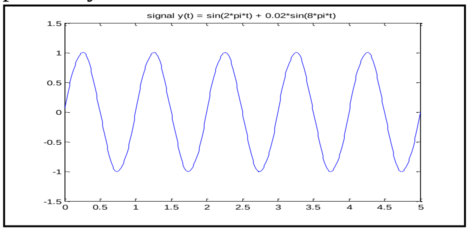

Figure 5: Signal y(t)

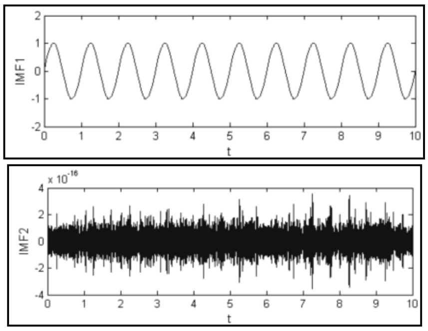

Figure 6: EMD decomposition of y(t) [9]

On comparison of Figure 6 & 7 it can clearly be seen that the standard EMD method collapsed.IMF 2 represents very low signal level 10^{-16}, while the EWT decomposition of the same signal gives its components. Although the second component suffers a little mode mixing, but with EWT one gets to know which frequencies are present in the signal.

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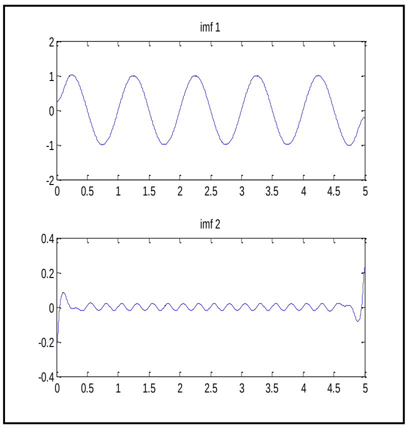

Figure 7: EWT Decomposition of signal x(t)

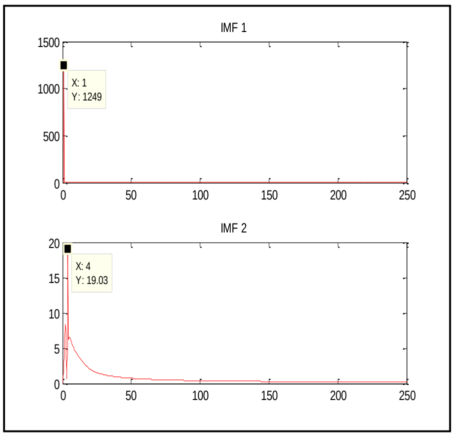

Figure 8: Magnitude spectrum of the components obtained in figure 7.

As concluded from the table 1, signal 1 which suffers from mode mixing in EMD, can be decomposed easily by EWT. While signal 2, where EMD method fails, EWT is able to extract components but in IMF 2, mode mixing occurs slightly.

Table 1: Parameters of IMF'S of x(t) and y(t) obtained by EWT Decomposition

Sign al	Magnitude at Frequency Peak		Phase difference (degrees)	Energy		
	True	Extract ed		True	Extracted	%
Signal x(t) = sin (2πt) + 0.1207 sin (6.6πt)						
Imf 1	1250	1250	-0.2078	3.1705e+006	3.1690e+ 006	99.9
Imf 2	94.62	13.91	0.0298	4.5527e+004	1.5470e+ 003	34.3
Signal y(t) = sin(2πt) + 0.02 sin(8πt)						
Imf 1	1250	1249	-0.1971	3.1250e+006	3.1210e+ 006	99.8
Imf2	25	19.03	0.0096	1.2500e+003	240.82	19.9

V. CONCLUSION

EMD & EWT both are promising adaptive time frequency representation techniques, where, EMD has already been explored in vast engineering and related applications and proves to be potential, it still lacks mathematical background. EWT is an emerging technique with immense potential to be explored. The solution for the mode mixing problem in the two harmonic signals obtained are provided by EWT, however the critical frequency limit for the difference in frequencies between two signals remain to be the same as proposed by Feldman.The frequency to amplitude ratio for two harmonics signals which is proved that will be lower for EWT can be explored further.

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