Solution of an Engineering Design Optimization System: A General Fuzzy Programming Technique and Intuitionistic Fuzzy Optimization Technique

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Abstract

The main objective of structural engineers through out design history has been to obtain structure under the prescribed design conditions which can not only withstand external loads safety but also achieve an economic solution. This paper we apply General Fuzzy Non-Linear Programming [GFNLP] technique as well as Intuitionistic Fuzzy optimization[IFO] technique to solve the problem of optimum design of plane truss structures. Our objective is to prove IFO method perform better than GFNLP method. This approach is illustrated on planer truss optimization model and the results are discussed.

Keywords: General Fuzzy non-linear programming, Intuitionistic fuzzy optimization, Engineering Design, Structural Optimization.

Mathematics Subject Classification Code: 90C15, 90C29, 90C70.

1. Introduction

The science and engineering in real-world problems are often not deterministic or non-crisp as people recognized. Fuzzy set theory [23] was a recent progress of describing certain non-crisp information with fuzzy arising in problems; since then, many fields ranging from sciences to industrial, medical and financial application had applied it successfully. From the point of view of engineering, most applications and developments with fuzzy theory belong to the category of measurement, manufacturing and control behavior.However; literatures reported engineering design and their applications with intutionistic fuzzy and fuzzy logic are uncommon in dealing with the fuzziness existing in the real-world problems.

Zadeh (1965) first gave the concept of fuzzy set theory. Later on Bellman and Zadeh (1970) used the fuzzy set theory to the decision making problem. Tanaka (1974) introduced the objective as fuzzy goal over α-cut of a fuzzy constraint set and Zimmerman (1978) gave the concept to an inventory and production problems. Wang et al. [21] first applied α-cut method to structural designs where the non-linear problems were solved with various design levels α, and then a sequence of solutions are obtained by setting different level-cut value of α. Rao [20] applied the same α-cut method to design a four–bar mechanism for function generating problem.Yeh and Hsu [22] followed the framework of Wang et al.[21] under different design level of α obtaining the optimum design level while the total cost is based on the failure possibility instead of the membership value of satisfaction.

Intuitionistic Fuzzy Set (IFS) was introduced by K.Atanassov (1986) and seems to be applicable to real world problems. The concept of IFS can be viewed as an alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. Thus it is expected that, IFS can be used to simulate human decision-making process and any activities requiring human expertise and knowledge that are inevitably imprecise or totally reliable. Here the degree of rejection and satisfaction are considered so that the sum of both values is always less than unity (1986). Atanassov also analyzed Intuitionistic fuzzy sets in more explicit way.Atanaossov (1989) discussed open problems in intuitionistic fuzzy sets theory. An interval valued intutionistic fuzzy sets was analyzed by Atanaossov and Gargov (1999).Atanassov and Kreinovich (1999) implemented Intuitionistic fuzzy interpretation of interval data. The temporal intuitionistic fuzzy sets are also discussed by Atanassov (1999).Intuitionistic fuzzy soft sets are also considered by Roy et al. [10].Rough intuitionistic fuzzy sets are analyzed by Rizvi et al.[17].Angelov(1997) implemented the optimization in an intuitionistic fuzzy environment. He (1995) also contributed in his another two important papers, based on Intuitionistic fuzzy optimization.Pramnik and Roy (2005) solved a vector optimization problem using Intuitionistic fuzzy goal programming. A transportation model is solved by Jana and Roy (2007) using multi-objective intuitionistic fuzzy linear programming.

The main objective of a structural engineering is to design structures which withstand external loads safely and at a minimum cost or weight [9,18 and 19].The desire to improve a design without compromising the structural integrity has been a strong driving force behind the development of various optimum design methods.

In this paper we consider a structural model is subject to geometry area, stress [1].First time we apply General Fuzzy Non-Linear Programming [GFNLP] technique to solve the above mention model. We also apply Intuitionistic Fuzzy Optimization [IFO] technique to solve the above mention model. In this paper our objective is to establish the fact numerically that IFO technique minimizes the weight more than FGNLP technique.

1.1 Preliminaries.

Definition 1.1.1. (FS) Let X is a set (space), with a generic element of X denoted by x, that is $X(x)$.Then a FS is defined as Equation

$$A=\{\langle x,\mu_A(x)\rangle / x\in X\}$$

Where $\mu_A : X \to [0,1]$ the membership function of FS A is, $\mu_A(x)\in[0,1]$ is the degree of membership of the element x to the set A.

Definition 1.1.2. (IFS) For a set X, an IFS A in the sense of Atanassov is given by equation

$$A=\{\langle x,\mu_A(x),\upsilon_A(x)\rangle / x\in X\}$$

Where the function $\mu_A : X \to [0,1]$ and $\upsilon_A : X \to [0,1]$ with the condition $0\le \mu_A(x)+\upsilon_A(x)\le 1$, $\forall x\in X$.

The numbers, $\mu_A(x)\in[0,1]$ and $\upsilon_A(x)\in[0,1]$,denote the degree of membership and the degree of non-membership of the element x to the set A, respectively. For each IFS A in

X, the amount $\pi_A(x) = 1 - (\mu_A(x) + \upsilon_A(x))$ is called the degree of indeterminacy (hesitation part) ,which may cater to membership value, non-membership value or both.

Definition 1.1.3: α-Level Set or α-cut of a Fuzzy Sets: The α-level set (or interval of confidence at level α or α-cut) of the fuzzy set $\tilde{A}$ of X is a crisp set A_α that contains all the elements of X that have membership values in $\tilde{A}$ greater than or equal to α i.e $\tilde{A} = \{x, \mu_{\tilde{A}}(x) \ge \alpha, x \in X, \alpha \in [0,1]\}$.

2. Mathematical Analysis

2.1. General Fuzzy Non-linear Programming (FNLP) Technique to solve Multi-Objective Non-Linear Programming Problem (MONLP):

A Multi-Objective Non-Linear Programming (MONPL) or Vector Minimization problem (VMP) may be taken in the following form:

$$\text{Min}\, f(x) = [f_1(x), f_2(x), \ldots\ldots, f_k(x)]^T \tag{2.1.1}$$

Subject to $x \in X = \{x \in R^n : g_j(x) \le \text{ or } = \text{ or } \ge b_j \text{ for } j = 1, 2, \ldots., m\}$ and $l_i \le x_i \le u_i$ $(i = 1, 2, 3, \ldots., n)$.

Zimmermann (1978) showed that fuzzy programming technique can be used to solve the multi-objective programming problem.

To solve MONLP problem, following steps are used:

Step 1: Solve the MONLP (2.1.1) as a single objective non-linear programming problem using only one objective at a time and ignoring the others, these solutions are known as ideal solution.

Step 2: From the result of step 1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

	$f_1(x)$	$f_2(x)$	...	$f_k(x)$
x^1	$f_1^*(x^1)$	$f_2^*(x^1)$	....	$f_k^*(x^1)$
x^2	$f_1^*(x^2)$	$f_2^*(x^2)$	...	$f_k^*(x^2)$
...	...	...	....	....
x^k	$f_1^*(x^k)$	$f_2^*(x^k)$	......	$f_k^*(x^k)$

Here $x^1, x^2, \ldots\ldots, x^k$ are the ideal solutions of the objectives $f_1(x)$, $f_2(x)$, ..., $f_k(x)$ respectively.

So $U_r = \max\left\{f_r\left(x^1\right), f_r\left(x^2\right), \ldots\ldots, f_r\left(x^k\right)\right\}$ and $L_r = f_r^*\left(x^r\right)$ for $r = 1, 2, \ldots\ldots, k$

Where U_r and L_r be upper and lower bounds of the r^{th} objective function $f_r(x)$ for $r = 1, 2, \ldots\ldots, k$.

Step 3: Using aspiration level of each objective of the MONLP (2.1.1) may be written as follows:

Find x so as to satisfy

$$f_r(x) \tilde{\leq} L_r \qquad \text{for } r = 1,2,3,\ldots\ldots,k$$

$$x \in X.$$

Here objective functions of (2.1.1) are considered as fuzzy constraints. These types of fuzzy constraints can be quantified by eliciting a corresponding membership function:

$$\mu_r\left[f_r(x)\right] = \begin{cases} 0 & \text{if } f_r(x) \geq U_r \\ \dfrac{U_r - f_r(x)}{U_r - L_r} & \text{if } L_r \leq f_r(x) \leq U_r \quad (r = 1, 2, 3, \ldots, k) \\ 1 & \text{if } f_r(x) \leq L_r \end{cases} \tag{2.1.2}$$

Having elicited the membership functions (as in (2.1.2)) $\mu_r[f_r(x)]$ for r=1,2,3,….,k. introduce a general aggregation function

$\mu_{\tilde{D}}(x) = G(\mu_1(f_1(x)), \mu_2(f_2(x)), \ldots\ldots, \mu_k(f_k(x)))$.

So a fuzzy multi-objective decision making problem can be defined as

$$\text{Max}\, \mu_{\tilde{D}}(x) \tag{2.1.3}$$

subject to $x \in X$.

Here we adopt fuzzy decision based on minimum operator (like Zimmermann's approach (1978).In this case (2.1.3) is known as Fuzzy Non Linear Programming Model (FNLPM).

Then the problem (2.1.3) using the membership function as in (2.1.2) according to max-min operator is reduces to

$$\text{Max } \alpha \tag{2.1.4}$$

Subject to $\mu_i[f_i(x)] \geq \alpha$ for i=1,2,…….,k

$x \in X, \alpha \in [0,1],$

Step 4: Solve (2.1.4) to get optimal solution.

2.2. Formulation of Intuitionistic Fuzzy Optimization (IFO)

When the degree of rejection (non-membership) is defined simultaneously with degree of acceptance (membership) of the objectives and when both of these degrees are not complementary to each other, then IF sets can be used as a more general tool for describing uncertainty.

To maximize the degree of acceptance of IF objectives and minimize the degree of rejection of IF objectives subject to constraints, we can write:

$$\max \mu_i(x),\ x \in R,\ i = 1, 2, \ldots\ldots.., k+n. \tag{2.2.1}$$

$$\min \upsilon_i(x),\ x \in R\ , i = 1, 2, \ldots\ldots.., k+n.$$

Subject to

$$\begin{gathered} \upsilon_i(x) \geq 0 \\ \mu_i(x) \geq \upsilon_i(x) \\ \mu_i(x) + \upsilon_i(x) < 1 \\ x \geq 0 \end{gathered}$$

Where $\mu_i(x)$ denotes the degree of membership function of (X) to i^{th} IF sets and $\upsilon_i(x)$ denote the degree of non-membership (rejection) of (X) from the i^{th} IF sets.

2.3. An Intuitionistic Fuzzy Approach for Solving Engineering Design Optimization Model (EDOM) with Linear Membership and Non-Membership Functions:

To define the membership function of EDOM problem, let L_k^{acc} and U_k^{acc} be the lower and upper bounds of the k^{th} objective function. These values are determined as follows:

Calculate the individual minimum value of each objective function as a single objective IP subject to the given set of constraints. Let $x_1^*, x_2^*, x_3^*, \ldots\ldots, x_k^*$ be the respective optimal solution for the k different objective and evaluate each objective function at all these k optimal solution. It is assumed that here at least two of these solutions are different for which the k^{th} objective function has different bounded values. For each objective, find lower bound (minimum value) L_k^{acc} and the upper bound (maximum value) U_k^{acc}. But in intuitionistic fuzzy optimization (IFO), the degree of rejection (non-membership) and degree of acceptance (membership) are considered so that the sum of both values is less than one. To define membership function of EDOM problem, let L_k^{rej} and U_k^{rej} be the lower and upper bound of the objective function $Z_k(x)$ where $L_k^{acc} \le L_k^{rej} \le U_k^{rej} \le U_k^{acc}$.When the upper and lower bounds for each objective are specified then we form IF model and then convert it into a crisp model.

The linear membership and non-membership function for the objective $Z_k(x)$ is defined as:

$$\mu_k\left(Z_k(x)\right) = \begin{cases} 1 & if\ Z_k(x) \le L_k^{acc} \\ \dfrac{U_k^{acc} - Z_k(x)}{U_k^{acc} - L_k^{acc}} & if\ L_k^{acc} \le Z_k(x) \le U_k^{acc} \\ 0 & if\ Z_k(x) \ge U_k^{acc} \end{cases} \tag{2.3.1}$$

$$\upsilon_k\left(Z_k(x)\right) = \begin{cases} 1 & if\ Z_k(x) \ge U_k^{rej} \\ \dfrac{Z_k(x) - L_k^{rej}}{U_k^{rej} - L_k^{rej}} & if\ L_k^{rej} \le Z_k(x) \le U_k^{rej} \\ 0 & if\ Z_k(x) \le L_k^{rej} \end{cases} \tag{2.3.2}$$

The picture of linear membership and non-membership functions of the objective goal is given below.

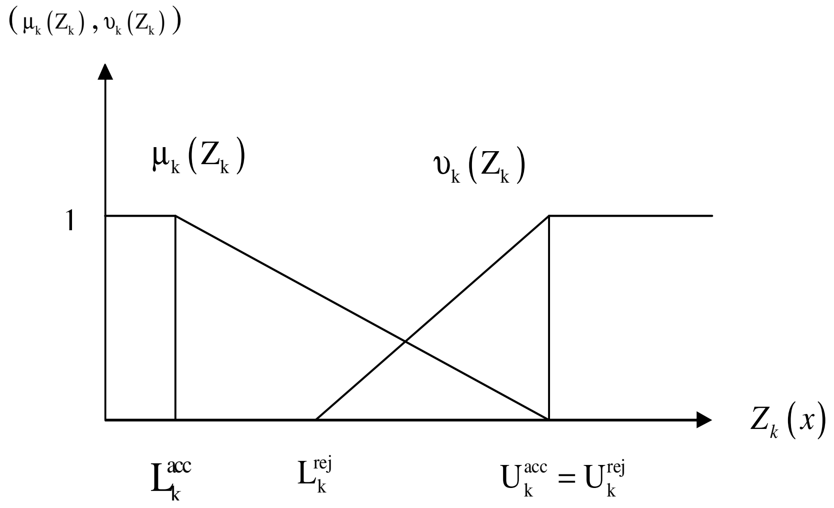

Figure-1: Linear membership and non-membership functions for objective goal.

In case of minimization problem, the lower bound for non-membership function (rejection) is always greater than that of the membership function (acceptance).

Then the solution of the EDOM problem is summarized in the following steps:

Step-1. Pick the first objective function and solve it as a single objective IP subject to the constraints .Continue the process K-times for K different objective functions. If all the solutions (i.e $x_1^* = x_2^* = x_3^* = \ldots\ldots\ldots = x_k^*$. i =1,2,3,……,m. j = 1,2 ,3,……..,n) same ,then one of them is the optimal compromise solution and go to Step-6.otherwise go to Step-2. However this rarely happens due to the conflicting objective functions. Then the Intuitionistic fuzzy goals take the form $Z_k(x) \tilde{\le} L_k(x_k^*)$ $k = 1, 2, 3, \ldots\ldots, K$.

Step-2. To build membership function, goal and tolerance should be determined at first. Using the ideal solution, obtain in step-1, we find the values of all the objective functions at each ideal solution and construct pay-off matrix as follows:

$$\begin{bmatrix} Z_1(x_1^*) & Z_2(x_1^*) & \ldots\ldots & \ldots\ldots & Z_k(x_1^*) \\ Z_1(x_2^*) & Z_2(x_2^*) & \ldots\ldots & \ldots\ldots & Z_k(x_2^*) \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ Z_1(x_{k-1}^*) & Z_2(x_{k-1}^*) & \ldots\ldots & \ldots\ldots & Z_k(x_{k-1}^*) \\ Z_1(x_k^*) & Z_2(x_k^*) & \ldots\ldots & \ldots\ldots & Z_k(x_k^*) \end{bmatrix}$$

Step-3. From step-2. we find the upper and the lower bounds of each objectives for the degree of acceptance and rejection corresponding to the set of solutions as follows:

$U_k^{acc} = \max\left\{Z_k\left(x_r^*\right)\right\}$ $for\ 1 \le r \le K$ and $L_k^{acc} = \min\left\{Z_k\left(x_r^*\right)\right\}$ $for\ 1 \le r \le K$ for degree of acceptance of objectives. We present upper bound and lower bound for the degree of rejection of objectives as follows:

$\mathrm{L}_k^{rej} = \mathrm{L}_k^{acc} + t\left(\mathrm{U}_k^{acc} - \mathrm{L}_k^{acc}\right)$ with $\mathrm{U}_k^{rej} = \mathrm{U}_k^{acc}$ where $0 < t < 1$,t is presented by decision maker.

Step-4. The initial intuitionistic fuzzy model becomes (in terms of aspiration levels with each objective)

Find $\left\{x_{ij},\ i = 1,2,3,....,m.\ \text{ and }\ j = 1,2,3,.......,n\right\}$

So as to satisfy

$Z_k \le L_k^{acc}$ with tolerance $\left(U_k^{acc} - L_k^{acc}\right)$ for the degree of acceptance ,for k=1,2,3,….,K.

and $Z_k \ge U_k^{rej}$ with tolerance $\left(U_k^{rej} - L_k^{rej}\right)$ for the degree of rejection, for k=1,2,3,…,K.

Step-5 .Now constructs the membership (acceptance) and non-membership (rejection) functions of objective $Z_k\left(x\right)$ by (2.4.1) and (2.4.2).

Step-6. Then the following Intuitionistic Fuzzy Optimization model can be written as

$$\max \mu_k\left(x\right),\ x \in R,\ k = 1,2,........,K.$$

$$\min \upsilon_k\left(x\right),\ x \in R\ ,k = 1,2,........,K.$$

Subject to

$$\begin{aligned}
&\upsilon_k\left(x\right) \ge 0 \\
&\mu_k\left(x\right) \ge \upsilon_k\left(x\right) \\
&\mu_k\left(x\right) + \upsilon_k\left(x\right) < 1 \\
&x \ge 0
\end{aligned}$$

Then the above problem of equation can be reduced following Angelov (1997) to the following form:

$$
\begin{aligned}
&\max(\alpha-\beta);\\
&\textit{subject to}\\
&Z_k(x)\le U_k^{acc}-\alpha\left(U_k^{acc}-L_k^{acc}\right);\\
&Z_k(x)\le L_k^{rej}+\beta\left(U_k^{rej}-L_k^{rej}\right);\\
&\alpha\ge\beta;\\
&\alpha+\beta<1;\\
&\beta\ge 0;\\
&x\ge 0;
\end{aligned}
\tag{2.3.3}
$$

Step-7. Solve equation (2.3.3) by using appropriate mathematical programming algorithm to get an optimal solution and evaluate the K objective function at these optimal compromise solutions.

Step-8. Stop.

3.APPLICATION

A two-bar truss shown in Fig.2 is designed to support the loading condition Consider the following data Nodal load (P) =100 KN; Volume density (γ)=7.7 KN/m^3;Length (l)=2000 mm;Width(x_B)=1000 mm; Allowable tensile stress($[\sigma_t]$)=130 MPa with maximum allowable tolerance 20 MPa;Allowable compressive stress($[\sigma_c]$)=90 MPa with maximum allowable tolerance 10 MPa;Cross-sectional area of bar 1(A_1)=(0 $mm^2 \le A_1 \le 1000\, mm^2$);Cross-sectional area of bar 2 (A_2)=(0 $mm^2 \le A_2 \le 1000\, mm^2$);Y coordinate of node B(y_B)=(500 $mm \le y_B \le 1500\, mm$);

The structure is subject to constraints in geometry, area, stress [1]. Obviously, this is fuzzy optimization problem.

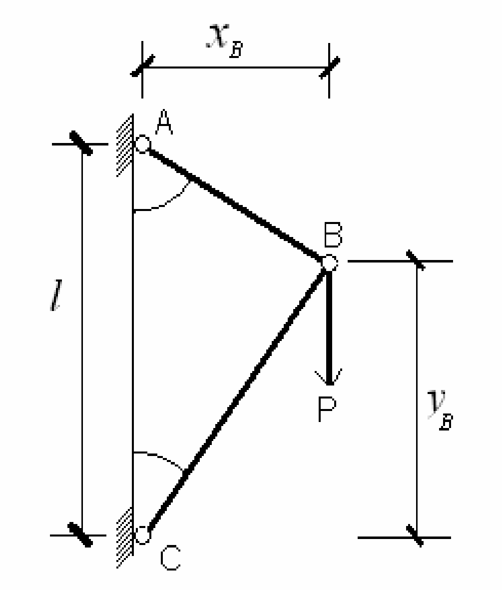

Figure-2: Design of the two-bar planar truss

The Fuzzy Optimization model of the two-bar truss is as follows:

$$\left.\begin{aligned}
&\min W(A_1, A_2, y_B) = 7.7\left(A_1\sqrt{1+(2-y_B)^2} + A_2\sqrt{1+y_B^2}\right)\\
&\text{subject to. } G_1(A_1, A_2, y_B) \equiv \frac{100\sqrt{1+(2-y_B)^2}}{2A_1} \tilde{\le} 130 \;\text{ with tolerance } 20;\\
&\qquad G_2(A_1, A_2, y_B) \equiv \frac{100\sqrt{1+y_B^2}}{2A_2} \tilde{\le} 90 \;\text{ with tolerance } 10;\\
&\qquad\qquad 0.5 \le y_B \le 1.5 \qquad A_1 > 0; A_2 > 0;
\end{aligned}\right\} \tag{3.1.1}$$

According to Werner's approache (1987), we consider two sub-problems. One sub-problem is an optimization problem without tolerance of constraints and other sub-problem is an optimization problem with maximum allowable tolerance of constraints.

They are

Sub-Problem-1:

$$\left.\begin{aligned} &\min W(A_1,A_2,y_B)=7.7\left(A_1\sqrt{1+(2-y_B)^2}+A_2\sqrt{1+y_B^2}\right)\\ &\text{subject to. } G_1(A_1,A_2,y_B)\equiv\frac{100\sqrt{1+(2-y_B)^2}}{2A_1}\tilde{\le}130\;;\\ &\qquad G_2(A_1,A_2,y_B)\equiv\frac{100\sqrt{1+y_B^2}}{2A_2}\tilde{\le}90\;;\\ &\qquad 0.5\le y_B\le 1.5 \qquad A_1>0;A_2>0; \end{aligned}\right\} \tag{3.1.2a}$$

Sub-Problem-2:

$$\left.\begin{aligned} &\min W(A_1,A_2,y_B)=7.7\left(A_1\sqrt{1+(2-y_B)^2}+A_2\sqrt{1+y_B^2}\right)\\ &\text{subject to. } G_1(A_1,A_2,y_B)\equiv\frac{100\sqrt{1+(2-y_B)^2}}{2A_1}\tilde{\le}150\;;\\ &\qquad G_2(A_1,A_2,y_B)\equiv\frac{100\sqrt{1+y_B^2}}{2A_2}\tilde{\le}100\;;\\ &\qquad 0.5\le y_B\le 1.5 \qquad A_1>0;A_2>0; \end{aligned}\right\} \tag{3.1.2b}$$

Solving these two sub-problems we have lower bound and upper bound of weight W. i.e. $125.7\,\mathrm{N}\le W\le 142.3\,\mathrm{N}$.

So the model with fuzzy constraints (3.1.1) reduces to the following fuzzy optimization problem

Find A_1, A_2 and y_B

Such that

$$W(A_1,A_2,y_B)\equiv 7.7\left(A_1\sqrt{1+(2-y_B)^2}+A_2\sqrt{1+y_B^2}\right)\tilde{\le}125.7 \text{ with tolerance 16.6 N.}$$

$$G_1(A_1,A_2,y_B)\equiv\frac{100\sqrt{1+(2-y_B)^2}}{2A_1}\tilde{\le}130 \text{ with tolerance 20;}$$

$$G_2(A_1,A_2,y_B)\equiv\frac{100\sqrt{1+y_B^2}}{2A_2}\tilde{\le}90 \text{ with tolerance 10;}$$

$$0.5\le y_B\le 1.5 \qquad A_1>0;A_2>0;$$

So we can apply the max-min operator to obtain the optimal decision.i.e

$$\begin{aligned} &\text{Max } \alpha \\ \text{subject to } &\mu_W(A_1, A_2, y_B) \geq \alpha; \\ &\mu_{G_1}(A_1, A_2, y_B) \geq \alpha; \\ &\mu_{G_2}(A_1, A_2, y_B) \geq \alpha; \\ &\alpha \in [0,1]; \end{aligned} \tag{3.1.3}$$

Where the linear membership function and the corresponding figure for weight $W(A_1, A_2, y_B)$ is

$$\mu_W(A_1, A_2, y_B) = \begin{cases} 1 & \text{if } W(A_1, A_2, y_B) \leq 125.7 \\ \dfrac{142.3 - W(A_1, A_2, y_B)}{16.6} & \text{if } 125.7 \leq W(A_1, A_2, y_B) \leq 142.3 \\ 0 & \text{if } W(A_1, A_2, y_B) \geq 142.3 \end{cases} \tag{3.1.3a}$$

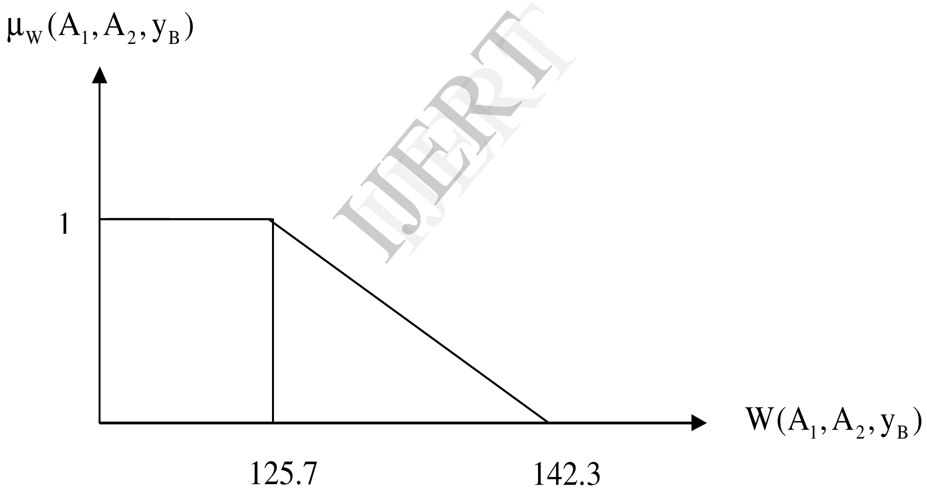

Figure-3: Linear membership function for $W(A_1, A_2, y_B)$

And the linear membership function for first constraint $G_1(A_1, A_2, y_B)$ and the corresponding figure is

$$\mu_{G_1}(A_1, A_2, y_B) = \begin{cases} 1 & \text{if } G_1(A_1, A_2, y_B) \leq 130 \\ \dfrac{150 - G_1(A_1, A_2, y_B)}{20} & \text{if } 130 \leq G_1(A_1, A_2, y_B) \leq 150 \\ 0 & \text{if } G_1(A_1, A_2, y_B) \geq 150 \end{cases} \tag{3.1.3b}$$

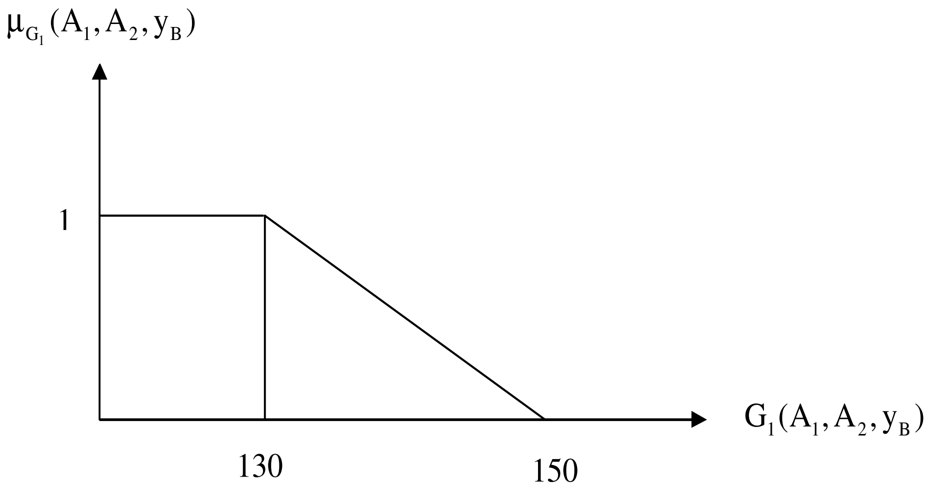

Figure-4: Linear membership function for $G_1(A_1, A_2, y_B)$

And the linear membership function for second constraint $G_2(A_1, A_2, y_B)$ and the corresponding figure is

$$\mu_{G_2}(A_1, A_2, y_B) = \begin{cases} 1 & \text{if } G_2(A_1, A_2, y_B) \le 90 \\ \dfrac{100 - G_2(A_1, A_2, y_B)}{10} & \text{if } 90 \le G_2(A_1, A_2, y_B) \le 100 \\ 0 & \text{if } G_2(A_1, A_2, y_B) \ge 100 \end{cases} \qquad (3.1.3c)$$

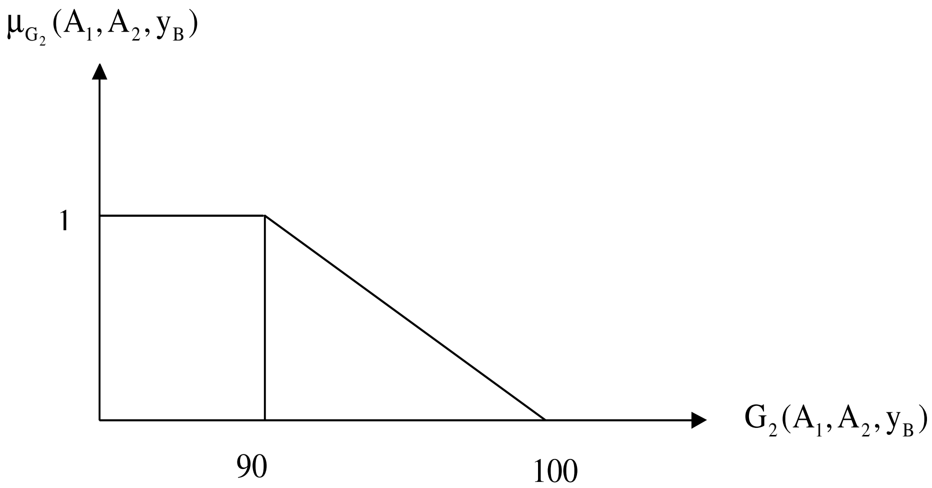

Figure-5: Linear membership function for $G_2(A_1, A_2, y_B)$

Then the problem becomes.

$$\text{Max } \alpha$$

subject to

$$7.7\left(A_1\sqrt{1+(2+y_B)^2}+A_2\sqrt{1+y_B^2}\right)+16.6\alpha \le 142.3;$$
$$\frac{100\sqrt{1+(2+y_B)^2}}{2A_1}+20\alpha \le 150;$$
$$\frac{100\sqrt{1+y_B^2}}{2A_2}+10\alpha \le 100; \qquad (3.1.4)$$
$$0.5 \le y_B \le 1.5; \qquad \alpha \in [0,1];$$
$$A_1 > 0; A_2 > 0;$$

Solution of the above model by General Fuzzy Non-Linear Programming [GFNLP] Technique in section 2.1 and we get the following results are obtain in Table-1:

Table-1

Design variable $A_1(mm^2)$	**Design variable** $A_2(mm^2)$	**Y coordinate of node B** $Y_B(m)$	Aspiration Level	**Weight** $W(N)$
556.5	677.8	.81	$\alpha = .51331$	133.78

To solve the model (3.1.1) by Intuitionistic Fuzzy Optimization (IFO) Technique,

$$Max\ \alpha - \beta$$

subject to

$$\mu_W(A_1,A_2,y_B) \ge \alpha; \quad \upsilon_W(A_1,A_2,y_B) \le \beta;$$
$$\mu_{G_1}(A_1,A_2,y_B) \ge \alpha; \quad \upsilon_{G_1}(A_1,A_2,y_B) \le \beta; \qquad (3.1.5)$$
$$\mu_{G_2}(A_1,A_2,y_B) \ge \alpha; \quad \upsilon_{G_2}(A_1,A_2,y_B) \le \beta;$$
$$\alpha + \beta \le 1$$
$$\alpha \in [0,1]; \ \beta \in [0,1]$$

Where the membership function $\mu_W(A_1, A_2, y_B)$ for weight $W(A_1, A_2, y_B)$ is defined in (3.1.3a) and the non-membership function $\upsilon_W(A_1, A_2, y_B)$ for weight $W(A_1, A_2, y_B)$ and its figure is defined as

$$\upsilon_W(A_1, A_2, y_B) = \begin{cases} 0 & \text{if } W(A_1, A_2, y_B) \le 125.7 \\ \dfrac{W(A_1, A_2, y_B) - 125.7}{5} & \text{if } 125.7 \le W(A_1, A_2, y_B) \le 130.7 \\ 1 & \text{if } W(A_1, A_2, y_B) \ge 130.7 \end{cases} \qquad (3.1.5a)$$

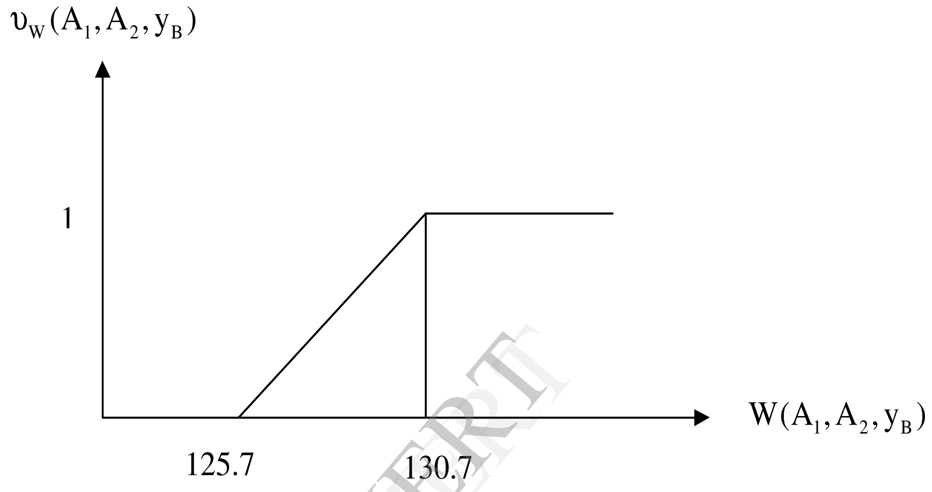

Figure-6: Linear non-membership function for $W(A_1, A_2, y_B)$

And the membership function $\mu_{G_1}(A_1, A_2, y_B)$ for first constraint $G_1(A_1, A_2, y_B)$ is defined in (3.1.3b) and the linear non-membership function $\upsilon_{G_1}(A_1, A_2, y_B)$ for first constraint $G_1(A_1, A_2, y_B)$ and its figure is defined as

$$\upsilon_{G_1}(A_1, A_2, y_B) = \begin{cases} 0 & \text{if } G_1(A_1, A_2, y_B) \le 130 \\ \dfrac{G_1(A_1, A_2, y_B) - 130}{19} & \text{if } 130 \le G_1(A_1, A_2, y_B) \le 149 \\ 1 & \text{if } G_1(A_1, A_2, y_B) \ge 149 \end{cases} \qquad (3.1.5b)$$

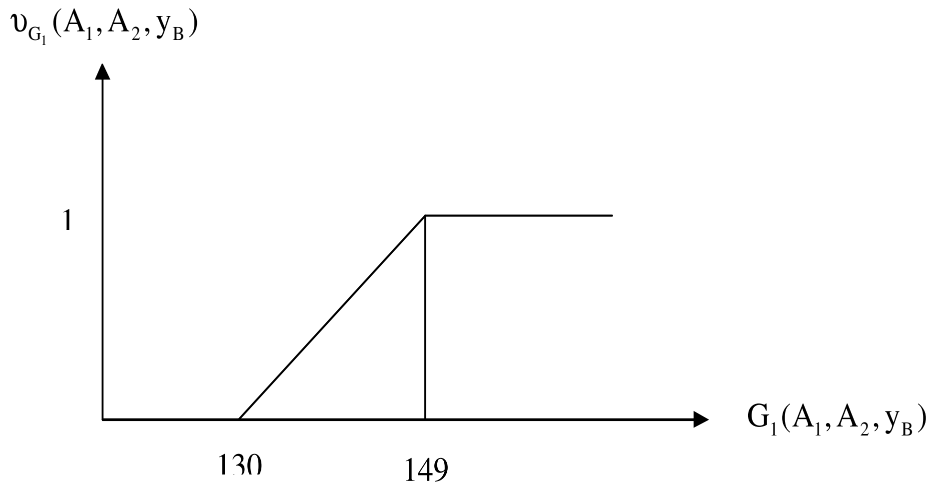

Figure-7: Linear non-membership function for $G_1(A_1, A_2, y_B)$

And the membership function $\mu_{G_2}(A_1, A_2, y_B)$ for second constraint $G_2(A_1, A_2, y_B)$ is defined in (3.1.3c) and the linear non-membership function $\upsilon_{G_2}(A_1, A_2, y_B)$ for second constraint $G_2(A_1, A_2, y_B)$ and its figure is defined as

$$\upsilon_{G_2}(A_1, A_2, y_B) = \begin{cases} 0 & \text{if } G_2(A_1, A_2, y_B) \le 90 \\ \dfrac{G_2(A_1, A_2, y_B) - 90}{9} & \text{if } 90 \le G_2(A_1, A_2, y_B) \le 99 \\ 1 & \text{if } G_2(A_1, A_2, y_B) \ge 99 \end{cases} \tag{3.1.5c}$$

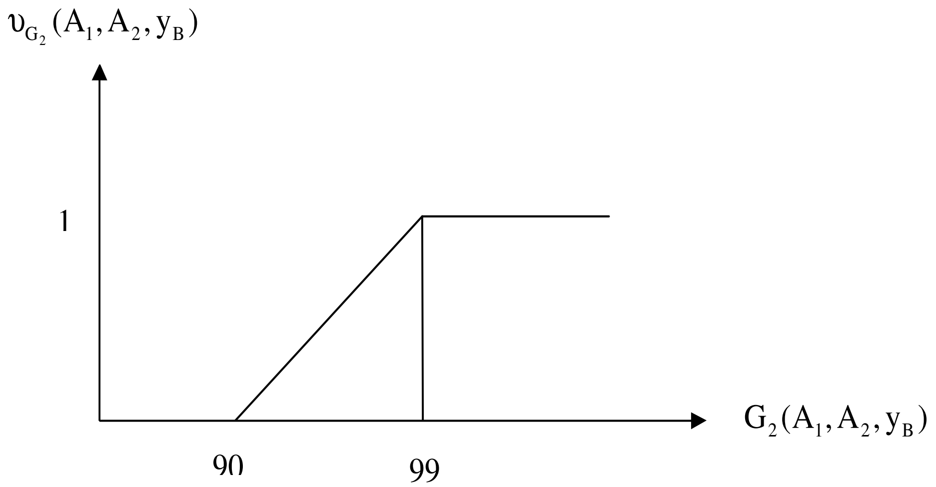

Figure-8: Linear non-membership function for $G_2(A_1, A_2, y_B)$

Then the problem becomes.

$$
\begin{aligned}
& \text{Max } \alpha - \beta \\
& \text{subject to} \\
& 7.7\left(A_1\sqrt{1+(2+y_B)^2} + A_2\sqrt{1+y_B^2}\right) + 16.6\alpha \le 142.3; \\
& \frac{100\sqrt{1+(2+y_B)^2}}{2A_1} + 20\alpha \le 150; \\
& \frac{100\sqrt{1+y_B^2}}{2A_2} + 10\alpha \le 100; \\
& 7.7\left(A_1\sqrt{1+(2+y_B)^2} + A_2\sqrt{1+y_B^2}\right) - 5\beta \le 125.7; \\
& \frac{100\sqrt{1+(2+y_B)^2}}{2A_1} - 19\beta \le 130; \\
& \frac{100\sqrt{1+y_B^2}}{2A_2} - 9\beta \le 90; \\
& 0.5 \le y_B \le 1.5; \quad \alpha + \beta \le 1; \\
& \alpha \in [0,1]; \ \beta \in [0,1]; \\
& A_1 > 0; A_2 > 0;
\end{aligned}
\tag{3.1.6}
$$

Solution of the model (3.1.1) by Intuitionistic Fuzzy Optimization (IFO) is obtained in table-2:

Table-2

Design variable $A_1^*(mm^2)$	Design variable $A_2^*(mm^2)$	Y coordinate of node B $Y_B^*(m)$	α	β	Weight $W(N)$
537.5	659.5	.80	0.2023	0.7977	129.6

4. Analyzing the above tables (Table-1 and table-2) and the following observation can be made:

We compare TABLE-1 and TABLE-2 and the solution of the Weight of the planer truss bar is minimized in case of Intuitionistic Fuzzy Optimization (IFO) Technique then General Fuzzy Non-linear Programming [GFNLP] .We finally concludes that IFO technique performs better than GFNLP technique.

5. Conclusions and Future Scope of Research

In this paper we consider 'two bar truss' model and is solved by GFNLP method, firstly. Besides this elaborate solution IFO method is also used for improvement of the solutions. We established that, objective of this paper is that, IFO technique usually perform better than GFNLP technique. We conclude that IFO minimizes weight more than GFNLP technique.

This two bar truss model can also be analyzed by Geometric Programming as well as Fuzzy Geometric Programming technique can also be applied to obtain possible improve solutions of this model.

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