Soil Interaction Of Laterally Loaded Free Head Long Pile Embedded In Layered Sand Medium

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Abstract

In this paper, some aspects of behavior of laterally loaded free head single pile in cohesionless soil (with both homogeneous and layered sand beds) have been studied based on experiments and numerical methods. The experimental results are supplemented by adopting numerical techniques. The analysis has been done by PLAXIS-3D Foundation version 2.1 software for single pile with variation of slenderness ratio and embedment ratio of pile (ratio of embedded length of pile in a particular soil layer to the pile length) and also stiffness of soil layers. The results are found to agree very well with the experimental results. Further a numerical study has been carried out by finite difference approach to evaluate the coefficient of modulus of horizontal sub-grade reaction (η_h). Its variation with other soil-pile parameters, such as slenderness ratio and embedment ratio of pile and relative density of sand (used as foundation medium) were also studied.

Key words: Laterally-loaded pile, layered soil-medium, finite difference, finite element, modulus of sub-grade reaction

1. Introduction

In civil engineering practice piles are subjected to a variety of loading conditions due to earthquake, wind, sea wave and the like. Thus it is understood that response of piles under lateral loading has a great importance in analysis and design of piles. Analysis of lateral resistance of vertical piles with non dimensional relative stiffness factors has been done to predict the behavior of piles (Reese and Matlock 1956). Elastic analysis has been extended to piles (Davisson and Gill 1963) and subsequently design charts for prediction of lateral response of piles using theoretical and experimental studies were created (Broms 1965). Biswas et al (2012, 2013) carried out an experimental and numerical analysis using cylindrical hollow model cast iron piles of slenderness ratio of 15, 20 and 25 respectively, where the experimental model test results were found to be in close proximity (within 10%) with the 3D finite element models which were created in PLAXIS 3D FOUNDATION version 2.1 software. The PLAXIS 3D results pertaining to piles of higher slenderness ratios (30, 42, 50 and 55) were subsequently used to find the modulus of horizontal sub-grade reaction (η_h) with relevant finite difference formulations.

With this in view, a numerical study has been carried out in which the value of η_h and its normalized form that is the flexibility ratio (Barber 1953) is found out for laterally loaded piles in homogenous and layered cohesionless soil medium and their variation with other soil-pile parameters, such as relative density, slenderness ratio of pile for homogenous soil and ratio of top layer thickness to embedded pile length for Layered Soil. The soil intrinsic parameters and material properties of cast iron (material used in hollow cylindrical model pile) were taken accordingly to those considered by Biswas et al (2012, 2013).

2. Numerical Study

The numerical study was carried out by PLAXIS 3D finite element software. Variation of deflection and moments of pile at different depths were obtained for different 3D finite element models. Further finite difference formulations were used to evaluate the co-efficient of modulus of sub grade reaction (η_h) from the above results. (Biswas et al 2013)

Using PLAXIS-3D same results would have been obtained indirectly by evaluating the stress at the Gaussian integration points or stress points as done earlier by (Kim and Jeong 2011). However, this would have taken a larger computational time than the method adopted for the present study. Thus the numerical part of the study comprises of two analyses:

1. Finite Element Analysis by PLAXIS-3D Foundation version 2.1 software.

2. Application of Finite Difference Method on data obtained from PLAXIS Analysis.

2.1 Soil modelling by PLAXIS 3D

The whole soil continuum has been divided into a number of 15-noded wedge elements to form the 3D finite element mesh. The 15-noded elements were constructed with 8-noded quadrilaterals in vertical direction and 6-noded triangles in horizontal direction. Consequently interface elements were modeled as 16-node (consisting of eight pairs of nodes) interface elements. The depth of soil continuum was taken as 1.7 times the length of the pile (Kim and Jeong 2011) for numerical modelling (Fig1). These dimensions were considered adequate to eliminate the boundary effects on the performance of pile.

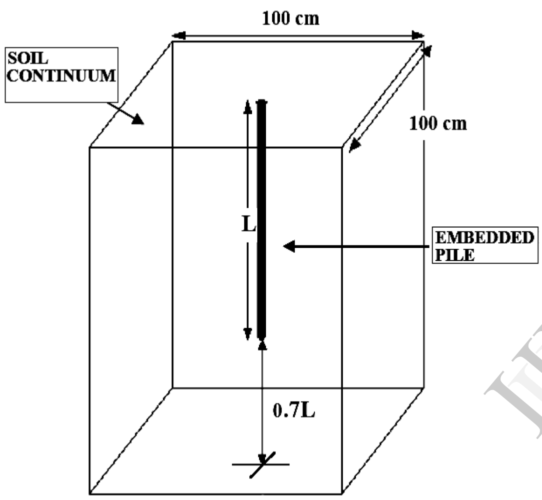

Fig1 Schematic diagram of the model

To model the behaviour of soil surrounding the pile, Mohr-Coulomb failure criteria have been used. This elasto-plastic model depends on the soil parameters, friction angle "φ′" and cohesion intercept "c′". Young's modulus "E′", change in elastic modulus with depth "$E_{increment}$" and Poisson's ratio "ν′". These parameters helped to obtain stress-strain behaviour of the soil and they were taken according to Biswas et al. (2012). The pile was modeled with linear elastic elements following Hooke's law. The linear elastic model included two elastic stiffness parameters viz. Young's modulus "E" and Poisson's ratio "µ" which were also considered from Biswas et al. (2012). A total number of 54 cases of 3D Finite Element models were considered for analysis with variation in slenderness ratio and relative density for sand and ratio of top layer thickness to embedded pile length for Layered Soil exist.

2.2 Finite Difference Method

From the analysis of PLAXIS 3D models the variation of moments with respect to the depth of pile and the displacement profile of the pile for gradual increment of horizontal load are obtained in each case of analysis. On these results the finite difference approach is further infused to obtain horizontal modulus of sub-grade reaction (η_h). The process has been illustrated as follows:

The pile was divided into number of nodes (n) having the node to node length of 'h' units; and the boundary conditions used in this study [Poulos and Davis (1980)] are briefly outlined below.

1. For long floating free head pile the value of end moments that is M_1 (moment at extreme top of the pile) and M_{n+1} (moment at extreme bottom end of the pile) will be equal to zero.
2. It is known that the soil induced stress in cohesionless soil medium varies with the depth as the node '1' is situated at the ground level so it can be assumed that the soil reaction per unit length of the pile at node '1' will be negligible, so w_1=0 (as no overburden is present).

   Another boundary condition incorporated in the present analysis is furnished below:
3. For pile under consideration at the pile head shear force at the node '1' will be equal to the horizontal load acting on it.

The coefficient of modulus of subgrade reaction 'k_h' having units of force/length^3 at a node 'n' can be obtained from the expression given below.

$$k_h = \frac{(M_{n+1} - 2m_n + M_{n-1})}{dyh^2} \quad (1)$$

The coefficient modulus of sub-grade reaction 'k_h' was obtained from Equation (1) through a computer program developed for the purpose which was coded in C++ programming language. The flowchart provided by Biswas et al. (2012) was followed during the writing of computer codes.

The value of k_h was calculated for each incremental load up to the assumed fixity point (Chin et al. 2010). The fixity point was calculated for each incremental load corresponding to a depth where horizontal deflection was negligible.

It was observed that the values of k_h obtained from finite difference calculations were found in close proximity (at a variation of 10%) with those obtained indirectly from PLAXIS 3D. The proximity was further increased by increasing the number of nodes in which the pile was divided.

2.4 Failure Load Criteria

Both for experimental and PLAXIS models a common failure criterion was considered which is explained as follows.

The value of k_h corresponding to each depth down to the fixity point for each incremental load was calculated to the failure load. (Reese and Matlock, 1960) proposed an empirical equation to calculate horizontal pile head deflection corresponding the ultimate soil resistance of the pile. The equation is given as:-

$$Y_u = \frac{3b}{80} \quad (2)$$

Where, b = Width of the pile (in inch) and Y_u = Ultimate deflection (in inch).

As the above deflection corresponds to ultimate soil resistance the load corresponding to it has been taken as the ultimate failure load for the present study.

So, $$Y_u = \frac{3\times(\frac{24}{25.4})}{80}$$

= 0.0354 inch = **0.9 mm**

(This ultimate deflection accounts about 3.75% of the pile diameter)

5. Results and Discussion

5.1 Soil interaction parameters: k_h and η_h

From the results of PLAXIS 3D model analysis of all the 54 cases an attempt has been made to study the variation of the soil interaction parameter for the cohesionless soil medium with different soil-pile parameters.

Main soil interaction parameter is the horizontal sub-grade reaction, K (force/length^2), however the ratio of the former with the diameter or width of pile is known as coefficient of horizontal sub-grade reaction, k_h (force/length3). Further the product of k_h with the ratio of diameter or width to the depth is known as coefficient of modulus of horizontal sub-grade reaction, η_h (force/length3). The modulus of horizontal sub-grade reaction, η_h is used as an integral soil interaction parameter in case of cohesionless soil medium as it has a constant value for a particular cohesionless soil medium.

Using the data obtained from the PLAXIS 3D models and subsequently with the finite difference approach k_h vs. depth graphs were obtained. After obtaining the graph an average line of variation for values of k_h with respect to depth is drawn as was previously done by Biswas et al. (2013).

In all the cases of homogeneous and layered soil systems it is observed that k_h increases with depth and the variation is linear. The curves appear to be much dispersed from the average straight line at larger depth. From the average straight line the coefficient of modulus of horizontal sub-grade reaction for sand η_h is calculated in accordance with the method suggested by Poulos and Davis (1980):

$$k_h = \eta_h \times \frac{z}{B} \quad (3)$$

Where, η_h is coefficient of modulus of subgrade reaction in case of sand (Force/length^3), z is depth of the pile (length) and B is breadth of the pile or the diameter of the pile (length), which in this case is the pile diameter.

5.2 Variation of Coefficient of Horizontal Modulus of Sub-grade Reaction (η_h) With Soil-Pile Parameters

The variation of coefficient of horizontal modulus of sub-grade reaction (η_h) is studied with respect to the following parameters both for homogeneous and layered soil:-

1. Relative density (33%, 44%, 62%, 67%, 74% and 90%)
2. Slenderness ratio (30, 42, 50 and 55)
3. Ratio of top layer thickness to embedded pile length (10%, 20%, 33%, 50%, 67% and 100%).

The figures between the parentheses indicate the values of the parameters considered in this study.

5.2.1 Homogenous Soil

5.2.1.1 Coefficient of horizontal modulus of subgrade reaction (η_h) vs. Relative Density

From the numerical results coefficient of horizontal modulus of sub-grade reaction (η_h) vs. relative density graphs have been plotted as shown in Fig 2 in case of homogeneous soil continuum for different values of slenderness ratio.

From the graph it is noted that in case of loose to medium compactness of the sand medium the values of η_h do not vary much as the relative density decreases. The variation is not prominent for change in slenderness ratio within the range of study. This is due to the fact that as less relative density decreases the stiffness of the soil pile system. With lesser stiffness of the system the effect of slenderness ratio is not much pronounced whereas for denser soil the stiffness of the system is more and the effect of slenderness ratio appears to be more pronounced.

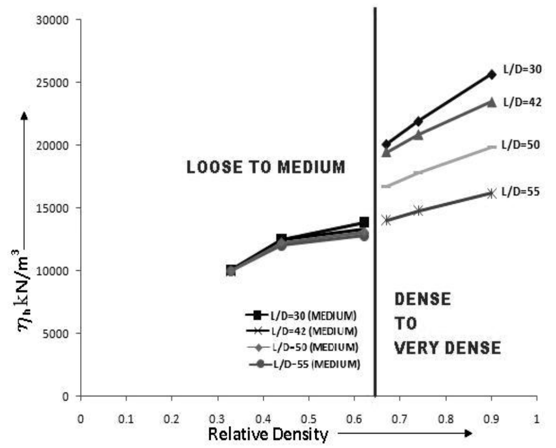

Fig 2 Represents a typical graph showing variation of 'Relative density' with 'η_h' for homogeneous soil medium.

5.2.1.2 Coefficient of horizontal modulus of sub-grade reaction (η_h) vs. Slenderness ratio

A combined graph showing the change of coefficient of horizontal modulus of sub-grade reaction (η_h) vs. slenderness Ratio (L/D) for different compactness of sand is presented in Fig 3.

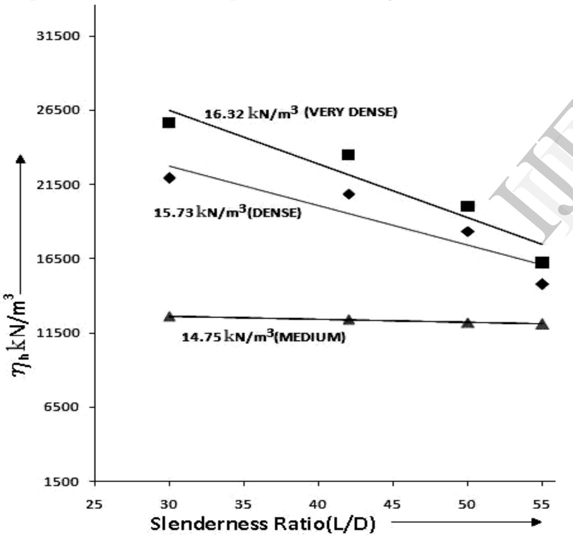

Fig 3 Typical graph showing 'Slenderness ratio' vs. 'η_h' for homogeneous soil medium

The figure shows that with the increase in slenderness ratio, decrease in the value of η_h takes place but the rate of decrease depends on the compactness of sand. Sand with dense to very dense compactness shows an appreciable rate of decrease whereas sand having medium or low compaction shows very little rate of decrease. From the figure it is clear that as the slenderness ratio increases the η_h decreases. With decrease of the stiffness of the soil the values of η_h become lower since the stiffness of the system reduces.

5.2.2 Layered Soil

In this study two types of sand layer have been used, i.e. Sand A and Sand B.

Sand A: Relative density for top layer is 44 and for bottom layer is 74%.

Sand B: Relative density for top layer is 44 and for bottom layer is 90%.

5.2.2.1 Coefficient of horizontal modulus of subgrade reaction (η_h) vs. Ratio of top layer thickness to embedded pile length for Layered Soil

i) For Sand A and Sand B when slenderness ratio is fixed:

Variation for coefficient of horizontal modulus of subgrade reaction (η_h) with Ratio of top layer thickness to embedded pile length for two combinations of density variations that is the medium to dense and medium to very dense compactness is represented respectively in Fig 4.

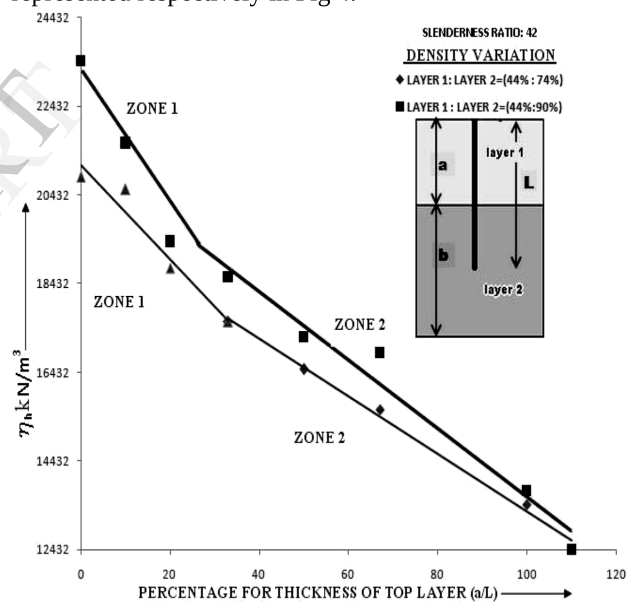

Fig 4 Typical graph showing Percentage for thickness of top layer for two different layer cases vs. 'η_h'

ZONE1 and 2 marked in figure represents the difference in the nature of slopes. It is observed from Fig4 that as the percent top layer thickness with respect to total embedded length of pile increases η_h decreases. This is due to the fact that the weaker top layer thickness is increasing which decrease the system stiffness represented by η_h. Finally the two curves (for Sand A and Sand B) approach each other when the value of percent top layer thickness is increasing and the soil is tending to be homogeneous.

ii) For different values of slenderness ratio with fixed relative densities of upper layer and lower layers.

Using the numerical results another graph (Fig 5) is plotted which shows a comparison of variation for coefficient of horizontal modulus of sub-grade reaction (η_h) with the change in thickness of the top layer for varied slenderness ratio for Sand A. In this case also the variation is linear and separate zones 1 and 2 have been marked in the figure indicating the variation of slope of the lines. It is noted from the Fig 5 that the magnitude of η_h decreases as the upper layer of weaker soil thickness increases. However this rate of increase is much predominant in case of ZONE1.

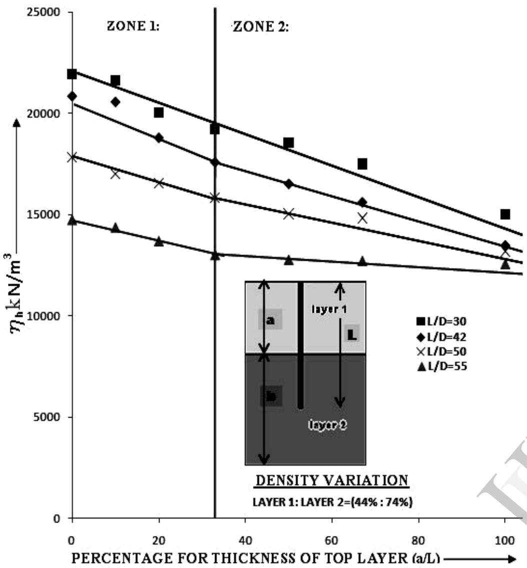

Fig 5 Typical graph showing Percentage for thickness of top layer for varied L/D vs. 'η_h'

It can also be observed that as the compactness of the bottom layer increases from dense to very dense (Fig 4) the rate of increase in coefficient of horizontal modulus of subgrade reaction (η_h) exhibits a higher rate of increase in ZONE 1. It reveals that if the top layer thickness is such that the fixity point enters well inside the bottom layer then there will be decrease in overall head deflection since the pile undergoes bending like a cantilever which is considered to be fixed at the fixity point. The head deflection of pile depends on the position of the fixity point. If it is within the strong stratum rate of increase of η_h with decrease of ratio of top layer thickness to pile length will be more. Otherwise if the fixity point lies inside the weaker stratum the rate will not appreciably increase. In Fig 5 it can be observed that as the slenderness ratio of pile decreases, due to the decrease in pile stiffness, the slope of the graph between ZONE 1 and 2 tends to become identical. It indicates that the decrease in slenderness ratio of pile decreases the system stiffness in such a way that even the effect of increase of sand density is becoming insignificant.

5.3 Normalized Form of η_h

The normalized form of η_h proposed by Barber 1953 has been considered in this study is given by:

$$\text{Flexibility Ratio (FR)} = \frac{\eta_h L^5}{E_p I_p} \qquad (4)$$

E_p and I_p are the elastic modulus and the moment of inertia of model pile respectively. An attempt has been made to examine the variation of flexibility ratio with different parameters like relative density, slenderness ratio, and ratio of top layer thickness to embedded length.

5.3.1 Homogeneous Soil

5.3.1.1 Flexibility Ratio vs Relative Density

From the numerical results described earlier flexibility ratio vs. relative density with varied slenderness ratio have been plotted in Fig 6. It appears from the figure that for each slenderness ratio and for both ranges of sand densities, flexibility ratio increases with slenderness ratio. This is quite justified since the factor by which η_h has been multiplied represents the flexibility of pile. Increase of slenderness ratio results in increase in flexibility, hence variation is as observed in the figure.

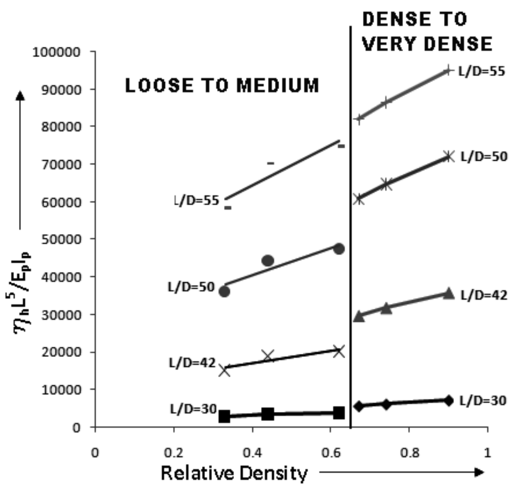

Fig 6 Variation of η_h with increase in relative density

5.3.1.2 Flexibility Ratio vs Slenderness Ratio

The graph shown in Fig 7 represents a the variation of flexibility ratio vs. slenderness ratio for different sand densities which are indicative of its degree of compaction, e.g. medium, dense, very dense respectively of the sand medium considered.

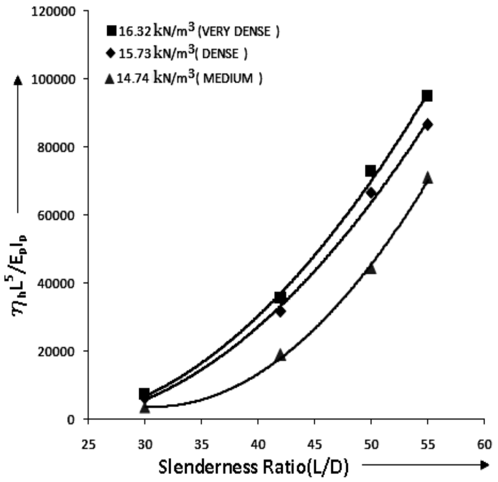

Fig 7 Variation of Flexibility Ratio with increase in Slenderness Ratio

It is observed from the graph that for each compactness (i.e. density) flexibility ratio increases with increase of slenderness ratio. This is attributed to increase in flexibility of pile with increase of slenderness ratio. Further it is observed that for a fixed slenderness ratio flexibility ratio increases with sand density. This is due to the fact that η_h increases the flexibility ratio when pile parameters remain the same as is seen from equation (4).

5.3.2 Layered Soil

Two types of sand layer (Sand A and Sand B) have been used in this study.

Sand A: Relative density for top layer is 44 and for bottom layer is 74%.

Sand B: Relative density for top layer is 44 and for bottom layer is 90%.

5.3.2.1 Flexibility Ratio vs. Ratio of top layer thickness to embedded pile length

The variation of flexibility ratio vs. the ratio of top layer thickness to embedded pile length has been shown in Fig 8. It is observed from Fig 8 that there is a marked difference between the curves for Sand A and Sand B in Zone 1 (stiffer slope), but in case of Zone 2 (flatter slope) for Sand A and Sand B the curves are becoming closer to each other and tends to meet at a point. This indicates that the effect of stronger sand of bottom layer is becoming less and less as percent of pile length in layer 1 increases with respect to embedded pile length. However the differences in the slope between the zones are not as sharp as was observed in Fig 5 since flexibility ratio is non dimensional form of η_h involving other parameters.

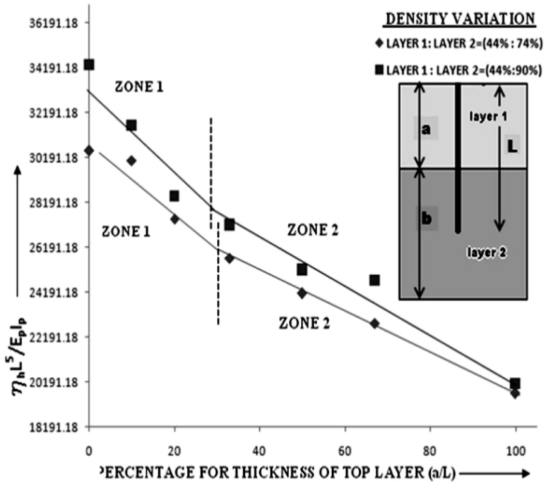

Fig 8 Flexibility ratio vs. the variation of thickness of top layer graphs have been plotted for a fixed slenderness ratio

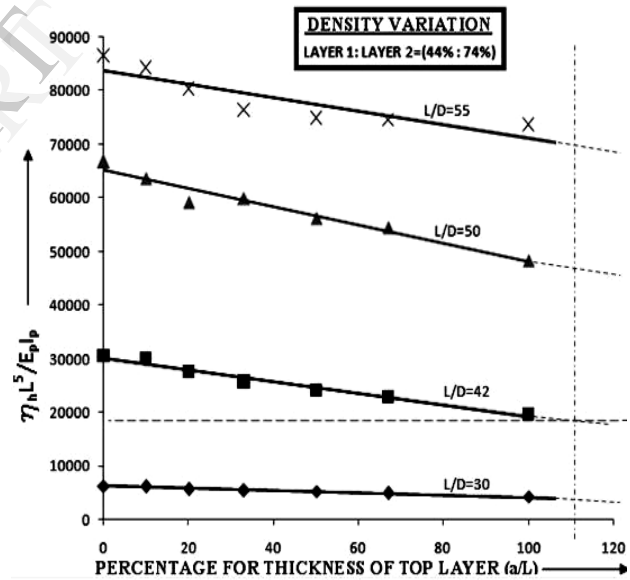

Fig 9 Comparison of variation for Flexibility ratio due to the change in thickness of the top layer for varied Slenderness Ratio

Flexibility ratio vs. percent of top layer thickness to embedded pile length has been plotted in Fig 9. It can be inferred from the figure that for a constant slenderness ratio the value of flexibility ratio decreases with increase in thickness of top layer. But this rate of decrease of the flexibility ratio increases with the increase in slenderness ratio. If the curves representing different values of slenderness ratio as shown in Fig 9 is extrapolated beyond 100% (as shown by the dotted lines), then it can be observed that at 110% the value of flexibility ratio shown by them corresponds to the value of η_h which is in fact the value corresponding to a homogeneous layer of medium degree of compactness. If the dotted line as shown in the figure is followed corresponding to the line showing the variation for slenderness ratio 42, the value of η_h obtained at 110% is 18195, using Equation 1, the η_h is 12434.6 KN/m^3 which according to Fig 6 represents the value corresponding to the Relative Density of 44% (medium degree of compaction), which appears to be reasonable. This indicates that with a virtual thickness of top layer equal to 110% of pile embedded length, the effect is same as that of a homogeneous soil layer with relative density of 44%. Thus the bottom layer looses to produce any effect in soil pile stiffness when the top layer thickness is quite large.

6. Conclusions

The following conclusions may be drawn from the present study:

1. The value of coefficient of horizontal modulus of sub-grade reaction (η_h) increases with the increase in the compactness of the sand.
2. With the increase in slenderness ratio a decrease in the value of coefficient of horizontal modulus of sub-grade reaction (η_h) takes place. However the rate of decrease depends on the compactness of sand.
3. Coefficient of horizontal modulus of sub-grade reaction (η_h) increases as the upper layer thickness of weaker sand decreases in case of a two layered soil system. However, the rate of increase in the magnitude of coefficient of horizontal modulus of sub-grade reaction (η_h) depends on the slenderness ratio.
4. The Flexibility Ratio (normalized form of η_h) increases with the increase in the compactness of the sand.
5. With the increase in slenderness ratio there is a decrease in Flexibility Ratio. However the rate of decrease depends on the compactness of sand.
6. The Flexibility Ratio increases as the upper layer thickness of weaker sand decreases in case of a two layered system. However, the rate of increase in the magnitude of coefficient of horizontal modulus of sub-grade reaction (η_h) depends on the slenderness ratio of pile.

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