

# Soil Constitutive Models and Their Application in Geotechnical Engineering: A Review

Kh Mohd Najmu Saquib Wani<sup>1</sup>  
Post Graduate Student,  
Dept. of Civil Engineering  
NIT Srinagar, Kashmir, India 190006

Rakshanda Showkat<sup>2</sup>  
Post Graduate Student,  
Dept. of Civil Engineering  
NIT Srinagar, Kashmir, India 190006

**Abstract:** Various constitutive models have been developed for modeling the stress strain behavior of soils and apply such models in finite element modeling for application in geotechnical engineering and also for analysis of soil structure problems under different loading conditions. Simple as well as complex models have been formulated based upon the mechanical principle (hooks law of linear elasticity and columbs law of perfect plasticity). However soils are not entirely linearly elastic and perfectly plastic for the entire range of loading. In fact, actual behavior of soils is very complicated to understand and it shows range of behaviors under different conditions. Hence, different models have been proposed to describe its response. Moreover, no model can completely describe the complex behavior of soils. This paper presents brief introduction of various soil models and particularly the cam clay Model and also their application.

**Key words:** *Finite element, Cam clay Model, Application*

## INTRODUCTION:

Soils being complex materials consist of solid grains in contact with each other and voids present in between may be filled either by water or air. The solid grains transmit normal and shear forces and this solid Skelton behaves in a complex fashion depending upon factors like permeability, void ratio etc.

Soils exhibit a complicated behavior whenever subjected to stresses. Soils mostly show non linear, anisotropic, time dependent response under different loading conditions. They undergo plastic deformation and is inconsistent in dilatancy. They also undergo small strain stiffness at small strain levels and when subjected to stress reversal. Different aspects of soil behavior have to be taken into consideration (Brinkgreve 2005).

1. Influence of water on soil from effective stress and pore pressures.
2. Factors influencing soil stiffness like stress path, stress level, soil density, strain levels.
3. Factor influencing soil strength like age and soil density, consolidation ratio, undrained behavior, loading speed.
4. Compaction, dilatancy etc.

Moreover, the failure of soil under three dimensional state of stress is extremely complicated. Various criteria have been proposed to explain failure condition under this state. With the advancement in numerical methods like development of finite element method, it has been possible to analyze and predict the complex behavior of soils and soil-structure interaction problems. Such analysis depends on relation between stress and strain of various materials. In numerical methods, this relation of stress and strain of a given material is represented by **constitutive Model** which models the behavior of soil in a single element. The main aim of these constitutive models is to simulate soil behavior with sufficient accuracy under all loading conditions.

Constitutive models have developed over a period of time, from being simple to more complex in order to capture the behavior of soil under complex loading conditions. These models have been formulated based on the principles of continuum mechanics and also numerical evaluation of the models with respect to the facility which can be implemented in computer calculations (Chen 1985). Different types of models for simulation of soil behavior are Hookes model, Mohr-Coulumb Model, (modified) cam clay Model, Hyperelastic Model, Hypoelastic Model, Plaxis hardening soil Model etc.

## HOOKE'S MODEL:

It is a linear elastic model which is based upon Hooke's law of linear elasticity. This model consists of two basic parameters viz modulus of elasticity (E) and poisons ratio  $\mu$ . Since the soil behavior is highly non linear and irreversible, this model is insufficient to capture the essential features of soil however, this model can be used to model the stiff volumes in soil like the concrete walls.

## MOHR-COULUMB MODEL:

It is a linear elastic perfectly plastic model and represents first order approximation of soil behavior .The graph of Mohr columb model is shown in Fig. 1

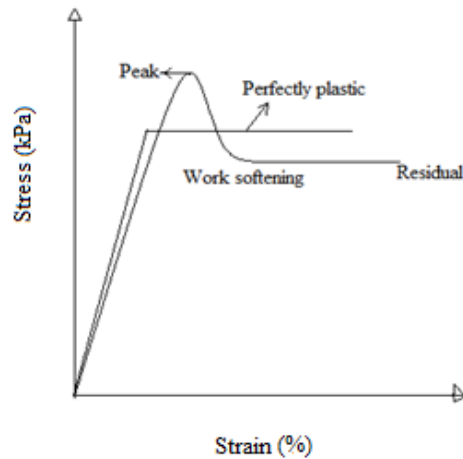


Fig. 1 Elastic perfectly plastic assumption of Mohr Coulomb Model

From Fig.1, it is clear that material behaves linearly in the elastic range, defined by two parameters like Young's modulus ( $E$ ) and Poisson's ratio  $\mu$ . For defining the failure criteria, parameters are friction angle  $\phi$ , cohesion intercept  $C$ . Also a parameter to describe the flow rule is dilatancy angle  $\Psi$  which is used to model the irreversible volume change due to the shearing.

In plastic theory, flow rule is used for plastic strain rates. In order to evaluate whether or not the plasticity occurs, a yield function  $f$  is introduced which is a function of stress and strain. The condition  $f = 0$  is related to the plastic yielding. In

principal stress space, condition  $f = 0$  can be represented as a surface as shown in Fig. 2. Mohr Coulomb model is simple and applicable to three dimensional stress space model to describe plastic behavior. This model finds its application in stability analysis of dams, slopes, embankments and shallow foundations.

In this model, the failure behavior of soil is captured under drained condition, but the effective stress path followed in undrained materials shows considerable variation from observations. Thus in undrained analysis, it is preferred to use undrained shear parameters  $\phi = 0^\circ$ .

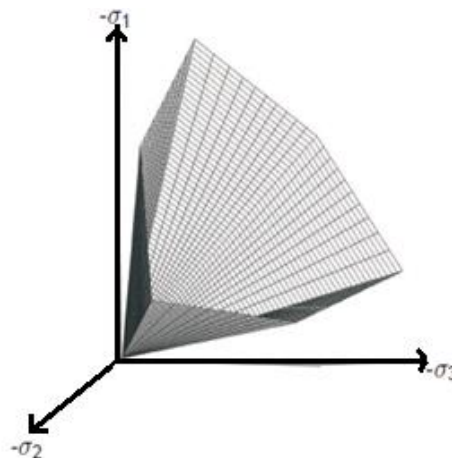


Fig. 2 Mohr Coulomb yield surface in principal stress space

#### HYPERBOLIC MODEL (DUNCAN-CHANG MODEL):

Since the mathematical behavior of soils is highly non linear and also shows stress dependency in their stiffness. Duncan-Chang Model also known as hyperbolic Model (Duncan-Chang 1970) is based on stress strain

curve in drained triaxial compression tests of clays and sands and can be approximated by a hyperbolic function as shown in Fig. 3.

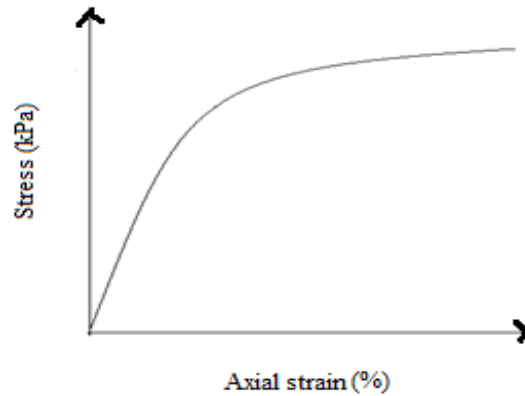


Fig. 3 Hyperbolic stress strain curve

It is an incremental non linear stress dependent model having both loading and unloading modulus. The failure criterion is based on Mohr columb Model and also it is formulated using power law functions (Ohde 1939).

Under the loading condition ( $\sigma_1/\sigma_3 > 0$ ), plastic deformation continues till stress point is on yield surface , otherwise, the stress state must drop below yield value, and in this case all deformations are elastic which occurs under unloading ( $\sigma_1/\sigma_3 < 0$ ).

#### HARDENING SOIL MODEL:

The hardening soil Model (Brinkgreve and Vermer 1997) is an advanced model for simulation of soil behavior. It is a true second order model for soils in general for any type of application (Brinkgreve 2005).The background of this model is the hyperbolic relationship between vertical strain and deviatoric stress in primary loading. It uses theory of plasticity and includes soil dilatancy and a yield cap.

The model involves two types of hardening namely shear hardening and compression hardening. Shear hardening is used to model irreversible strains due to

primary deviatoric loading and compression hardening is used to model irreversible plastic strains due to primary compression in oedometer loading and isotropic loading. Hardening is assumed to be isotropic depending on both plastic shear and volumetric strain. Due to the involvement of two types of hardening, it is used for problems which involve reduction of mean effective stress and mobilization of shear strength like excavation and tunnel construction projects.

In contrast to Mohr Columb Model, yield surface of hardening soil model is not fixed in principal stress space, but it can expand due to plastic strain as shown in Fig. 4. This model accounts for stress dependent stiffness moduli i.e, stiffness increases with pressure. It also uses power law for formulation of stress dependent stiffness. Here, the soil stiffness is described accurately by using three different stiffness namely,

1. Triaxial loading stiffness  $E_{50}$
2. Triaxial unloading stiffness  $E_{ur}$ .
3. Oedometer loading stiffness  $E_{oed}$ .

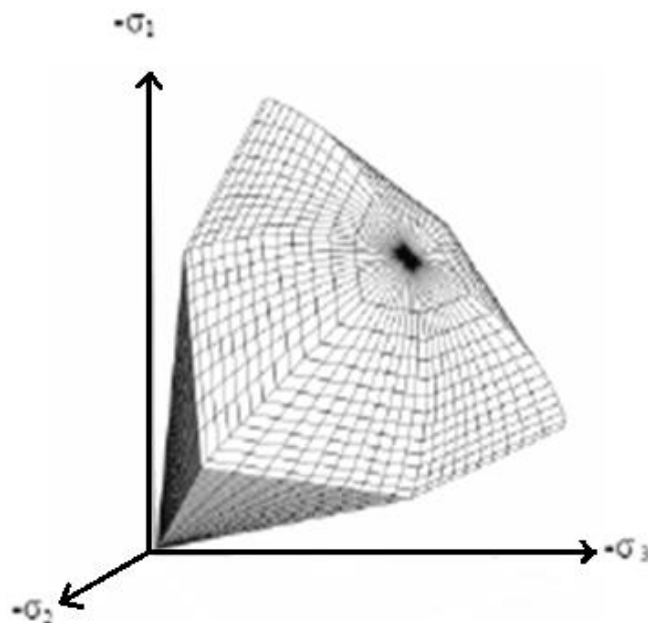


Fig. 4 Yield surface of hardening soil Model

#### HYPERELASTIC MODEL:

Hyperelastic or green elastic Model is a type of constitutive model for ideally elastic materials, where the stress is a function of current strain and not a function of history of strain i.e, it depends on current state of deformation. The Hyperelastic material is also called as Cauchy-elastic material. A Cauchy-elastic material is one in which stress at each point is determined by current state of deformation with respect to an arbitrary reference configuration. Thus, the stress computation in a Cauchy elastic material is independent of history, path of deformation and also the time taken to achieve deformation. Also Cauchy elastic material exhibits energy dissipation violating energy dissipation properties of elastic material.

When an external force is applied to an elastic material in its natural state, the body undergoes deformation and reaches a different energy state. When the external force is removed, body regains its original state and there is no energy dissipation. Materials for which work performed is independent of load path are Hyperelastic materials. Thus Hyperelastic materials are elastic material which derives stress strain relation (as shown in Fig. 5) from a strain energy density function. For deduction of this energy function, it is assumed material is isotropic, constant volume and incompressible. The most common functions of deformation energy are the Mooney-Rivlin Model, Neo-Hookean Model, Odgen Model (Odgen 1972) Yeoh Model (Yeoh 1993), Gent Model (Gent 1996).

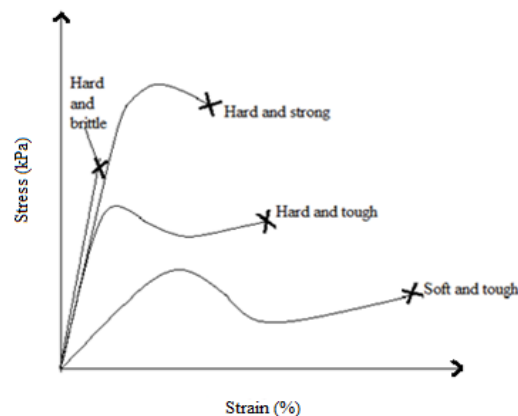


Fig. 5 Stress strain curves for different materials using hyper elasticity

#### HYPOELASTIC MODEL:

Hypoelasticity is used to model materials that exhibit nonlinear, but reversible stress strain behavior even at small strain. Here also, the strain in material depends only on stress applied rather than history of loading and the rate of loading. In this model, the stress is a non linear function of strain, even though strains are small as shown in Fig. 6. Because strains are small this is true whatever stress measure we adopt (Cauchy stress) and is true whatever strain measure we adopt (Lagrangian strain or infinitesimal strain).

Truesdell (1955) proposed a rate theory on the basis of Cauchy formulation for such materials. From this theory, incremental stress strain laws can be developed. The stress strain curve for such materials is shown in Fig. 6 (A F Bowler). This curve is developed based on assumption that strains and rotations are assumed to be small. Thus, deformation is characterized using infinitesimal strain tensor.

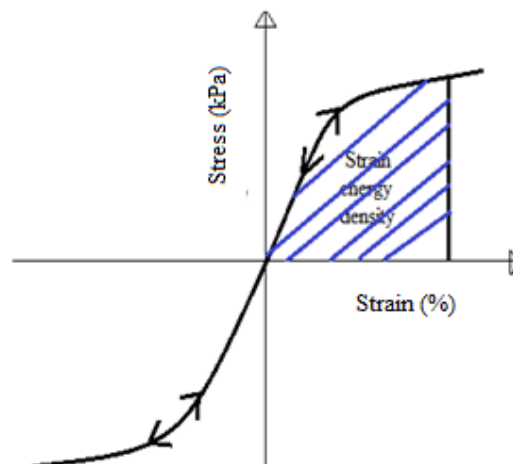


Fig. 6 Stress strain curve of hypoelastic materials

Chen (1985) discussed that the path-independent behavior implied in the previous secant type of stress-strain formulation can be improved by the hypoelastic formulation in which the incremental stress and strain tensors are linearly related through variable tangent material response moduli that are functions of the stress or strain state. In the simplest case of hypoelastic Models, the incremental stress-strain relations are formulated directly as simple extensions of the isotropic linear elastic model with the elastic constants replaced by variable tangential moduli which are taken to be functions of the stress and/or strain invariants.

#### (MODIFIED) CAM CLAY MODEL:

The cam clay Model was developed by researchers at Cambridge University for past thirty years (Roscoe 1970). Roscoe et al. (1963) utilized the strain hardening theory to develop stress strain model for normally consolidated or over consolidated soils in triaxial test known as cam clay Model (Schofield and wroth 1968). Also cam clay Model was based on assumption that soil is isotropic, elasto-plastic, deforms as continuum, and it is not affected by creep. Burland (1968) was responsible for the modification of original cam clay Model where the yield surface is

described by an ellipse i.e, three dimensional stress states (Roscoe and Burland 1968) and hence modified cam clay Model. Both cam clay and modified cam clay Model describe three important aspects of soil behavior.

1. Strength
2. Compression or dilatancy (the volume change that occurs with shearing).
3. Critical state at which soil elements can experience unlimited distortion without any changes in stress or volume.

The modified cam clay Model is an elastic plastic strain hardening model where the non linear behavior is modeled by means of hardening plasticity. Formulation of modified cam clay Model is based on plastic theory which makes it possible to predict volume changes due to different loadings using an associated flow rule. This model is based on critical state theory.

In critical state theory, state of soil is characterized by three parameters:

1. Effective mean stress  $p'$ .
2. Deviatoric stress  $q$
3. Specific volume  $v$ .

Where,

$$p' = \frac{(\sigma'_1 + \sigma'_2 + \sigma'_3)}{3}$$

$$q = \frac{1}{\sqrt{2}} \sqrt{(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2}$$

The specific volume is defined as  $v=1+e$  where  $e$ = void ratio of soil.

Following are the components of critical state theory:

#### 1. VIRGIN COMPRESSION LINE AND SWELLING LINES:

The models assume that under isotropic stress conditions, when a soft soil sample is slowly compressed under perfectly drained conditions, the relationship between specific volume  $v$  and logarithm of mean effective stress  $p'$  consists of a straight line known as normal compression line or virgin compression line and upon reloading and unloading the sample, we get swelling lines as shown in Fig. 7.

The virgin consolidation line is given as:

$$v = N - \lambda \ln p'$$

While the equation for swelling line has the form as:

$$v = v_s - \kappa \ln p'$$

The values  $\lambda$ ,  $\kappa$ ,  $N$  are the characteristic properties of a particular soil,  $\lambda$  is the slope of NCL on  $v - \ln p'$  plane, while  $\kappa$  is the slope of swelling line  $N$  is the specific volume of NCL at unit pressure.

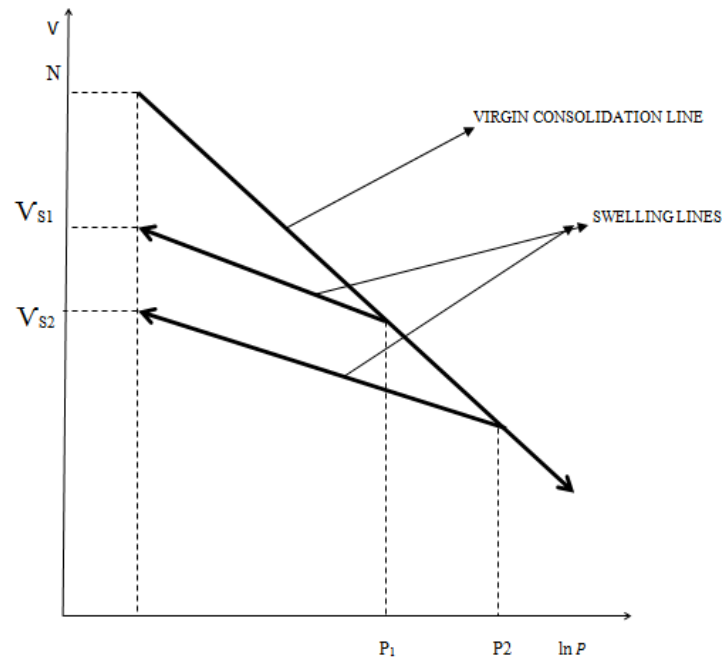


Fig. 7 Behavior of soil sample under isotropic compression

## 2. CRITICAL STATE LINE:

When the soil sample is continuously sheared, it will eventually reach a state where further deformations can occur without any change in stress or volume i.e, soil distorts at constant stress without any change in volume and this state is known as critical state which is characterized by critical state line (CSL). Critical state line is always parallel to NCL in  $v - \ln p'$  space as shown in Fig. 8:

At critical state,  $q = Mp'$  where  $M$  is the slope of straight line passing through origin in  $P' - q$  plane.

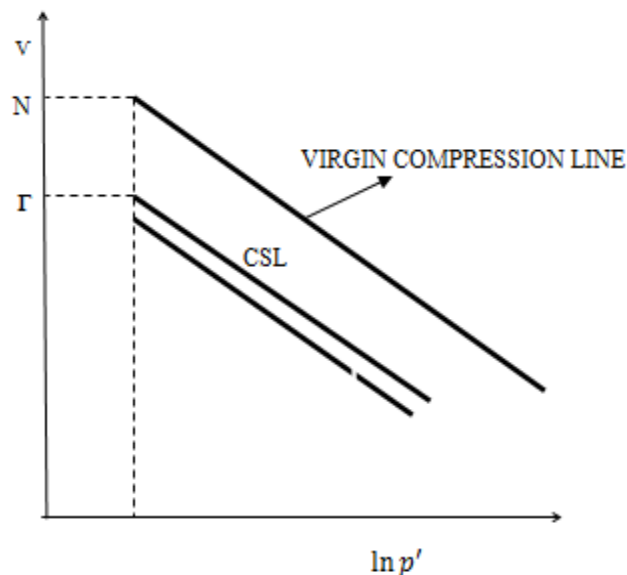


Fig. 8 Location of CSL with respect to virgin compression line

Further, at critical state,

$$v = \Gamma - \lambda \ln p'$$

Where,  $\Gamma$  is the specific volume of swelling line at unit pressure.

For cam clay Model, equation of CSL is as:

$$\Gamma = N - (\lambda - \kappa)$$

And for modified cam clay Model, relationship between these parameters is as:

$$\Gamma = N - (\lambda - \kappa) \ln 2$$

### 3. YIELD FUNCTIONS:

The yield function of cam clay and modified cam clay models are determined from following equations:

For cam clay model:

$$q + Mp' \ln \frac{p'}{p'_0} = 0$$

For modified cam clay model:

$$\frac{q^2}{p'^2} + M^2 \left( 1 - \frac{p'_0}{p'} \right) = 0$$

In the  $p' - q$  space, the CC yield surface is a logarithmic curve while for MCC, the yield surface plots as an elliptical curve as shown in Fig. 9. The parameter  $p'_0$  (known as yield stress or preconsolidation pressure) controls the size of yield surface. The parameter  $M$  is the slope of CSL in  $p' - q$  plane. A characteristic of CSL is that it intersects the yield surface at a point at which the maximum value of  $q$  is obtained.

In three dimensional space defined by  $v - p' - q$ , the yield surface for CC and MCC formulation is known as state boundary surface which is shown in Fig. 10.

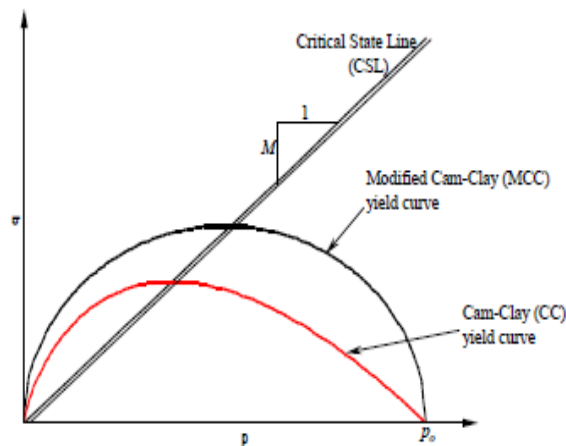


Fig. 9 Yield surface of Cam clay and Modified Cam clay Model

This model is more suitable to describe deformations than failure especially for normally consolidated soils. These models find applications in embankments and foundations. Despite some success in modifying the standard cam clay in 1980's Yu (1995, 1998) found some limitations of this model:

1. Yield surfaces adopted in many critical state models significantly overestimate failure stresses on dry side.

2. Since the model is based on associated flow rule, it is unable to predict the peak in deviator stress before critical state is approached in undrained tests.
3. Due to inability to predict observed softening and dilatancy of dense sands, this model does not work well for modeling granular materials.



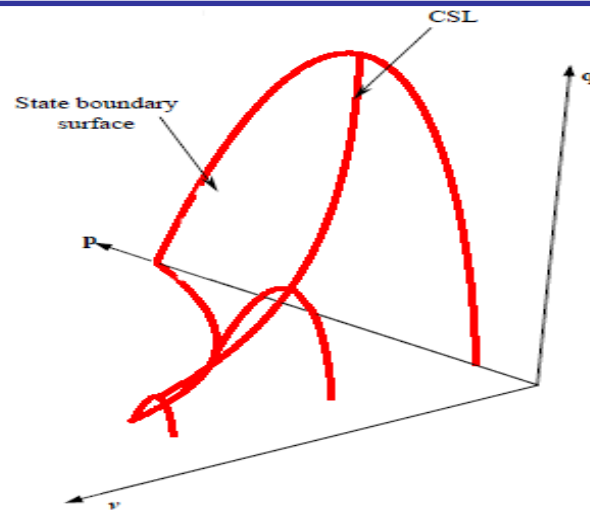


Fig. 10 State boundary surface of modified cam clay

#### APPLICATIONS OF CONSTITUTIVE MODELS:

Various types of geotechnical problems can be analyzed using the constitutive models. The following types of geotechnical applications have been found (Duncan 1994).

- Soil structure interaction
- Soil reinforcement and anchorage.
- Tunnels
- Dams
- Embankments

- Settlements due to fluid extraction.
- Natural and unbraced cut slopes

Thus it has been possible to analyze and predict the behavior of any type of complex soil structure and soil structure interaction problems. These models find great advantage in places where there is an interaction between stresses and soil volume changes plays a dominant role as shown in Fig. 11.

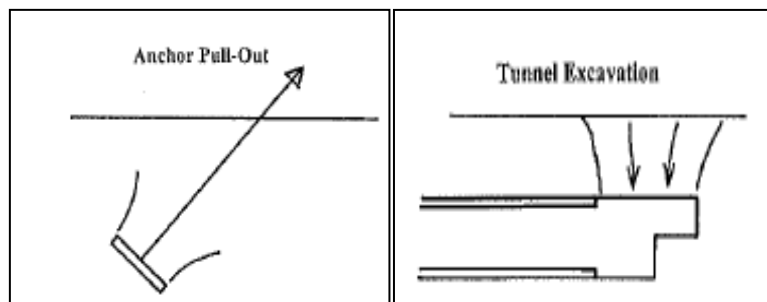


Fig. 11 Structures in which soil behavior plays an important role

Another type of problem depicting the importance of soil behavior is shown in Fig. 12. Due to filling of water reservoir, the upstream shell of rockfill dam may collapse

due to wetting. This produces displacements of crest which can be better analyzed by modeling of soil behavior.

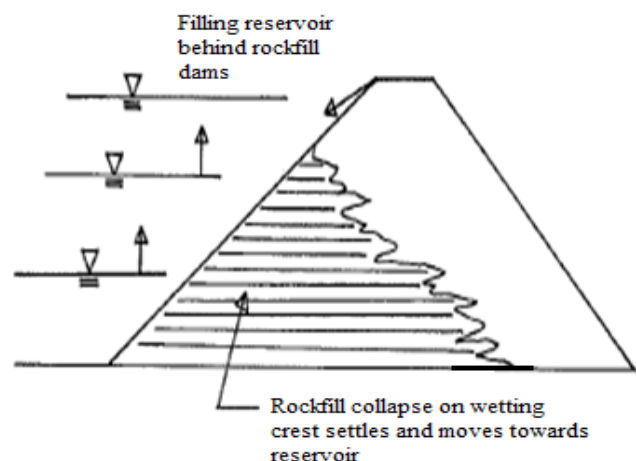


Fig. 12 Filling of water reservoir behind a rockfill dam with the resulting crest displacement due to wetting and collapse of rockfill material in upstream shell.



Also, since most of the embankments have been built on soft soils, the constitutive laws employed to describe their behavior are: linear and non linear elasticity, elastoplasticity without strain hardening; elastoplasticity with strain hardening and elastoplasticity. When the embankment is described by finite elements, the most widespread constitutive law is isotropic linear elasticity (55%), followed by perfect elastoplasticity (36%) and non-linear elasticity (9%).

The constitutive models used for describe soil behavior in case of tunnels includes the following types of laws: linear and non-linear elasticity, elastoplasticity without strain hardening; elastoplasticity with strain hardening and elastoviscoplasticity. Generally speaking, the most widespread constitutive law is the Mohr-Coulomb perfect elastoplasticity Model with isotropic linear elasticity. Among the elastoplastic laws with strain hardening, the Cam-clay Models remains the most widely used.

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