Small Signal Stability Analysis of Multi-Machine Power Systems Interfaced with Micro Grid

D. Padma Subramanian  
EEE Department  
Professor and Dean, Faculty of Electrical Engineering  
Director- Centre for Robotics and Automation  
AMET University, Kanathur, Chennai, India

K. Harinee  
EEE Department  
Jerusalem College of Engineering  
Chennai, India

Abstract— This paper work presents a study of small signal stability analysis of multi-machine power systems interfaced with micro grid. Modeling of DFIG for Wind Energy Conversion System (WECS), marine current energy system and, and PV module is presented. A procedure for incorporating Wind Energy Conversion System, Marine Current Energy System and PV system into multi-machine power systems is presented. A program is developed in MATLAB environment and effectiveness of developed program is tested in a standard IEEE-9 bus system. The small signal stability of the multi-machine power systems interfaced with micro grid is analyzed and the results are presented.


I. INTRODUCTION

Renewable energy is one of the sizzling themes in the entire world today due to the fast and huge consumption of fossil fuels. The ocean covers more than 70% surface of the earth, the wind energy above the sea surface and oceanic energy under the seawater can be captured simultaneously to generate large electric power[2,4,11,12]. The wind energy and oceanic energy can be effectively integrated together to deliver electric power to the loads. The utilization of marine current turbines offers an exciting proposition for the extraction of energy from marine currents[1]. Wind, tidal and PV system has a higher reliability for maintaining a continuous power than any other individual sources[2,14]. Compared to external grid, micro-grid is a single controllable unit, that link multiple distributed power generation sources into a small network. The modelling of PV cell in simplified equivalent circuit with output elements and the stability of PV system using Eigen-value analysis was discussed by Hun-Chul Seo . Shan ying li presented the analysis of small signal stability of grid-connected doubly fed induction generators .The detailed model of grid-connected DFIG wind turbine is firstly established, and the Eigen-values are classified and characterized based on participation factors.

This paper presents a modeling of power system and micro grid for small signal stability analysis, step by step procedure for small signal stability analysis of micro grid interfaced with multi-machine system and state space model for multi-machine system with micro grid. The small signal stability analysis of multi-machine system with micro grid is presented using Eigen-value analysis.

This paper comprises of the following sections: Section 2 deals with the modeling of power system. In section 3, the small signal stability analysis of multi-machine system with micro grid is presented. In section 4, the results and discussion of the test system are presented. Conclusion is presented in section 5.

II. POWER SYSTEM MODELING

In this section, modeling of synchronous generator in power system and micro grid components for small signal stability analysis are presented.

A. Synchronous Generator Model

In multi-machine model, a synchronous machine or group of synchronous machines connected to a larger system through one or more power lines as shown in Fig-1.

Fig-1: General m machine n bus system
Synchronous Generators in multi-machine power systems are modeled as classical machine model and variable voltage behind transient reactance model.

Neglecting saliency, the stator of a synchronous machine is represented by the equivalent circuit shown in Fig-2. The block diagram of excitation system is shown in Fig-3.

The linearized equations of classical machine model for the small signal stability analysis in state variable form are represented by the following equations[6,7,8]

\[
\Delta \dot{\omega}_{12} = \frac{-K_s}{2\lambda} \Delta \omega_{12} - \left( \frac{T_d}{\lambda n} - \frac{T_s}{\lambda n} \right) \Delta \delta_{12} - \left( \frac{\lambda n}{2\lambda} - \frac{\lambda n}{2\lambda} \right) \Delta E'_{q1}
\]

(3)

\[
\Delta \delta_{12} = \omega_d \Delta \omega_{12}
\]

(4)

\[
\Delta E'_{q1} = E_1 \Delta \delta_{12} + E_{11} \Delta E'_{q1} - \frac{K_D}{T_d} \Delta X_1
\]

(5)

\[
\Delta X_1 = \frac{P_{11}}{T_r} \Delta \delta_{12} + \frac{P_{11}}{T_r} \Delta E'_{q1} - \frac{1}{T_R} \Delta X_1
\]

(6)

Where \( \delta \) and \( \omega \) refers rotor angle and speed, \( E'_{q1} \) is quadrature voltage behind transient reactance, \( X_1 \) is terminal voltage transducer, \( H \) is inertia constant, \( K_D \) is damping coefficient.

Where,

\[
E_{11} = -\frac{2}{T_{do1}} \left[ 1 - B_{11} (X_{d1} - X_{d1}') \right]
\]

(7)

\[
E_{12} = -\frac{1}{T_{do1}} (X_{d1} - X_{q1}) (G_{12} \cos \delta_{12} + B_{12} \sin \delta_{12}) E_{q10}
\]

(8)

\[
P_{11} = \frac{V_{q10}}{V_{q10}} (1 + d_{11} X_{d1}) - \frac{V_{d10}}{V_{q10}} q_{11} X_{q1}
\]

(9)

\[
P_{13} = \frac{V_{q10}}{V_{q10}} (d_{13} X_{d1} - \frac{V_{d10}}{V_{q10}} q_{13} X_{q1})
\]

(10)

\[
d_{11} = B_{11}
\]

(11)

\[
d_{13} = -(G_{12} \cos \delta_{12} + B_{12} \sin \delta_{12}) E_{q10}
\]

(12)

Where \( X_{d1} \) refers direct axis synchronous reactance, \( X'_{d1} \) is direct axis transient reactance, \( T_{do1} \) is direct axis open circuit transient time constant and \( X_{q1} \) refers quadrature axis synchronous reactance, \( B \) is susceptance and \( G \) is conductance.

**B. Doubly Fed Induction Generator Model for Wind and Marine Farm**

DFIG is an induction-type generator. The d-axis and q-axis equivalent circuit of doubly fed induction generator for wind and marine farm is shown in Fig-4 (a) and (b).
1) Electrical Equations

The linearized electrical differential equations of DFIG in wind farm for small signal stability analysis in state variable form are represented by the following equations:

\[
\Delta i_{dsc} = \frac{\dot{\omega}}{2} \left[ -r_s \Delta i_{qsc} + a_{ls} \Delta i_{qsc} + \frac{\omega_{eB} - \omega}{2} \Delta i_{qsc} + a_{ls} \Delta i_{dsc} + N \Delta \omega \right] \tag{13}
\]

\[
\Delta i_{qsc} = \frac{\dot{\omega}}{2} \left[ -r_s \Delta i_{qsc} - a_{ls} \Delta i_{dsc} - \omega_{eB} \Delta i_{qsc} - \frac{\omega_{eB} - \omega}{2} \Delta i_{dsc} + N \Delta \omega \right] \tag{14}
\]

\[
\Delta i_{dsw} = \frac{\dot{\omega}}{2} \left[ -a_{d} \Delta i_{dsw} - \frac{\omega_{eB} - \omega}{2} \Delta i_{dsw} - \frac{\omega_{eB} - \omega}{2} \Delta i_{dsw} + N \Delta \omega \right] \tag{15}
\]

\[
\Delta i_{qsw} = \frac{\dot{\omega}}{2} \left[ -a_{q} \Delta i_{qsw} + \frac{\omega_{eB} - \omega}{2} \Delta i_{qsw} - \frac{\omega_{eB} - \omega}{2} \Delta i_{qsw} + N \Delta \omega \right] \tag{16}
\]

Where, $\omega_{eB}$ - electrical base speed, $\omega$ - synchronous speed, $i_{qsc}$ and $i_{dsc}$ - q-axis and d-axis voltage of stator, $i_{dsw}$ and $i_{qsw}$ - q-axis and d-axis current of stator for wind farm, $i_{qsw}$ and $i_{dsw}$ - q-axis and d-axis current of rotor for wind farm, $L_s$ and $L_r$ - stator and rotor inductances for wind farm, $L_m$ - mutual inductances for wind farm, $r_s$ and $r_r$ - rotor and stator resistances for wind farm.

The linearized electrical differential equations of DFIG in marine farm for small signal stability analysis in state variable form are represented by the following equations:

\[
\Delta i_{dsm} = \frac{\dot{\omega}}{2} \left[ -r_s \Delta i_{qsm} + a_{ls} \Delta i_{qsm} + \frac{\omega_{eB} - \omega}{2} \Delta i_{qsm} + a_{ls} \Delta i_{dsm} + N \Delta \omega \right] \tag{17}
\]

\[
\Delta i_{qsm} = \frac{\dot{\omega}}{2} \left[ -r_s \Delta i_{qsm} - a_{ls} \Delta i_{dsm} - \omega_{eB} \Delta i_{qsm} - \frac{\omega_{eB} - \omega}{2} \Delta i_{dsm} + N \Delta \omega \right] \tag{18}
\]

\[
\Delta i_{dsw} = \frac{\dot{\omega}}{2} \left[ -a_{d} \Delta i_{dsw} - \frac{\omega_{eB} - \omega}{2} \Delta i_{dsw} - \frac{\omega_{eB} - \omega}{2} \Delta i_{dsw} + N \Delta \omega \right] \tag{19}
\]

\[
\Delta i_{qsw} = \frac{\dot{\omega}}{2} \left[ -a_{q} \Delta i_{qsw} + \frac{\omega_{eB} - \omega}{2} \Delta i_{qsw} - \frac{\omega_{eB} - \omega}{2} \Delta i_{qsw} + N \Delta \omega \right] \tag{20}
\]

Where, $\omega_{eB}$ - electrical base speed, $\omega$ - synchronous speed, $v_{qsc}$ and $v_{dsc}$ - q-axis and d-axis voltage of stator, $v_{qsm}$ and $v_{dsm}$ - q-axis and d-axis current of stator for marine farm, $i_{qsw}$ and $i_{dsw}$ - q-axis and d-axis current of rotor for marine farm, $L_s$ and $L_r$ - stator and rotor inductances for marine farm, $L_m$ - mutual inductances for marine farm, $r_s$ and $r_r$ - rotor and stator resistances for marine farm.

2) Mechanical Equations

A two-mass drive train model is used to get a more accurate response from the wind turbine and marine turbine. Two mass drive train model is shown in Fig-5.

The linearized mechanical differential equations of DFIG in wind farm for small signal stability analysis in state variable form are represented by the following equations:

\[
\Delta \dot{\omega}_w = \frac{\Delta T_{rw}}{2H_w} - \frac{D_{gw}}{2H_w} \Delta \omega_w - \frac{K_{gw}}{2H_w} \Delta \theta_{gw} \tag{21}
\]

\[
\Delta \dot{\theta}_{gw} = -\frac{D_{gw}}{2H_w} \Delta \omega_w + \frac{K_{gw}}{2H_w} \Delta \theta_{gw} - \frac{\Delta T_{gw}}{2H_w} \tag{22}
\]

\[
\Delta \dot{\theta}_{hgw} = \omega_b \Delta \omega_w - \omega_b \Delta \omega_w \tag{23}
\]

Where, $\omega_w$ and $\omega_g$ - turbine and rotor speeds for wind farm, $\theta_{hgw}$ - shaft torsional angle for wind farm, $D_{gw}$ and $K_{gw}$ - drive train damping coefficient and shaft stiffness for wind farm, $H_w$ and $H_{gw}$ - turbine and generator inertia for wind farm, $T_{cw}$ - electrical torque for wind farm.
The linearized mechanical differential equations of DFIG in marine farm for small signal stability analysis in state variable form are represented by the following equations

\[
\Delta \omega_{hm} = \frac{\Delta T_{mm}}{2H_{hm}} - \frac{D_{hgm}}{2H_{hm}} \Delta \omega_{gm} - \frac{K_{hgm}}{2H_{hm}} \Delta \theta_{hgm}
\]

\[
\Delta \omega_{gm} = \frac{D_{gmm}}{2H_{gm}} \Delta \omega_{gm} + \frac{K_{gmm}}{2H_{gm}} \Delta \theta_{gmm} - \frac{\Delta T_{em}}{2H_{gm}}
\]

\[
\Delta \theta_{gm} = -\frac{D_{gmm}}{2H_{gm}} \Delta \omega_{gm} - \frac{K_{gmm}}{2H_{gm}} \Delta \theta_{gmm}
\]

Where, \( \omega_{hm} \) and \( \omega_{gm} \) – turbine and rotor speeds for marine farm, \( \theta_{hgm} \) - shaft torsional angle for marine farm, \( D_{hgm} \) and \( K_{hgm} \) - drive train damping coefficient and shaft stiffness for marine farm, \( H_{hm} \) and \( H_{gm} \) – turbine and generator inertia for marine farm, \( T_{mm} \) – mechanical torque for marine farm, \( T_{em} \) – electrical torque for marine farm.

C. PV Cell Model

PV arrays are built up with combined series/parallel combinations of PV solar cells, which are usually represented by a simplified equivalent circuit model such as the one given in Fig-6.

![Fig-6: Simplified equivalent circuit of PV cell](image)

The linearized differential equations of PV cell for small signal stability analysis in state variable form are represented by the following equations

\[
\frac{d\Delta I_C}{dt} = -\frac{1}{\alpha L} \left( \frac{1}{I_{ph} - I_C + I_0} \right) \Delta I_C - \frac{1}{L} \Delta V_C
\]

\[
\frac{d\Delta V_C}{dt} = \frac{1}{C} \Delta I_C + \frac{P}{CV_C^2} \Delta V_C
\]

Where, \( R_s \) - array resistance, \( I_{ph} \), \( I_0 \), \( \alpha \) are constants, \( I_C \) and \( V_C \) are current and voltage through PV cell.

D. Load Model

The load is modeled as constant impedance load. The load \( (P + jQ) \) is represented as constant impedance load. The equation of load for small signal stability analysis are represented by equation (29).

\[
Z_L = V_L / I_L
\]

Where \( V_L = \) Load Voltage, \( I_L = \) Load Current, \( Z_L = \) Load impedance.

III. SMALL SIGNAL STABILITY ANALYSIS

Small signal stability analysis is performed by linearizing the system equations at the operating point and Eigen-value analysis. The system equations are described in the following general form

\[
X = f(X, Z, U), Z = g(X, U)
\]

Where \( X, Z \) and \( U \) are the vectors of state variables, control variables and input variables respectively. After performing the linearization, the following relation is derived.

\[
\Delta \dot{X} = A\Delta X + BU
\]

The state variable \( X \) is given as

Classical machine model:

\[
X = \left[ \begin{array}{c} \dot{\omega}_{hm} \\ \dot{\omega}_{gm} \\ \dot{\theta}_{hgm} \end{array} \right]
\]

Variable voltage behind transient reactance model

\[
X = \left[ \begin{array}{c} \dot{\omega}_{hm} \\ \dot{\omega}_{gm} \\ \dot{\theta}_{hgm} \\ \dot{\phi}_{hgm} \end{array} \right]
\]

Where \( A \) is the system state matrix. This matrix is then used for calculating the system Eigen-values.

IV. RESULTS AND DISCUSSION

In this section, results of small signal stability analysis of multi-machine power systems interfaced with micro grid carried out on standard IEEE 9 bus system.

A. IEEE-9 bus system

In this paper, standard IEEE-9 bus system is considered for small signal stability analysis. Data for the 9 bus system is provided in appendix. The single line diagram of micro grid interfaced standard IEEE-9 bus system is shown in Fig- 7.

The load is represented by constant impedance load. The losses in transmission lines are neglected. The mechanical power is assumed to be constant.
The small signal stability analysis is carried out by interfacing micro grid including WT, MCT and PV module between 7 and 8 bus of IEEE-9 bus system. The small signal stability analysis result with and without micro grid obtained for classical machine model and variable voltage behind transient reactance model in synchronous generator is tabulated in Table-1 and Table-2.

Fig-8: Single line diagram of micro grid interfaced in IEEE-9 bus system

### Table -1: Eigen-values for IEEE-9 bus system without micro grid

<table>
<thead>
<tr>
<th>No of variables</th>
<th>Eigen-values for classical machine model</th>
<th>Damping ratio</th>
<th>Eigen-values for variable voltage behind transient reactance model</th>
<th>Damping ratio</th>
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<tbody>
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<td></td>
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<tr>
<td>8</td>
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Table-2: Eigen-values for IEEE-9 bus system with micro grid

<table>
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</table>

From Table-3 and Table-4, it is observed that the system remains stable after interfacing micro grid into the system. Based on the Eigen-values, it is concluded that the small signal stability of the system is affected after interfacing micro grid into the system.

V. CONCLUSION

In the project work, the small signal stability analysis of multi-machine power systems interfaced with micro grid is presented. A program is developed using MATLAB. The effectiveness of the developed program is tested using a standard IEEE-9 bus system. The small signal stability results show that the system remains stable and stability of the system is affected after interfacing micro grid into the system.

APPENDIX

IEEE-9 BUS SYSTEM DATA

<table>
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<th>Generator Data</th>
</tr>
</thead>
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</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
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</table>
PV Generation System Data

e= 1.602*10^-19 C; K = 1.38*10^-23 J/K; P=130 w; Tc=298K; lph=5.14;l0=0.0002A; Rs=0.001ohm; L=1 ; C=10mH.

REFERENCES

Dr. D. Padma Subramanian received B.Tech. in Electrical and Electronics Engineering and M.Tech. in Electrical Power Systems with Honours from Govt. College of Engineering, Thrissur in 1990 and 1992 respectively. She has done full time doctoral research from College of Engineering, Guindy, Anna university during 2004-2007. She has a total of over 21 years of experience in the field of teaching industry and research. Dr. Padma Subramaian has more than 30 publications spanning around various international/ national journals and proceedings with good impact factor. She is serving as a member of editorial board for a number of international journals. She is an active researcher in the field of power systems non linear dynamics, FACTS applications to power systems, modeling and analysis of micro grid, grid integration issues of renewable energy sources and self healing networks for smart grid. She has completed couple of funded projects as Principal Investigator. She is a member of doctoral committee for research in various universities and approved supervisor for guiding Ph.D/M.S in many universities including Anna University. Presently she is working as Dean-Faculty of Electrical Engineering and Director- Centre for Robotics and Automation at AMET University, Kanathur, Chennai.