Slightly m-Precontinuous Multifunctions

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Abstract **- In this paper, we introduce a new class of functions namely slightly m- precontinuous multifunction. Further we obtain characterizations and relationships between upper/ lower slightly m- precontinuous multifunctions and other related multifunctions.**

Keywords - Upper slightly precontinuous, lower slightly precontinuous, upper slightly m-precontinuous, lower slightly m-precontinuous, upper m-precontinuous, lower m-precontinuous.
AMS Subject classification : 54C10, 54C08

1. INTRODUCTION

The notion of slightly continuous functions was introduced by R.C. Jain [2]. Nour [12] defined slightly semi-continuous functions as a weak form of slight continuous functions and obtained their properties. Further this concept is extended to develop the class of slightly m-continuous functions. Popa [14] and Smithson [23] studied the concept of weakly continuous multifunctions. In this paper, we introduce the notion of slightly m-precontinuous multifunction and investigate the relationships among m-precontinuity, weak m-precontinuity and slight m-precontinuity for multifunctions.

2. PRELIMINARIES

Throughout this paper, all spaces (X,τ) and (Y,σ) are always topological spaces. A subset A of a space X is said to be regular open (resp.regular closed), if $A=Int(Cl(A))$(resp. $A=Cl(Int(A))$), where Cl(A) and Int(A) denote the closure and interior of A . A subset A of a space X is called preopen if $A\subset Int(Cl(A))$.The complement of a preopen set is said to be preclosed. The family of all regular open (resp. regular closed, preopen, preclosed, clopen) sets of X is denoted by RO(X) (resp. RC(X), PO(X), PC(X), CO(X)). A subset A of a space X is said to be semi-open if $A\subset Cl(Int(A))$.The complement of a semi open set is called semi-closed. The union of all pre open sets of X contained in A is called the pre-interior of A and is denoted by pInt(A).A subset A of a space X is said to be clopen if it is both open and closed.

Throughout this paper, the spaces (X,τ) and (Y,σ)(or simply X and Y) denote topological spaces and $F:(X,\tau)\to(Y,\sigma)$ represents a multivalued function. For a multifunction $F:(X,\tau)\to(Y,\sigma)$, we shall denote the upper and lower inverse of a set B of a space Y by $F^+(B)$ and $F^-(B)$ respectively.

i.e. $F^+(B) = \{x\in X: F(x)\subset B\}$,$F^-(B) = \{x\in X: F(x)\cap B\neq\phi\}$.

Definition: 2.1[20]

A subfamily m_X of the power set P(X) of a nonempty set X is called a minimal structure (briefly m-structure) on X, if $\phi\in m_X$ and $X\in m_X$.

Definition: 2.2 [20]

By (X, m_X), we denote a nonempty set X with a minimal structure m_X on X and call it an m-space. Each member of m_X is said to be m_X -open (or briefly m-open). The complement of an m_X-open set is said to be m_X-closed (or briefly m-closed).

Remark: 2.3 [20]

Let (X,τ) be a topological space. Then the families τ, SO(X), PO(X), α (X), β(X), are all m-structures on X.

Definition: 2.4 [20]

A function $f:(X,\tau)\to(Y,\sigma)$ is said to be slightly continuous if for each point $x\in X$ and each clopen set V containing f(x), there exists an open set U containing x such that $f(U)\subset V$.

Definition: 2.5 [20]

A topological space (X,τ) is said to be extremally disconnected (briefly E.D), if the closure of each open set of X is open in X.

SLIGHTLY m-PRECONTINUOUS MULTIFUNCTIONS

Definition:3.1

A multifunction $F:(X,\tau)\to(Y,\sigma)$ is said to be

(a) Upper slightly precontinuous (resp. upper slightly continuous, upper slightly semi-continuous or upper faintly precontinuous, upper slightly β-continuous), if for each point $x\in X$ and each clopen set V of Y containing F(x), there exists an preopen (resp. open, semi-open, β-open) set U of X containing x such that $F(U)\subset V$.

(b) Lower slightly precontinuous (resp. lower slightly continuous, lower slightly semi-continuous or lower faintly precontinuous, lower slightly β-continuous), if for each point $x\in X$ and each clopen set V of Y such that $F(x)\cap V\neq\phi$, there exists an preopen (resp. open, semi-open, β-open) set U of X containing x such that $F(u)\cap V\neq\phi$, for each $u\in U$.

Definition: 3.2

A function $f:(X,m_X)\to(Y,\sigma)$, where (X,m_X) is a nonempty X with an minimal structure m_X and (Y,σ) is a topological space, is said to be slightly m-pre continuous, if for each $x\in X$ and each clopen set V of Y containing f(x), there exist a preopen set $U\in m_X$ containing x such that $f(U)\subset V$.

Definition: 3.3

A multifunction $F:(X,m_X)\rightarrow(Y,\sigma)$ is said to be

(a) Upper m-precontinuous(resp. upper almost m-precontinuous, upper weakly m-precontinuous), if for each point $x\in X$ and each open set V of Y containing F(x), there exists a preopen set $U\in m_X$ containing x such that $F(U)\subset V$(resp. $F(U)\subset Int(Cl(V))$, $F(U)\subset Cl(V)$).

(b) Lower m-precontinuous (lower almost m-precontinuous, lower weakly m-precontinuous), if for each point $x\in X$ and each open set V of Y such that $F(x)\cap V\neq\phi$, there exists a pre open set $U\in m_X$ containing x such that $F(u)\cap V\neq\phi$(resp. $F(u)\cap Int(Cl(V))\neq\phi$, $F(u)\cap Cl(V)\neq\phi$) for each $u\in U$.

Definition: 3.4

Let X be a nonempty set and m_X an m-structure on X. For a subset A of X, the m_X-preclosure of A and the m_X-preinterior of A are defined as follows.

1. $m_X\text{-}pCl(A)=\cap\{F:A\subset F, X-F\in m_X\}$.
2. $m_X\text{-}pInt(A)=\cup\{U:U\subset A, U\in m_X\}$.

Remark: 3.5

Let (X,τ) be a topological space and A be a subset of X. If $m_X=\tau$(resp.SO(X), PO(X), α(X)), then we have

1. m_X-Cl(A) = Cl(A)(resp.sCl(A),pCl(A), αCl(A)).
2. m_X-Int (A) = Int(A)(resp. sInt(A), pInt(A), αInt(A)).

Definition: 3.6

A multifunction $F:(X,m_X)\rightarrow(Y,\sigma)$, is said to be

(a) Upper slightly m-precontinuous, if for each point $x\in X$ and each colpen set V of Y containing F(x), there exist preopen set $U\in m_X$ containing x such that $F(U)\subset V$.

(b) Lower slightly m-precontinuous, if for each point $x\in X$ and each colpen set V of Y such that $F(x)\cap V\neq\phi$ there exists a preopen set $U\in m_X$ containing x such that $F(u)\cap V\neq\phi$ for each $u\in U$

Theorem: 3.7

For a multifunction $F:(X, m_X)\rightarrow(Y,\sigma)$ the following are equivalent.

1) F is upper slightly m-precontinuous
2) $F^+(V)=m_X\text{-}pInt(F^+(V))$ for each $V\in CO(Y)$
3) $F^-(V)=m_X\text{-}pCl(F^-(V))$ for each $V\in CO(Y)$

Proof:

(1)⇒(2): Let V be any clopen set of Y and $x\in F^+(V)$ then $F(x)\in V$ → (1).By (1), there exists a preopen set $U\in m_X$ containing x such that $F(U)\subset V$. Thus $x\in U\subset F^+(V)\Rightarrow x\in m_X\text{-}pInt(F^+(V))$ → (2).From eqn (1) & (2) we get,$F^+(V)\subset m_X\text{-}pInt(F^+(V))$.i.e. $m_X\text{-}pInt(F^+(V))\subset F^+(V)$, by lemma (3.1) of [20] $F^+(V)=m_X\text{-}pInt(F^+(V))$ for each $V\in CO(Y)$.

(2) ⇒ (3): Let K be any clopen set of Y. Then Y-K is clopen in Y.By (2), $F^+(V)=m_X\text{-}pInt(F^+(V))$.By lemma (3.1) of [20] we have, $X-F^-(K)=F^+(Y-K)=m_X\text{-}pInt(F^+(Y-K))=X-[m_X\text{-}pCl(F^-(K)]$.Therefore $F^-(K)=m_X\text{-}pCl(F^-K))$.

(3) ⇒ (2): Let B be any clopen set of Y. Then Y- B is clopen in Y.By (3) & lemma (3.1) of [20] we have, $X-F^+(B)=F^-(Y-B)=m_X\text{-}pCl(F^+(V))=X-[m_X\text{-}pInt(F^+(V))$.Therefore $F^+(B)=m_X\text{-}pInt(F^+(V))$.

(2) ⇒ (1): Let $x\in X$ and V be any clopen set of Y containing F(x). Then $x\in F^+(V)=m_X\text{-}pInt(F^+(V))$,there exists a preopen set $U\in m_X$ containing x such that $x\in U\subset F^+(V)$. Therefore we have, $x\in U$ and $U\in m_X$ and $f(U)\subset V$. Hence F is upper slightly m-precontinuous.

Theorem: 3.8

For a multi function $F:(X, m_X)\rightarrow(Y,\sigma)$ the following are equivalent.

1) F is lower slightly m-precontinuous.
2) $F^-(V)=m_X\text{-}pInt((F^-(V))$ for each $V\in CO(Y)$.
3) $F^+(V)=m_X\text{-}pCl(F^+(V))$ for each $V\in CO(Y)$

Proof:

(1) ⇒ (2): Let $V\in CO(Y)$ and $x\in F^-(V)$. Then $F(x)\cap V\neq\phi$.By (1), there exists a preopen set $U\in m_X$ containing x such that $F(u)\cap V\neq\phi$ for each $u\in U$.Therefore we have, $U\subset F^-(V)\Rightarrow x\in U\subset m_X\text{-}pInt(F^-(V))\Rightarrow x\in m_X\text{-}pInt(F^-(V))\Rightarrow F^-(V)\subset m_X\text{-}pInt(F^-(V))$ and by lemma (3.1) of [20] we have $F^-(V)=m_X\text{-}pInt(F^-(V))$.

(2) ⇒ (3): Let $V\in CO(Y)$.Then $Y-V\in CO(Y)$,by (2), we have,$X-F^+(V)=F^-(V)=F^-(Y-V)=m_X\text{-}pInt(F^-(Y-V))=X-[m_X\text{-}pCl(F^+(V))$.Hence $F^+(V)=m_X\text{-}pCl(F^+(V))$

(3)⇒ (1): Let $x\in X$ and $V\in CO(Y)$ such that $F(x)\cap V\neq\phi$,then $x\in F^-(V)$ and $x\notin X-F^-(V)=F^+(Y-V)$.By (3) we have,$x\notin m_X\text{-}pCl(F^+(V))$.By lemma (3.2) of [20], there exists preopen set $U\in m_X$ containing x such that $U\cap F^+(Y-V)=\phi$.Thus $U\subset F^-(V)$.Therefore $F(u)\cap V\neq\phi$ for each $u\in U$. Hence F is lower slightly m-precontinuous.

Theorem: 3.9

Let (Y,σ) be E.D. For a multifunction $F:(X, m_X)\rightarrow(Y,\sigma)$, the following are equivalent.

1) F is upper slightly m-precontinuous.
2) $m_X\text{-}pCl(F^-(V))\subset F^-(Cl(V))$ for every open set V of (Y,σ)
3) $F^+(Int(C))\subset m_X\text{-}pInt(F^+(C))$ for every closed set C of (Y,σ)

Proof:

(1) ⇒ (2): Let V be any open set of Y. Then $Cl(V)\in CO(Y)$,by theorem (3.7), we have,$F^-(Cl(V))=m_X\text{-}pCl(F^-(Cl(V)))$ and $F^-(V)\subset F^-(Cl(V))$,by lemma (3.1) of [20], we have, $m_X\text{-}pCl(F^-(V))\subset m_X\text{-}pCl(F^-(Cl(V)))=F^-(Cl(V))\Rightarrow m_X\text{-}pCl(F^-(V))\subset F^-(Cl(V))$

(2) ⇒ (3): Let C be any closed set of (Y,σ) and V = Y-C. Then V is open in (Y,σ). By lemma (3.1) of [20], we have, $X-[m_X\text{-}pInt(F^+(C))]=m_X\text{-}pCl(x-F^+(C))=m_X\text{-}pCl(F^-(Y-$

C))⊂ F^-(Y-Int(C)). Therefore X-[m_X-pInt(F^+(C))] = X-F^+(Int(C)) ⇒ F^+(Int(C))⊂ m_X-pInt(F^+(C))

(3) ⇒ (1): Let x∈X and let V∈CO(Y) containing F(x). Then By (3) we have, x∈F^+(V)= F^+(Int(V))⊂ m_X-pInt(F^+(V)).There exists a preopen set U∈m_X such that x∈U⊂F^+(V). Thus x∈U, U∈m_X and F(U)⊂V. Hence F is upper slightly m-precontinuous.

Theorem: 3.10

Let (Y,σ) be E .D. For a multifunction F:(X, m_X) →(Y,σ), the following are equivalent.

1) F is lower slightly m-precontinuous.

2) m_X-pCl(F^+(V))⊂ F^+(Cl(V)) for every open set V of (Y,σ)

3) F^-(Int(C))⊂ m_X-pInt(F^-(C)) for every closed set C of (Y,σ)

Proof:

(1) ⇒ (2): Let V be any open set of Y. Then Cl(V)∈CO(Y).By theorem (3.8), F^+(Cl(V)) = m_X-pCl(F^+(Cl(V))) and F^+(V)⊂ (F^+(Cl(V))).By lemma (3.1) of [20], we have m_X-pCl(F^+(V))⊂ m_X-pCl(F^+(Cl(V)))=F^+(Cl(V))⇒m_X-pCl(F^+(V))⊂ F^+(Cl(V))

(2) ⇒ (3): Let C be any closed set of (Y,σ) and V= Y-C, then V is open in (Y,σ)

By lemma (3.1) of [20], we have, X- [m_X-pInt (F^-(C))] = m_X-pCl(X-(F^-(C)))= m_X-pCl(F^+(Y-C))⊂ F^+(Cl(Y-C)) (by given (2))=F^+(Y-Int(C)).Therefore X- [m_X-pInt

(F^-(C))] = X- F^-(Int(C))⇒ F^-(Int(C))⊂ m_X-pInt(F^-(C))

(3) ⇒ (1): Let x∈X and V∈CO(Y) containing F(x)∩V ≠ ϕ.Let V = Y-C is open in Y. Let x∈F^+(C) and x∉X-F^+(C)∉F^-(Y-C).By (3) we have, x∉m_X-pInt (F^-(Y-C)),by theorem (3.8) we have, x∉m_X-pCl(F^+(Y-C)).By lemma (3.2) of [20], there exists a preopen set U∈m_X containing x such that U∩(F^+(Y-C)) = ϕ.Hence U⊂

F^-(V)and F (x)∩V ≠ ϕfor each u∈U. Hence F is lower slightly m-precontinuous.

Slight m-precontinuity and other forms m-precontinuity.

Theorem: 4.1

If multifunction F:(X, m_X)→(Y,σ) is upper weakly m-precontinuous, then it is upper slightly m-precontinuous.

Proof:

Let x∈X and V∈CO(Y) containing F(x).Since F is upper weakly m-precontinuous. There exists a preopen set U∈m_X containing x such that F(U)⊂Cl(V) = V. Hence F is upper slightly m-precontinuous.

Theorem: 4.2

If a multi function F:(X, m_X)→(Y,σ) is lower weakly m-precontinuous then it is lower slightly m-precontinuous.

Proof:

Let x∈X and V∈CO(Y) such that F(x)∩V ≠ ϕ.Since F is lower weakly m-precontinuous there exist a preopen set U∈m_X containing x such that F(U)∩Cl(V) ≠ ϕ, for each u∈U⇒ F(U)∩(V) ≠ ϕ, for each u∈U. Hence F is lower slightly m-precontinuous.

Lemma: 4.3

A multifunction F:(X, m_X)→(Y,σ) is upper almost m-precontinuous (resp.lower almost m-precontinuous) iff for each regular open set V containing F(x) (resp. meeting F(x)), there exist a preopen set U∈m_X containing x such that F(U)⊂V (resp. F(u)∩(V) ≠ ϕ, for every u∈U)

Proof:

Let x∈X and V be regular open set of Y containing F(x).Since F is upper almost m-precontinuous there exist a preopen set U∈m_X containing x such that F(U)⊂Int(Cl(V)) = V. Hence for each regular open set V containing F(x), there exist a preopen set U∈m_X containing x such that un F(U)⊂V. Conversely, there exist a preopen set U∈m_X containing x such that F(U)⊂V⊂Cl(V) ⇒ IntF(U)⊂Int(Cl(V)) ⇒F(U)⊂Int(Cl(V)).Hence F is upper almost m-precontinuous.

Theorem: 4.4

If a multi function F:(X, m_X)→ (Y,σ) is upper slightly m- precontinuous and (Y,σ) is E.D, then F is upper almost m- precontinuous.

Proof:

Let x∈X and V be any regular open set of (Y,σ) containing F (x). Then By lemma 5.6 of [13] we have, V ∈ CO(X).Since (Y,σ) is E.D and F is upper almost m- precontinuous, there exist a preopen set U∈ m_X containing x such that F(U)⊂V. By lemma (4.3), F is upper almost m- precontinuous.

Theorem: 4.5

If a multi function F: (X, m_X)→(Y,σ) is lower slightly m- precontinuous and (Y,σ) is E.D , then F is lower almost m- precontinuous.

Proof:

Let x∈X and V be any regular open set of (Y,σ) containing F(x). Then By lemma 5.6 of [13] we have, V∈CO(X).Since (Y,σ) is E.D. Since F is lower slightly m-precontinuous, there exist a pre open set U∈m_X containing x such that F(U)∩V≠ϕ for each u∈U. By lemma (4.4), F is lower almost m- pre continuous.

REFERENCES

[1] T.Banzaru, More remarks on some forms of continuity for multifunctions, Bul.Stiint.Univ.Politeh.Timisser Mat.Fiz., 44(58) (1999), 34-39.

[2] R.C.Jain, The role of regularly open sets in general topology Ph.D. Thesis. Meerut Univ., 1980.

[3] A.S.Mashhour, I.A.Hasanein and S.N.EI-Deeb, a note on semi-continuity and precontinuity, Indian J.Pure Appl.Math. 13 (1982), 1119-1123.

[4] A.S.Mashhour.M.E.Abd EI.monsef and S.N.EI-Deep, on precontinuous and weak precontinuous mappings, Proc.Math.Phys.Soc.Egypt 53(1982), 47-53.

[5] A.S.Mashhour. M.E.A.EI-monsef, I.A.Hasanein, on pretopological spaces, Bull. Math. Soc.Sci. RSR.28 (1984), 39-45.

[6] H.Maki, K.C.Rao and A.Nagoor Gani, On generalizing Semi-open sets and preopen sets. Pure Appl.Math. Sci. 49(1999), 17-29.

[7] T.Noiri, Characterizations of extremally disconnected spaces, Indian J.Pure Appl.Math., 19 (1988), 325-329.
[8] T.Noiri and V.Popa, On upper and lower weakly quasi continuous multifunctions, Rev.Roumaine Math.Pures Appl., 37(1992), 499-508.
[9] T.Noiri and V.Popa, almost weakly continuous multifunctions, Demonstratio Math, 26(1993), 363-368.
[10] A.A.Nasef and T.Noini, some weak forms of almost continuity, Acta. Math. Hungar, 74(1997), 211-219.
[11] T.Noiri, On slightly β-continuous functions, internet, J.Math. Math. Sci., 28(2001), 469- 478.
[12] T.M.Nour, slightly semi-continuous function, Bull.Calcutta Math.Soc., 87 (1995), 187-190.
[13] M.C.Pal and P.Bhattacharyya, Faint precontinuous functions, Soochow J.Math., 21(1995), 273-289.
[14] V.Popa, weakly continuous Multifunctions, Boll.Un.Math.Ital. (5), 15 (1978), 379-388.
[15] V.Popa, On some properties of weakly continuous multifunctions, Univ.Bacau, Stud.Cerc.Math., 1980, 121-127.
[16] V.Popa and T.Noiri, Properties of upper and lower weakly quasi continuous Multifunctions, univ.Bacau,Stud.Cerc.St.Ser.Mat.Inform, 25 (1998), 24-30.
[17] V.Popa and T.Noiri, Almost m-continuous functions, Math.Notal 40 (1999/ 2002), 75-94.
[18] V.Popa and T.Noiri, On M-continuous functions, Anal.Univ.”Dunarea de Jos” Galati, Ser.Math.Fiz.Mec.Teor.18 (23) (2000), 31-41.
[19] V.Popa and T.Noiri, Slightly m-continuous functions, Radovi Math, 10 (2001), 1-14.
[20] V.Popa and T.Noiri, Slightly m-continuous multifunctions, Bull Math, 1(2006).
[21] V.Popa and J.Noiri On upper and lower weakly α-continuous Multifunctions, Novi sad J.Math., 32 (2002), 7-24.
[22] V.Popa and T.Noiri, A unified theory of weak continuity for multifunctions, (submitted)