Sliding Mode based Level Controller with First Order Plus Delay Time (FOPDT) Modeling

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Abstract— Level control is a significant parameter in industries like chemical plants, nuclear reactors and even for various biomedical applications. Sliding Mode Controller (SMC) which belongs to the class of Variable Structure System (VSS) can be used to accurately control the level compared to conventional ones especially PID controller. A single cylindrical tank system is first modeled in real time using Process Reaction Curve method. For this the system is put in manual mode and operated using UT35 controller module. The Sliding Mode Controller for this system is then designed by First Order Plus Delay Time (FOPDT) approximation. The step responses of the system for SMC and PID controllers are obtained using SIMULINK and the results are compared.

Keywords— SMC; PID; VSS; FOPDT

I. INTRODUCTION

A control system which usually refers to a set of interconnected equipments intended to manage commands, direct or regulate the behaviour of other devices or systems has an inevitable role in modern industries. The control strategies adopted ranges from simple linear controllers to the most modern techniques like fuzzy logic based control [1], model predictive control (MPC), sliding mode control and the like. Among them Sliding Mode Control (SMC) is a robust and simple procedure to synthesize controllers for linear and nonlinear processes. The idea behind SMC is to define a surface along which the process can slide to its desired final value. The structure of the controller is intentionally altered as its state crosses the surface in accordance with a prescribed control law. This yields better stability and tracking performance. The concept of Sliding Mode came into existence in early 1950’s for systems represented by single-input high-order differential equations. As said by Gao in the early transactions [2] both the sliding mode and reaching mode controls can be done by trial and error for simple systems. In the late fifties, the concept of sliding mode control appeared in the Russian literature. It was Emelyanov who first observed that due to altering the structure in the course of controlling a process, the properties could be attained which were not inherent in any of the individual structures [3]. The survey paper by Utkin [3] introduced this concept in the English literature. After this, the theory has been extended in various directions. The technique became popular because of its application to a wide class of systems containing discontinuous control elements such as relays. As per Oscar Camacho [4], Sliding Mode Control (SMC) is a simple procedure to synthesize controllers for linear and nonlinear processes. To develop a Sliding Mode Controller, SMC, knowledge of the process model relating the controlled variable, X(t), to the manipulated variable, U(t), is necessary. The controller can be designed for a wide range of level systems from a first order single tank system to higher order models consisting of four tanks. The application of SMC in single tank system is illustrated in the following sections. Unlike conventional first order model, the tank is modeled as a First Order Plus Delay Time (FOPDT) system [3]. This gives a better approximation and facilitate a perfect controller design. The Taylor series approximation is used in further reduction steps which adds to the ease of controller design. Implementation of SMC in single cylindrical tank system is demonstrated by S.Harivardhagini and Dr.A.D.Rajkumar in [5]. The journal deals with the level control of single tank (cylindrical) using Sliding Mode strategy and using LABVIEW as a user interface platform. Here the modeling of the tank is done considering mass flow equations and the control law is derived. The controller is designed for both static and dynamic types. The following sections reveal a new controller design based on FOPDT which is a good approximation compared to simple first order approximation.

II. SLIDING MODE CONTROLLER

Sliding Mode Control is a technique derived from Variable Structure Control (VSC) which was originally studied by [6]. The controller designed using the SMC method is particularly appealing due to its ability to deal with nonlinear systems and time-varying systems. The robustness to the uncertainties becomes an important aspect in designing any control system. The idea behind SMC is to define a surface along which the process can slide to its desired final value. Figure 1 depicts the SMC objective [7] where e(t) is the tracking error, that is, the difference between the reference value or set point, R(t), and the output measurement, X(t), or e(t) = R(t) - X(t): is a tuning parameter, which helps to define S(t) (sliding surface). The S(t) is an integral-differential equation acting on the tracking-error expression. Once the sliding surface has been selected, attention must be turned to design of the control law that drives the controlled variable to its reference value [8][9].
The SMC control law, $U(t)$, consists of two additive parts; a continuous part, $U_C(t)$, and a discontinuous part, $U_D(t)$ [3].

The continuous part is given by

$$U_C(t) = \int [X(t)R(t)]$$

where $[X(t)R(t)]$ is a function of the controlled variable, and the reference value. The discontinuous part, $U_D(t)$, incorporates a nonlinear element that includes the switching element of the control law. This part of the controller is discontinuous across the sliding surface and is given as

$$K_D \frac{S(t)}{S(t)} + \delta$$

where $K_D$ is the tuning parameter responsible for the reaching mode. $\delta$ is a tuning parameter used to reduce the chattering problem. Chattering is a high-frequency oscillation around the desired equilibrium point. It is undesirable in practice, because it involves high control activity and also can excite high frequency dynamics ignored in the modeling of the system. In summary, the control law usually results in a fast motion to bring the state onto the sliding surface, and a slower motion to proceed until a desired state is reached.

### III. SYSTEM MODELLING.

#### A. Cylindrical tank system

A cylindrical tank system whose level is to be controlled is first modeled. The actual plant is as shown in figure 2. The specifications of the system are as follows:

<table>
<thead>
<tr>
<th>INSTRUMENT USED</th>
<th>SPECIFICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylindrical Tank</td>
<td>Height: 35 cm, Dia: 10 cm, Vol: 10.995</td>
</tr>
<tr>
<td>Controller</td>
<td>UT 35 Digital Indicating Controller</td>
</tr>
<tr>
<td>I-P Converter</td>
<td>Input: 4-20 mA, Output: 3-15 psi</td>
</tr>
<tr>
<td>Rotameter</td>
<td>10-100 LPH</td>
</tr>
<tr>
<td>Reservoir Tank</td>
<td>Material: Stainless Steel, Capacity: 30 L</td>
</tr>
<tr>
<td>Level Transmitter</td>
<td>Capacitive Type, 2 wire, Output: 4-20 mA</td>
</tr>
<tr>
<td>Control Valve</td>
<td>Input: 3-15 psi, linear type</td>
</tr>
<tr>
<td>Pump</td>
<td>Submersible water pump 750 LPH</td>
</tr>
</tbody>
</table>

The schematic of the level control system is as shown in figure 3.
• Put the controller in manual mode.
• Wait until the process value reaches steady state or as close as possible (stable and not changing).
• Introduce a small disturbance (step the output of the PID controller) - The step must be big enough to see a significant change in the process value.
• Collect data and plot.
• Repeat: making the step in the opposite direction.

IV. SLIDING MODE CONTROLLER DESIGN

The next step is to design the sliding mode controller for the system modeled using process reaction curve method. The design illustrated by Oscar Camacho[3] is followed. It involves Taylor series approximation and further reduction so as to obtain the control law which is nothing but the controller output. For this the system of the form \( \frac{X(S)}{U(S)} = \frac{K}{(1 + \tau S)(1 + t_0 S)} \) (1)

Where \( X(S) \) is the plant output and \( U(S) \) is the controller output(which is the input to the plant)

This is written in differential equation form as

\[
t_0 \tau \frac{d^2 X(t)}{dt^2} + (t_0 + \tau) \frac{dX(t)}{dt} + X(t) = KU(t) \tag{2}
\]

Next, the sliding surface must be designed whose general equation is given as

\[
S(t) = \left( \frac{d}{dt} + \lambda \right)^n \int e(t) dt \tag{3}
\]

Where \( e(t) \) is the tracking error and \( n \) is the order of the system.Here the order of the system is 2 from equation (1).

By subsequent cross multiplication and substitution, the controller output is obtained as

\[
U_c(t) = \left( \frac{t_0 \tau}{K} \right) \left( \frac{t_0 + \tau}{t_0 \tau} - \lambda_1 \right) \frac{dX(t)}{dt} + \frac{X(t)}{t_0 \tau} + \lambda_0 e(t) \tag{4}
\]

This is the continuous part of the control law since the control law consists of both continuous and discontinuous part defined as

\[
U(t) = U_c(t) + U_d(t) \tag{5}
\]

The discontinuous part \( U_d(t) \) is given as

\[
K_D \frac{S(t)}{\left| S(t) \right| + \delta} \tag{6}
\]

Where \( \delta \) is the chattering suppression factor. Putting \( \lambda_1 = \frac{t_0 + \tau}{t_0 \tau} \) and combining equations (4),(5) and (6) the controller output is obtained as

\[
U(t) = \left( \frac{t_0 \tau}{K} \right) \left( \frac{X(t)}{t_0 \tau} + \lambda_0 e(t) \right) + K_D \frac{S(t)}{\left| S(t) \right| + \delta} \tag{7}
\]

which includes both continuous and discontinuous parts.

Hence for the proposed system with transfer function given as \( \frac{4.885e^{-1.03S}}{1 + 83.97S} \) the controller output is obtained as

\[
U(t) = (17.705) \left( \frac{X(t)}{86.4895} + 0.24146 e(t) \right) + 295.994 \frac{S(t)}{\left| S(t) \right| + 238.51} \tag{8}
\]

V. SIMULATION RESULTS

The simulations to obtain the step response of the system for PID and Sliding Mode Control are done in SIMULINK and the responses are compared as shown below.

The process reaction curve obtained from the real time model is as shown below.
The PID values obtained on tuning are $kp=1.962$, $ki=0.042$ and $kd=0.6299$. The step response for the PID controller is shown below.

Fig. 5. SIMULINK model for PID control

The PID values obtained on tuning are $kp=1.962$, $ki=0.042$ and $kd=0.6299$. The step response for the PID controller is shown below.

Fig. 6. Step response with PID control

It is seen that the step response with sliding mode controller is faster and settles quickly. Also the steady state error is also negligible compared to the conventional PID controller. The overshoot in the response with SMC is due to the discontinuous part in the control law (required for switching). The responses with SMC and PID controllers are compared as shown below.

Fig. 8. Step response with Sliding Mode Control

The time domain specifications are compared and tabulated.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>PID</th>
<th>SMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time (sec)</td>
<td>24.2</td>
<td>5.38</td>
</tr>
<tr>
<td>Settling Time (sec)</td>
<td>85</td>
<td>12</td>
</tr>
<tr>
<td>ISE</td>
<td>4.622</td>
<td>2.243</td>
</tr>
<tr>
<td>IAE</td>
<td>10.27</td>
<td>5.394</td>
</tr>
</tbody>
</table>

The response of the level controller is greatly improved with Sliding Mode Controller which is manifested as the appreciable reduction in rise time and settling time. Also the Integral Square Error (ISE) and Integral Absolute Error (IAE) were also reduced to half compared to PID controller based response. But the overshoot is comparatively higher with Sliding Mode Control action which is contributed by the discontinuous part of the control law. This can be reduced by the use of terminal sliding mode controller.
VI. CONCLUSION AND FUTURE WORK

The Sliding Mode Controller being a versatile controller strategy could improve the response of the proposed level control system and the FOPDT approximation could effectively realise the system compared to linear model. The flexibility of SMC becomes more evident when used with higher order systems. It also eliminates the need of a tuning procedure which is used for PID controller. The effect of SMC on non-linear systems should also be evaluated and this can be achieved by designing the controller for systems like conical tanks which are non-linear. The analysis can further be extended to terminal sliding mode controller than can surpass the overshoot and chattering problems associated with standard sliding mode controllers.

REFERENCES


