# Simultaneous Identification of Segmented Vascular True Vessels From Retinal Replica using Graph Tracer Method 

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#### Abstract

Identification of retinal blood vessels morphology reflects the heart disease like hypertension, coronary heart disease etc. The contingency of the retinal vessels helps to detect the cardiovascular condition of human body. The aim of this paper is to find the optimal path of the true vessels from the segmented vessel graph. The major challenges in this identification are crossover and bifurcation that leads to ambiguity when one vessel is tracked on a one-at-a-time basis. To avoid this, the vessel tracking, vessel segmentation and vessel identification are preformed simultaneously. Hence the complete vessel of the retina is determined.


Keywords- cardiovascular structure, ophthalmology, vessel segmentation, skeletonization

## I. Introduction

The snapshot of retinal replica shows the condition of human body. Appropriately, the contingency of retinal vessels helps to detect the cardiovascular condition. The structure and properties of the retinal vessels help in the diagnosis of heart diseases. For example, the correspondent retinal artery and retinal vein can be distinguished from the retinal replica by analysing the oxygen content in those retinal vessels. The computation is interrelated with the heart diseases like hypertension, coronary heart disease and stroke [1]-[3]. For this computation, specific vessels must be exactly tracked using the retinal replica. The major problems in this computation are the uncertainties due to crossover and vessel bifurcation.

Fig. 1 is an example of retinal replica where the retinal vessels I (denoted as red) and II (denoted as blue) crossover (denoted within white circles). Fig. 1 (a) shows both the vessel crossing with each other. The problem is that the crossovers are identified as bifurcations where both the vessels are treated as same vessel. Fig. 1 (b) shows the variation between the two vessels where the vessel structures are exactly identified denoted as blue and red.

To avoid the uncertainties during the identification of bifurcations and crossovers in the retinal vessels like tracking the adjacent vessel segment of one vessel segment as its own segment, the linking of these vessel segments should be properly identified. Fig. 2 is an example; here the bifurcation of vessel I is denoted as red dots and bifurcation of vessel II are
denoted as blue dots. Fig. 2 (a) shows the crossovers of adjacent vessel segment of one vessel is identified as bifurcations of its own vessel segment which are denoted as red dots within a circle. This wrong identification is because the vessel I is tracked without the recognition of vessel II, where the red dots within the circle are considered as bifurcations rather crossovers. Therefore, it may result in a wide range of difference in the measurement of the vessels. Fig. 2 (b) shows the correct identification of bifurcation of vessel I and II is achieved by simultaneous identification. This results in correct identification because the data about the other vessels can be used during the time of identification of one vessel.

Here, the data of the retinal vascular structure which is segmented is used so that all the true vessels from the retinal replica are identified correct. Initially, the segmented vascular structure is represented as a graph and the true vessels from the retinal replica are identified on the basis of identification of the optimal path from a graph.

(a)

(b)

Fig 1. From vessel I and II crossovers are denoted within the white circles. (a) Wrong vessel identification (b) Correct vessel identification.

(a)

(b)

Fig. 2 (a) crossovers are identified as bifurcations (b) Correct identification of bifurcation.

## II. RELATED WORK

The vascular structure segmentation and the distinct vessels are found by joining the various segments of the vascular structure which shows the complete vessel is included in the extraction of retinal vessel. The vessel tracking refers to the performance of vessel identification and segmentation at the same time [5]-[8]. For vessel tracking the initial point of the vessel should be determined prior, where each of the vessels is tracked by finding the immediate next vessel point repeatedly with a function that takes the intensity of each pixels and the orientation in the vicinity of that particular point in the image which is currently considered. With the help of the intensity details, the crossovers and bifurcations are determined. When the vessel tracking is performed on a one at a time basis there occurs a lot of ambiguities since this approach does not provide the required information.

(a)

(b)

Fig 3. (a) Zone of Interest and structure of vessel. (b) Segmented Line Image.
The vessel identification can also be treated as a post processing step to vessel segmentation [9]-[11]. Before the vessels where individually identified the user had to resolve the connectivity of crossover and bifurcation as that of in [9]. The central vein is identified using a graph formulation [10] along with Dijkstra's shortest path algorithm.

Joshi et al. [11] identified the vessel on a one at a time basis and measured their method on a set of around 15 images. These methods resulted in wrong vessel identification because of selecting the right vessel segment which connects at either crossover or bifurcation that requires details from the other vessel. Al-Diri et al. [12] identified the crossovers and the segments that are locally joined at the crossovers to give a vascular network used some expert rules.

This work aims on identification of vessel which is done as a post processing step to vessel segmentation. Here the multiple vessel are identified simultaneously which uses the information of the structure to identify if joining of one vessel segment to another lead to wrong identification.


Fig 4. Example of Junctions

## III. PRELIMINARIES

The zone of interest is defined in the retinal replica. The circular ring is bounded by another two circles of radii $2 r$ and $5 r$ [Fig. 3(a)], Where $r$ is the optic disc (OD) radius. A number of clinical studies are used for measurement from this zone [3],[4]. The vessel identification is started from the pixel which is nearer to the circle with radius $2 r$. The root pixels are denoted in yellow in Fig. 3(a).

The method of existing vessel segmentation is applied to the procedure of skeletonization [9], [13], so that the lined image is obtained in the zone of interest [Fig. 3(b)]. The line image shows the vessel structure connectivity.

Let A be all white pixels of the line image where the two pixels $a_{i}, a_{j} \in A$ are adjacent $\operatorname{adj}\left(a_{i}, a_{j}\right)$, iff $a_{j} \in n 8\left(a_{i}\right)$, where $n 8(a)=\{a 1, a 2, a 3, a 4, a 5, a 6, a 7, a 8\}$ is the eight neighbourhood pixels of a [Fig. 4(a)].

Pixels $a_{i}, a_{j} \in A$ are connected where connected $\left(a_{i}, a_{j}\right)$, if $\operatorname{adj}\left(a_{i}, a_{j}\right)$ or $a_{c} \in A-\left\{a_{i}, a_{j}\right\}$ which refers to the connected pixels.

The sequence $\{a 1, a 2, a 3, a 4, a 5, a 6, a 7, a 8\}$ is considered to be the clockwise sequence of the eight neighbourhood pixels of the pixel $A$. And $x(p)$ is considered to be the transitions from black to nonblack in the above sequence of the neighbourhood pixels.


Fig 5. Segment pixels.
Consider $w 8(a) \subseteq n 8(p)$ is the white pixels neighbouring a. The junction pixels in $A$ is $Y_{a}=\{a \in A / x n(a)>2 v / w 8(a) /$ $>3\}$. The junction is the set of all connected pixels where $j u n$ $\subseteq Y_{a}$ such that $\forall a_{i}, a_{j \neq i} \in j u n$, connected $\left(a_{i}, a_{j}\right)$, where connected is restricted to $Y_{p}$. Then, all the junctions in $A$ is $j u n_{p}$.

Fig. 4 is an example of the junction pixels. Fig. 4(a), w8(a) $=\{a 2, a 4, a 6\}$ and $x n(a)=3$ is because of the transitions (al.a2), (a3, a4) and (a5, a6). This is a case where the junction pixel is the shaded pixel a with $x n(p)>2$. In Fig. $4(b)$, the crossing numbers of 2 is in the top an left neighbours belonging to the shaded pixel.

From Fig. 4(c), the junction pixels are all the shaded pixels since each of the pixels have atleast more than 3 white pixels in their 8-neighbourhood.

The sequence of unique white pixels $\{a 1, \ldots \ldots, a n\}$ in $A$ is the segment seg such that it gets satisfied with the folloving conditions true:

1. $n>0$ and $\forall i \in[1, n], a_{i} \in J_{P}$
2. $n>1 \Rightarrow \forall i \in[1, n-1], \operatorname{adj}\left(a_{i,}, a_{i+1}\right)$ 3. $\forall I \in\{1, n\}, /=1 \vee \exists a_{j} \in J_{a}$
3. $n>2 \Rightarrow \forall i \in[2, n-1], x n\left(a_{i}\right)=2$.

The pixels $a_{l}$ and $a_{n}$ are the end pixels of the seg. Consider $S E G_{a}$ as the set of all segments in $A$ and $N_{a}=A-Y_{a}$ where $N_{a}$ is the nonjunction pixels which are the part of segments. seg $\epsilon$ $S E G_{a}$ is adjacent to the junction jun where adj (seg, jun), if $\exists$ $a_{j} \in j u n$ s.t. $\operatorname{adj}\left(a_{j}, a_{l}\right) \vee \operatorname{adj}\left(a_{j}, a_{n}\right)$. The two segments sega, $\operatorname{seg}_{b} \in S E G_{P}$ are adjacent, $\operatorname{adj}\left(\operatorname{seg}_{a}, \operatorname{seg}_{b}\right)$ if $\exists j u n \in J_{p}$ s.t. $\operatorname{adj}\left(\operatorname{seg}_{a}, \operatorname{seg}_{b}\right){ }^{\wedge} \operatorname{adj}\left(\operatorname{seg}_{b}, J\right)$.

The examples of segments, end pixels and junction pixels according to junction and segment for any region in the line image is shown in Fig. 5. The segments are referred by the connected white pixels.

## IV. VESSEL TRACKING

Here the proposed method focuses on identification of true vessels in the retina and to represent those vessels in binary tree form in order to get the measurements of subsequent vessels. This is done in two steps: A) Correct identification of crossovers B) Determination of optimal forest.

## A) Correct Identification of Crossovers

Generally, there are chances for the vessels to cross each other in many cases, at a particular point or at a shared segments. The crossover which are occurring at particular point are known to be former crossover points and the other is the latter crossover points.

Junction point of Crossover: There are some set of pixels A in white in a line image, and a junction jun $\epsilon J_{a}$ is said to be a crossover point iff the number of segments which are adjacent to C is almost greater than or equal to 4,i.e., cover(jun) will be true iff $\left|\left\{\operatorname{seg} \in S E G_{a} \mid \operatorname{adj}(\operatorname{seg}, J)\right\}\right| \mathrm{b} \geq 4$.

When two different vessels takes the same segment then it leads to crossover segment as indicated in Fig. 6(a). The set of pixels in white A of a line image, a single part i.e., segment $|\operatorname{seg}| E S E G_{a}$ is said to be candidate crossover segment when


Fig 6. Example of crossover segment.
$|\operatorname{seg}|<\operatorname{Lim}$ and $\exists J_{1}, J_{2} \in J_{a}$ s.t. $\operatorname{adj}\left(\operatorname{seg}, J_{1}\right)^{\wedge} \operatorname{adj}\left(\right.$ seg, $\left.J_{2}\right)$ $\wedge \neg \operatorname{cover}\left(J_{1}\right)^{\wedge} \neg \operatorname{cover}\left(J_{2}\right)$. L is the parameter to limit candidates to short segments.
The short segments between two segments are not necessary true crossover segments. So there is a directional change
between adjacent segments and their intensity values of pixels has been proposed to distinguish crossover segments.

Changing of Directions among the segments: Given two segments $\operatorname{seg} g_{a}$ and $\operatorname{seg} g_{b}$ that are very $a_{a}$ adjacent to a common junction. Let $a_{a}$ and $a_{b}$ be the end points of $\operatorname{seg}_{a}$ and $\operatorname{seg}_{b}$ that are closed to each other. Let $v e c_{a}$ be a vector that starts on $\operatorname{seg} g_{a}$ and ends on $a_{a}$ and $v e c_{b}$ be a vector that starts from $a_{b}$ and end on $a_{b}$. Then, the directional change between $\operatorname{seg}_{a}$ and $\operatorname{seg}_{b}$ is produced by

$$
\Delta D(s a, s b)=\cos -1(v a \cdot v b \nmid v a \| v b \mid)
$$

where $\Delta D\left(\operatorname{seg}_{a}, \operatorname{seg}_{b}\right) \in\left[0^{\circ}, 180^{\circ}\right]$.
Initially, $\Delta D\left(\operatorname{seg}_{a}\right.$, $\left.^{\text {seg }}{ }_{b}\right)$ determines the magnitude of a change in the vessel direction when we go from $\operatorname{seg}_{a}$ to $\operatorname{seg}_{b}$. Thus the details about the angles which representing change of direction among the various segments are considered.

Crossover Segment : Given a candidate segment segment between two junctions jun ${ }_{1}$ and $j u n_{2}$, let $S E G i=\left\{\right.$ seg $_{a} \epsilon$ $S E G_{P} \mid a d j\left(s_{a}, j u n_{i}\right)^{\wedge}$ seg $_{a}=$ segment $\}$ for $i \in\{1,2\}$. Each $S E G_{i}$ having two segments taking the same junction as one end pixel of segment. Let $A=\{\mathrm{seg}\} \cup S_{1} \cup S_{2}$ and $\Phi=\left\{\left\{\mathrm{seg}_{a}\right.\right.$, segment, seg $\left._{b}\right\rangle \mid s_{a} \in S E G_{1}$, seg $\left._{b} \in S E G_{2}\right\}$. Then segment is a crossover segment, i.e., cover (segment) is true, when the below conditions are achieved:


Fig. 7. Segment 3 is a crossover segment, when $4^{\text {th }}$ segment is not a crossover segment, because it does not satisfy the condition 1 .

1) $\forall$ seg, seg $^{\prime} \in \operatorname{SEG} G_{i}, i \in\{1,2\}, \Delta D\left(\right.$ seg, seg $\left.{ }^{\prime}\right)>30^{\circ}$
2) $\mid$ segment $\mid \leq \operatorname{Lim}_{\theta} \Rightarrow$

$$
\begin{gathered}
{\left[\exists \operatorname{seg}_{a}, \operatorname{seg}_{b} \in S E G_{1}, \operatorname{seg}_{c}, \operatorname{seg}_{d} \in S E G 2,\right.} \\
\text { s.t. } \left.\Delta D\left(\operatorname{seg}_{a}, \operatorname{seg}_{c}\right)<30^{\circ} \wedge \Delta D\left(\operatorname{seg}_{b}, \operatorname{seg}_{d}\right)<30^{\circ}\right] \\
\vee \min _{\varphi \in \Phi}[\operatorname{sd}(M(\varphi))+\operatorname{sd}(M(A 1-\varphi))]<\operatorname{sd}(M(A 1))
\end{gathered}
$$

3) $\mid \mathrm{seg} />L_{\theta} \Rightarrow$
$\left[\forall\right.$ seg $\in S E G_{1} \cup \operatorname{SEG}_{2}, \Delta D($ segment, seg $\left.)<\theta_{\text {low }}\right]$
V[Vseg $\in S E G 1 \cup S E G 2, \Delta D($ segment, seg $)<\theta$ high
$\left.\Lambda \min _{\varphi \in \varphi}[s d(M(\varphi))+s d(M(A-\varphi))]<s d(M(A))\right]$
Here $\mu($ seg $)$ is used as the mean intensity of the pixels in segment ses, and the bag $M(S E G)=\{\mu($ seg $) \mid$ seg $\epsilon S E G\}$ for the given a set of segments $S E G$, and sdv is the standard deviation of the numbers in $M(S E G)$.
Condition 1: It is used when segment is at a bifurcation. For example, segment 4 in Fig. 7 is not a cross over segment due to the small directional change between segments 1 and 5 .
Condition 2: It is used when the length of segment is too short to find the change in direction. In this situation, we check if
the adjacent segments of segment forms a reasonable cross pattern, i.e., if there occurs some pairing of the segments in $S E G 1$ with those in $S E G 2$ such that their directional change are less than $30^{\circ}$. Or, we partition A1 into two such that the sum of the $s d v$ of both partitions is minimum. If this minimum is less than the $s d v$ of all the segments in A1, then segment is a cross over segment.

Condition 3: It says that if the length of segment is long enough and the directional change between segment and each of its adjacent segment is less than $\theta_{\text {low }}$, then segment is a crossover segment. Otherwise, if directional change is less than $\theta_{\text {high }}$, we compare the $s d v$ of the segments' intensity values as in Condition 2.
See that $\theta_{\text {low }}$ and $\theta_{\text {high }}$ can be determined empirically. For our experiments, we set $\theta_{\text {low }}=65^{\circ}$ and $\theta_{\text {high }}=85^{\circ}$.

## B) Determination of Optimal Forest

The optimization constraint to find the appropriate vessel tees from the formed graph which is used in the segment graph.
$P$ is the set of white pixels in the line image, a segment graph $G_{p}=\left(S E G_{a}, E D G E_{a}\right)$, where each of the vertex in $S E G_{P}$ is considered to be a segment and edge $e d g e_{i, j}=\left(\operatorname{seg}_{i}, \operatorname{seg}_{j}\right) \in$ $E D G E_{P}$ is present if $\operatorname{adj}\left(\operatorname{seg}_{i}, s e g_{j}\right), \operatorname{seg}_{i}, \operatorname{seg}_{j} \in S E G_{P}, i \neq j$.

Generally, $G_{P}$ includes the sub graphs which are disconnected and independent but can be processed parallel. The aim is to obtain binary trees from the segment graph where binary tree matches to a vessel in the retinal replica.


Fig 8. Crossover segments are identified and indicated in arrows.
The segment graph $G_{P}=\left(S E G_{a}, E D G E_{a}\right)$, each vessel is said to be a binary tree, $B T=\left(\operatorname{seg}_{\text {root }}, V E R T E X_{B T}, E D G E_{B T}\right)$ such that $\operatorname{seg}_{\text {root }}$ is root node, $\operatorname{root}(B T)=\operatorname{seg}_{\text {root }}, V E R T E X_{B T} \subseteq$ $S E G_{B T}$ and $E D G E_{B T} \subseteq E D G E_{a}$. This set of all binary trees are referred to as a forest.

The blood vessels are represented in a binary tree as it only bifurcates whereas the root pixels are the identified end points of the segment which is nearer to the inner circle of the zone of interest. The root segment that contains a unique pixel corresponds to root of each tree. Fig 9. depicts the segment graph and binary trees which corresponds to the vessels. The focus of simultaneous identification is formulate as a constraint optimization problem (COP).

The segment graph $G_{P}=\left(S E G_{P}, E D G E_{P}\right)$, and a set of root segments $S E G_{\text {root }}$, forest $_{P}$ is the set of possible forests from $G_{P}$
for each root segment in $S E G_{\text {root }}$. Optimal forest $F^{*} \in$ forest $_{\text {a }}$, which corresponds to the vessels in $G_{P}$ is given by
$F^{*}=\operatorname{argmin} \operatorname{cost}(F)$
${ }^{F} \in$ forest $_{p}$
Some constraints are considered for the optimal forest which are as follows:

1) Unique roots to each tree
$\forall B T_{1} \in F, \forall B T_{2} \in F-\left\{B T_{1}\right\} . \operatorname{Root}\left(B T_{2}\right) \notin V E R T E X_{B T I}$
2) The change of direction between the parent and child segments within the threshold
$\forall B T \in F .\left(\operatorname{seg}_{\text {a }} \operatorname{seg}_{c}\right) \in E D G E_{B T},\left[\operatorname{seg}_{\text {a }} \operatorname{Lim}_{\theta} \wedge \operatorname{seg}_{c}>\operatorname{Lim}_{\theta}\right]$
$\Rightarrow \Delta D\left(\operatorname{seg}_{a}, \operatorname{seg}_{c}\right)<135$.
3) The segment that is appearing in more than one segment is considered to be a crossover segment.
$\forall$ seg $\in S E G_{P} /\left\{B T \in F,\left(\right.\right.$ seg, $\left.\in V E R T E X_{B T}\right\} />1$ $\Rightarrow$ cover (seg).
4) With a minimum directional change, a parent segment at crossover junction should be connected to a child.
$\forall B T \in F,\left(\operatorname{seg}_{a}, \operatorname{seg}_{c}\right) \in E D G E_{B T}, \exists J \in j u n_{P}$ s.t. cover $(J) \wedge$ $\operatorname{adj}\left(\operatorname{seg}_{a}, J\right) \wedge \operatorname{adj}\left(\operatorname{seg}_{c}, J\right) \Rightarrow \operatorname{child}\left(B T . \operatorname{seg}_{a}\right)=1$ ${ }^{\wedge} \operatorname{seg}_{c}=\operatorname{argmin} \Delta D\left(\operatorname{seg}_{a}, \operatorname{seg}\right)$
where $z=\left\{\operatorname{seg} \in S E G_{a}-\left\{\operatorname{seg}_{a}\right\} / \operatorname{adj}(\operatorname{seg}, J)\right\}$.


Fig 9. (a) Example of the segment graph related to the segments (b)Example of forest of binary trees related to two vessels rooted at the segments.
5) The crossover is the only child that have one child with minimum directional change.
$\forall B T \in F,\left(\operatorname{seg}_{a}, \operatorname{seg}_{x}\right),\left(\operatorname{seg}_{x}, \operatorname{seg}_{c}\right) \in E D G E_{B T}, \operatorname{cover}\left(\operatorname{seg}_{x}\right)$ $\Rightarrow \operatorname{child}\left(B T, \operatorname{seg}_{a}\right)=1 \wedge \operatorname{seg}_{c}=\operatorname{argmin} \Delta D\left(\operatorname{seg}_{a}, \operatorname{seg}\right)$

$$
\operatorname{seg}_{\epsilon}^{L}
$$

where $L=\left\{\operatorname{seg} /\left(\operatorname{seg}_{x}, \operatorname{seg}\right) \in E D G E_{A} \wedge \operatorname{seg} \neq \operatorname{seg}_{a} \wedge\right.$ $\neg \operatorname{adj}\left(\operatorname{seg}^{\prime}, \operatorname{seg}_{a}\right)$ ).
6) The crossover segments cannot be the leaf segments
$\forall B T \in F, \operatorname{seg} \in V E R T E X_{B T}, \operatorname{leaf}(B T, \operatorname{seg}) \Rightarrow \neg \operatorname{cover}(\operatorname{seg})$.
Each of the binary tree $B T \in F$ must not contain any cycles and a particular segment should not appear more than once in a vessel. For any vessel BT, the set of bifurcation is,
$Y_{B T}=\left\{\left(\operatorname{seg}_{y}, \operatorname{seg}_{1}, \operatorname{seg}_{2}\right) / \operatorname{seg}_{y}, \operatorname{seg}_{1}, \operatorname{seg}_{2} \in \operatorname{VERTEX} X_{B T}\right.$ ^
$\left.\left(\operatorname{seg}_{y}, \operatorname{seg}_{1}\right),\left(\operatorname{seg}_{y}, \operatorname{seg}_{2}\right) \in E D G E_{T}\right\}$.
The set of single parent-child nodes in $B T$ is,
$I_{B T}=\left\{\left(\operatorname{seg}_{a}, \operatorname{seg}_{c}\right) / \operatorname{seg}_{a}, \operatorname{seg}_{c} \in \operatorname{VERTEX} X_{B T} \wedge\right.$ child $\left(\operatorname{seg}_{a}\right)=$
$1^{\wedge}\left[\left(\neg \operatorname{cover}\left(\operatorname{seg}_{p}\right)^{\wedge} \neg \operatorname{cover}\left(\operatorname{seg}_{c}\right)^{\wedge}\left(\operatorname{seg}_{a}, \operatorname{seg}_{c}\right) \in E D G E_{B T}\right) V\right.$
$\left(\exists\left(\operatorname{seg}_{a}, \operatorname{seg}_{m}\right),\left(\operatorname{seg}_{m}, \operatorname{seg}_{c}\right) \in E D G E_{B T}\right.$ s.t. $\operatorname{cover}\left(\operatorname{seg}_{m}\right) \wedge$ child $\left.\left.\left.\left(\operatorname{seg}_{m}\right)=1\right)\right]\right\}$.

The following functions are defined with $\mathrm{Y}_{\mathrm{BT}}$ and $\mathrm{I}_{\mathrm{BT}}$ :

$$
\begin{aligned}
& \Gamma_{Y}(B T)=\sum_{(\text {segy, } \text { segl, seg } 2) 0.5[\Delta B T)}\left[\Delta D\left(\operatorname{seg}_{y}, \operatorname{seg}_{2}\right)+\Delta D\left(\operatorname{seg}_{y}, \operatorname{seg}_{1}\right)\right] \\
& \Gamma_{l}(B T)=\sum_{(\text {sega, segc })} \Delta D\left(\operatorname{seg}_{a}, \operatorname{seg}_{c}\right) .
\end{aligned}
$$

$\Gamma_{Y}(B T)$ adds the average of the parent-child directional changes at bifurcations in $B T$ and $\Delta D$ are considered as the child segments. $\Gamma_{1}(B T)$ adds the directional change between the parents where the tree has only one child segment. This helps to know the minimum directional change while the segments that connect at junctions is chosen. The cost function of the tree forests is defined as

$$
C f(F)=\sum_{J_{\in F}}\left[\Gamma_{1}(B T)+\Gamma_{y}(B T)\right]
$$

In many ways COP problems are differ from other graph problems based on their characteristics. Initially we have to assign the cost on forests for allowing weighting and fusing of multiple cost criteria instead of defining the cost on edges as in minimum spanning forest (MSF) [14], [15] and minimum spanning tree (MST). Then we have to find a forest from a connected graph, during MSF finds a tree in each of the graph [16]. After that we have to indicate that a vessel tree does not allow us to use the weighted-SAT formulation in [13] as it may produce broken vessels.

Candidate enumeration algorithm is used to solve the COP graph problems by utilizing the lower bound of the cost function to save the search space. The lower bound $L B_{\text {cost }}(F)$ is based on the below theorem.
Lower bound of cost theorem: For a given set of binary trees F and any vessel $B T \in F$, we build the vessel $B T^{\prime}$ by increasing one left node of $B T$, so it will have either one or two children. Let assume $F^{\prime}=F-\{B T\} \cup\left\{B T^{\prime}\right\}$. Then, $c f(F)$ $\leq c f\left(F^{\prime}\right)$, i.e., $c f(F)$ is the lower bound cost of any $F^{\prime}$ obtaining result from growing the vessels in $F$.

## Proof:

We increase the size of $I_{B T}$ by one, $Y_{B T}$ by one, or neither, but not both, this is done by adding new children to a leaf node. As $\Delta D$ has the codomain $\left[0^{\circ}, 180^{\circ}\right], \Gamma_{l}(B T) \leq \Gamma_{l}\left(b t^{\prime}\right) \wedge$ $\Gamma_{Y}(B T) \leq \Gamma_{Y}\left(B T^{\prime}\right)$. There by we got $c f(F) \leq c f\left(F^{\prime}\right)$. Fig. 10 states the details of the proposed tracing algorithm, GraphTracer.

The input is the segment graph here is $G_{P}$ with n root segments that are given in $S E G_{\text {root. }}$ Lines 1-6 initialize the global variables and call the recursive procedure Trace. $F[1 . . n]$ respects to the initial forest of $n$ vessels. $R[1 . . n]$ denotes a fringe stack for each of the vessel. $F_{\text {min }}$ and $c_{\text {min }}$ saves the minimum cost forest and their costs.

We have to update $F_{\text {min }}$ if $c f(F)<c_{\text {min }}$ (Line 7) when F satisfies all the constraints and cannot be grown further in Trace method. Otherwise we may have to prune descendant forests grown from F with the corresponding lower bound
$L B_{\text {cost }}(F)$ (at Line 9). In line 10 , the outer loop is ordering each vessel $B T \in[1, n]$ for further growth. $B T$ ranges from the processing index $i$ to $n$, tells that Trace does not enumerate additional duplicate forests.

The fringe stack of the each vessel, $R[B T]$, saves its current leaf nodes to be grown next. $R[B T]$ is used in the conjunction along with the loop at Line 11 to enumerate those vessels in a depth in a depth-first traversal order.

Here a subfunction FindChildren returns back the vessel paires ( $\mathrm{seg}_{\mathrm{l}}, \mathrm{seg}_{r}$ ) of possible children for the current fringe node $\operatorname{seg}_{B T}$. We set the value $\operatorname{seg}_{r}=\theta$. FindChildren is forward checking to avoid children pairs that cause constraints (1) (6) in the formulation of COP.

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Algorithm of Tracer:
Input : \(\mathrm{G}_{\mathrm{p}}=\left(\mathrm{SEG}_{2}, \mathrm{EDGE}_{3}\right), \mathrm{SEG}_{\text {rost }}\)
Output: \(\mathrm{F}_{\text {min }}\)
1. \(\mathrm{C} \leftarrow\) set of constraints (i) to (yi) in COP formulation.
2. \(\mathrm{F}[1 \ldots \mathrm{n}] \leftarrow<\) vessels of root nodes in \(\mathrm{SEG}_{\text {rose }}>\)
3. \(\mathrm{R}[1 \ldots \mathrm{n}] \leftarrow<\operatorname{root}(\mathrm{F}[\mathrm{BT}])>\) for \(\mathrm{BT} e[1 \mathrm{n}]\)
4. \(\mathrm{C}_{\min } \leftarrow \infty ; \mathrm{F}_{\min } \leftarrow \mathrm{F}[]\)
5. Trace (1,F,R)
6. retum \(F_{\text {min }}\)
    Procedure Trace (i, F, R)
7. if \(\operatorname{cost}(\mathrm{F})<\mathrm{c}_{\text {min }}\)
8. \(\mathrm{c}_{\min } \leftarrow \operatorname{cost}(\mathrm{F}) ; \mathrm{F}_{\min } \leftarrow \mathrm{F}\)
9. else if \(\operatorname{Lim} B_{\cos }(\mathrm{F}) \leq \mathrm{c}_{\min }\) then
10. for \(\mathrm{BT}=\mathrm{I}\) to n do
11. while \(\mathrm{R}[\mathrm{BT}] \neq 0\) do
12. segst \(\leftarrow \operatorname{pop}(R[B T])\)
13:\# valid children of \(\mathrm{s}_{\mathrm{T}}\) using forward checking
\(14 . \mathrm{N} \leftarrow\) Find children ( F , segT)
15. for each (seg segs) \(\in \mathrm{N}\) do
16. left \(\left(\operatorname{seg}_{T}\right) \leftarrow\) seg; ; ight \((\) seg.st \(\left.) \leftarrow \operatorname{seg}_{\sigma}\right)\) )
17. push(R [BT], segi)
18. if \(\operatorname{seg}_{g}=0\) then
19. Trace (BT,F, R)
20.else
21. push(R [BT], segr)
22. Trace (BT, F, R)
23. \(\operatorname{Pop}(\mathrm{R}[\mathrm{BT}])\)
24. endif
25. Pop(R [BT])
26. left \(\left(\right.\) seg \(\left._{5 T}\right) \leftarrow 0\); right \(\left(\right.\) seg \(\left._{5 T}\right) \leftarrow 0\)
27. end for
28. end while
29. end for
30. end if
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Fig 10. Graph Tracer Algorithm Details.
The time complexity of GraphTracer is exponential for the number of edges in $G_{a}$ and it is independent of the size of the retinal image as it deals only with the connectivity of the entire vessel segments without measuring pixel properties like intensity. In practical, FindChildren avoids so many combinations with help of the constraints presented.

## VI.EXPERIMENTAL RESULTS

The proposed method is measured from the retinal replica that is obtained from the ophthalmologist. The image is initially segmented where this process is considered to be the preprocessing step and the actual process of the proposed
method is considered to be the post processing step to segmentation.

We figure out our proposed method on retinal replicas in MATLAB (MATrix LABoratory) version - 7.8. Initially the segmented retinal replica is converted to a grey scale image to increase the intensity of the vessel region in the retinal replica. After the conversion of the image tracing of the vessel region is performed.

There are two tracing methods such as solo tracer and graph tracer method. Graph tracer method is considered to be more robust when compared with that of the solo tracer method. The graph tracer method is implemented here where the vessel tracking is performed in an accurate manner and the tracking of the vessels are completely done.

The converted image is identified with the presence of the vessel region. The retinal replica is divided into $5 \times 5$ matrix and checked for the connectivity of the pixels with each other so that the vessel region is identified accurately.

All tracers are given the sane line image, artery/vein labeling and use same method to compute the vessel diameters. We can calculate on both clean and noisy line replica. Noisy line replica is obtained using the vessel segmentation.
The tracer starts with the tracking process and continue in the vessel region. When there is any ambiguity during the tracking, a constraint is considered where the angle of the vessel region is identified and further tracking is performed for the vessel region with angle less than $30^{\circ}$.

Once the tracking of vessels is completed. Then the identification of crossover and bifurcation is performed where the optimal path of the vessel is identified and displayed.

Hence our proposed system concentrates on complete vessel identification, tracking of vessel simultaneously, identification of crossover and bifurcation. During the time of tracking a split of vessel is seen, which is the the directional change of segments. When split is not seen as crossover then it is fit as bifurcation and the tracer will follow the paths. From these results, we conclude that tracing all vessels simultaneously is better than tracing vessels individually without any knowledge of other vessels.

## VII.CONCLUSION

The technique to identify the true vessels from retinal images is presented. Here the identification of vessels is done accurately and completely that are reliable vascular morphology measurements for clinical assessment. This method is the post processing step to segmentation. The vascular structure of the retina is modelled as finding the optimal vessel forest from a graph with constraints on each vessel trees so that the correct identification and tracking of the retinal vessels are done. Every vessel trees are considered during the identification of the optimal forest and therefore this approach is aware of wrong linking of vessels. This method helps to make the clinical assessments even easier.

## REFERENCES

[1] C. Y.-L. Cheung, Y. Zheng,W. Hsu, M. L. Lee, Q. P. Lau, P. Mitchell, J. J. Wang, R. Klein, and T. Y. Wong, "Retinal vascular tortuosity, blood pressure, and cardiovascular risk factors," vol. 118, 2011.
[2] W. E. Hart, M. Goldbaum, P. Kube, and M. R. Nelson, "Automated measurement of retinal vascular tortuosity," in Proc. AMIA Fall Conf., 1997,pp. 459-463.
[3] T. Y. Wong, F. M. A. Islam, R. Klein, B. E. K. Klein, M. F. Cotch, C. Castro,A. R. Sharrett, and E. Shahar, "Retinal vascular caliber, cardiovascular risk factors, and inflammation: The multi-ethnic study of atherosclerosis (MESA)," vol. 47, 2006.
[4] Y.Yin, M.Adel, M. Guillaume, and S. Bourennane, "A probabilistic based method for tracking vessels in retinal images,". 2010.
[5] K. Mc Geechan, G. Liew, P. Macaskill, L. Irwig, R. Klein, B. E. K. Klein, J. J. Wang, P. Mitchell, J. R. Vingerling, P. T. V. M. Dejong, J. C. M. Witteman,M.M. B. Breteler, J. Shaw, P. Zimmet, and T. Y.Wong, "Metaanalysis: Retinal vessel caliber and risk for coronary heart disease," vol. 151, 2009.
[6] E. Grisan, A. Pesce, A. Giani, M. Foracchia, and A. Ruggeri, "A new tracking system for the robust extraction of retinal vessel structure," Sep. 2004.
[7] A. W. P. Foong, S.-M. Saw, J.-L. Loo, S. Shen, S.-C. Loon, M. Rosman, T. Aung, D. T. H. Tan, E. S. Tai, and T.Y.Wong, "Rationale and methodology for a population-based study of eye diseases in Malay people: SiMES," vol. 14, 2007.
[8] V. S. Joshi, M. K. Garvin, J. M. Reinhardt, and M. D. Abramoff, "Automated method for the identification and analysis of vascular tree structures in retinal vessel network," vol. 7963, 2011.
[9] B. Al-Diri, A. Hunter,D. Steel, andM.Habib, "Automated analysis of retinal vascular network connectivity," vol. 34, 2010.
[10] S. Garg, J. Sivaswamy, and S. Chandra, "Unsupervised curvature-based retinal vessel segmentation,". 2007.
[11] K.Rothaas,X.Jiang,And P.Rhiem,X.Jiang and P.Rhienm,"'Separation of the retinal vascular graph in arteries and veins based upon structural knowledge." vol.27,2009.
[12] C. Y.-L. Cheung, W. Hsu, M. L. Lee, J. J. Wang, P. Mitchell, Q. P. Lau, H. Hamzah, M. Ho, and T. Y. Wong, "A new method to measure peripheral retinal vascular caliber over an extended area," Microcirculation, vol. 17, 2010.
[13] C. Y.-L. Cheung, W. T. Tay, P. Mitchell, J. J. Wang, W. Hsu, M. L. Lee, Q. P. Lau, A. L. Zhu, R. Klein, S. M. Saw, and T. Y. Wong, "Quantitative and qualitative retinal micro vascular characteristics and blood pressure," J. Hypertens, vol. 29, 2011.
[14] J. Cousty, G. Bertrand, L. Najman, and M. Couprie, "Watershed cuts: Minimum spanning forests and the drop of water principle," IEEE Trans. Pattern Anal. Mach. Intell., vol. 31. 2009.
[15] Y. Tolias and S. Panas, "A fuzzy vessel tracking algorithm for retinal images based on fuzzy clustering," IEEE Trans. Med. Imag., vol. 17, no. 2. 1998.
[16] C. All'ene, J.-Y. Audibert, M. Couprie, and R. Keriven, "Some links between extremum spanning forests, watersheds and min-cuts," Imag. Vis. Comput., vol. 28, 2010.
[17] H. Li, W. Hsu, M. L. Lee, and T. Y. Wong, "Automatic grading of retinal vessel caliber," IEEE Trans. Biomed. Eng., vol. 52. 2005.
[18] R. L. Graham and P. Hell, "On the history of the minimum spanning tree problem," IEEE Ann. Hist. Comput., vol. 7. 1985.
[19] M. Martinez-Perez, A. Highes, A. Stanton, S. Thorn, N. Chapman, A. Bharath, and K. Parker, "Retinal vascular tree morphology: A semiautomatic quantification," IEEE Trans. Biomed. Eng., vol. 49. 2002.
[20] H. Azegrouz and E. Trucco, "Max-min central vein detection in retinal fundus images," in Proc. IEEE Int. Conf. Image Process.. 2006,

