Significant Characters of Fuzzy Soft Matrices

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***Abstract*-The reference function will play an important role in the definition of fuzzy soft matrices. In this paper, we intend to deal union, intersection, complement, addition, multiplication and trace of fuzzy soft matrices and its character by using reference function with example.**

***Keywords*-Soft set, Fuzzy soft set(FSS), Fuzzy soft matrices(FSM), Union, Intersection, Complement, Addition, Multiplication and Trace of fuzzy soft matrices.**

I. INTRODUCTION

In 1965, Zadeh [17] introduced the notion of fuzzy set theory. In 1999, soft set theory was firstly introduced by Molodtsov [7] as a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. In 2001, Maji, Biswas and Roy [5] studied the theory of soft sets initiated by Molodtsov [7] and developed several basic notions of soft set theory. In 2009,Ahmad and Kharal [1] developed the result of Maji [5].

In 2010, Cagman and Enginoglu [2] defined soft matrices which were a matrix representation of the soft sets and constructed a soft max-min decision making method. In 2011, Yong yang and Chenli Ji [16] successfully applied the proposed notion of fuzzy soft matrix in certain decision making problems. In 2011, Neog and Sut [14] have defined the "addition operation" for fuzzy soft matrices and an attempt has been made to apply our notion in solving a decision problem.

In 2012, Cagman and Enginoglu [3] defined fuzzy soft matrices and constructed decision making problem. In 2012,Rajarajeswari and Dhanalakshmi [9] introduced the application of similarity between two fuzzy soft sets based on distance. In 2012,Neog, Bora and Sut[15] Combined fuzzy soft set based on reference function with soft matrices. In 2012, Borah, Neog, Sut [6] extended fuzzy soft matrix theory and its application.

In 2013, Mamoni Dhar [8] applied this concept to fuzzy square matrix and developed some interesting properties as determinant, trace and so on. In 2013,Said Broumi, Florentin Smarandache and M.Dhar [10] introduced an application of fuzzy soft matrix based on reference function in decision making problem is given. In 2013, Md.Jalilul and T.K.Roy [4] have introduced different types of fuzzy soft matrices and some properties with proof.

In 2014, Dr.N.Sarala and Rajkumari [11] introduced intuitionistic fuzzy soft matrices in agriculture and issued[12] intuitionistic fuzzy soft matrices in medical diagnosis and also introduced [13] role model service rendered to orphans by using fuzzy soft matrices.

In this paper, we proposed fuzzy soft matrix theory and extended its characters in the field of fuzzy soft matrices.

II. PRELIMINARIES

In this we section, We recall some basic significant notion of fuzzy soft set and defined different types of fuzzy soft set.

2.1 Soft Set [7]

Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Let A⊆E. A pair (F_A,E) is called a soft set over U, where F_A is a mapping given by F_A :E→ P(U) Such that F_A(e) = φ if e∉ A.

Here F_A is called approximate function of the soft set (F_A,E). The set F_A(e) is called e-approximate value set which consist of related objects of the parameter e∈E. In other words, a soft set over U is a parameterized family of subsets of the universe U.

Example 2.1:

Let U={u_1, u_2, u_3, u_4} be a set of four types of rice and E={Costly(e_1),Ordinary(e_2),Cheap(e_3)} be the set of parameters. If A={ e_1 , e_2 }⊆E. Let $F_A(e_1)$={u_1, u_2, u_3, u_4} and $F_A(e_2)$={ u_1, u_3, u_4}. Then we write the soft set

(F_A,E) ={(e_1,{ u_1, u_2, u_3, u_4}),(e_2,{ u_1, u_3, u_4})}

over U which describe the "Quality of rice" Which Mr.Y is going to buy.

We may represent the soft set in the following form:

U	Costly(e_1)	Ordinary(e_2)	Cheap(e_3)
u_1	1	1	0
u_2	1	0	0
u_3	1	1	0
u_4	1	1	0

TABLE 2.1.1

2.2 Fuzzy Soft Set [5]

Let U be an initial universe set and E be a set of parameters. Let A⊆E. A pair ($\tilde{F}_A$,E) is called a fuzzy soft set (FSS)over U, where$\tilde{F}_A$ is a mapping given by, $\tilde{F}_A$:E→ I^U, where I^U denotes the collection of all fuzzy subsets of U.

Example 2.2:

Consider the **example 2.1.,** here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval [0,1]. Then

$(\tilde{F}_A,E) = \{\ \tilde{F}_A(e_1) = \{(u_1,0.6),(u_2,0.4),(u_3,0.2),(u_4,0.1)\},$
$\tilde{F}_A(e_2) = \{(u_1,0.8),(u_3,0.3),(u_4,0.5)\}\}$ is the fuzzy soft set representing the "Quality of rice" which Mr.Y is going to buy.

We may represent the fuzzy soft set in the following form:

U	Costly(e_1)	Ordinary(e_2)	Cheap(e_3)
u_1	0.6	0.8	0.0
u_2	0.4	0.0	0.0
u_3	0.2	0.3	0.0
u_4	0.1	0.5	0.0

TABLE 2.2.2

2.3 Fuzzy Soft Class[9]

Let U be an initial universe set and E be the set of attributes. Then the pair (U,E) denotes the collection of all fuzzy soft sets on U with attributes from E and is called a fuzzy soft class.

2.4 Fuzzy Soft Subset[9]

For two fuzzy soft sets $(\tilde{F}_A,E)$ and $(\tilde{G}_B,E)$ over a common universe U, we have $(\tilde{F}_A,E) \subseteq (\tilde{G}_B,E)$ if A⊂B and ∀ e∈A, $\tilde{F}_A$(e) is a fuzzy subset of $\tilde{G}_B$(e). i.e, $(\tilde{F}_A,E)$is a fuzzy soft subset of $(\tilde{G}_B,E)$.

2.5 Fuzzy Soft Complement set[14]

The complement of fuzzy soft set $(\tilde{F}_A,E)$ denoted by $(\tilde{F}_A,E)^\circ$ is defined by $(\tilde{F}_A,E)^\circ = (\tilde{F}_A^\circ,E)$, where $\tilde{F}_A^\circ$: E→I^U is a mapping given by $\tilde{F}_A^\circ$(e) = $[\tilde{F}_A(e)]^\circ$, ∀ e∈E.

2.6 Fuzzy Soft Null Set[9]

A fuzzy soft set $(\tilde{F}_A,E)$ over U is said to be null fuzzy soft set with respect to the parameter set E, denoted by $\tilde{\varphi}$, if $\tilde{F}_A(e) = \tilde{\varphi}$, ∀ e∈E.

2.7 Union of Fuzzy Soft Sets[15]

Union of two fuzzy soft sets $(\tilde{F}_A,E)$and $(\tilde{G}_B,E)$ in a soft class (U,E) is a fuzzy soft set $(\tilde{H}_C,E)$ Where C=A∩B and ∀ e∈C,

$$\tilde{H}_C(e) = \begin{cases} \tilde{F}_A(e), & if\ e \in A-B \\ \tilde{G}_B(e), & if\ e \in B-A \\ \tilde{F}_A(e)\,\tilde{\cup}\,\tilde{G}_B(e), & if\, e \in A\cap B \end{cases}$$

and is written as $(\tilde{F}_A,E)\ \tilde{\cup}\ (\tilde{G}_B,E) = (\tilde{H}_C,E)$.

2.8Intersection of Fuzzy Soft Sets[15]

Intersection of two fuzzy soft sets $(\tilde{F}_A,E)$ and $(\tilde{G}_B,E)$ in a soft class (U,E) is a fuzzy soft set $(\tilde{H}_C,E)$ Where C=A∩B and ∀ e∈E, $\tilde{H}_C(e) = \tilde{F}_A(e)$ or $\tilde{G}_B(e)$and is written as $(\tilde{F}_A,E)\ \tilde{\cap}\ (\tilde{G}_B,E) = (\tilde{H}_C,E)$.

Ahmad and Kharal [1] pointed out that generally $\tilde{F}_A(e)$ and $\tilde{G}_B(e)$ may not be identical. Moreover in order to avoid the degenerate case, he proposed that A∩B must be non-empty and thus revised the above definition as follows.

2.9 Intersection of Fuzzy Soft Sets Redefined[15]

Let $(\tilde{F}_A,E)$and $(\tilde{G}_B,E)$ be two fuzzy soft sets in a soft class (U,E) with A∩B $\neq \varphi$. Then intersection of two fuzzy soft sets $(\tilde{F}_A,E)$and $(\tilde{G}_B,E)$ in the fuzzy soft class (U,E) is a fuzzy soft set $(\tilde{H}_C,E)$ where C=A∩B and ∀ e∈C, $\tilde{H}_C(e) = \tilde{F}_A(e)\,\tilde{\cap}\,\tilde{G}_B(e)$.

III. CHARACTERIZATION OF FUZZY SOFT MATRICES

In this section, we introduce the notion of fuzzy soft matrices with several types based on reference function.

3.1 Fuzzy Soft Matrices:

Let U = $\{u_1,u_2,u_3,\cdots u_m\}$ be the universal set and E be the set of parameters given by E = $\{e_1,e_2,e_3,\cdots,e_n\}$. Then the fuzzy soft set $(\tilde{F}_A,E)$ can be expressed in matrix form as $\tilde{A} = [a_{ij}^{\tilde{A}}]_{m\times n}$ or simply by $[\ a_{ij}^{\tilde{A}}\ ]$, $i = 1,2,3,\cdots,m; j = 1,2,3,\cdots,n$ and $[\ a_{ij}^{\tilde{A}}\ ] = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]$; where $\mu_{ij}^{\tilde{A}}$ and $\gamma_{ij}^{\tilde{A}}$ represent the fuzzy membership function and fuzzy reference function U in the fuzzy set $\tilde{F}_A(e_j)$ so that $\delta_{ij}^{\tilde{A}} = \mu_{ij}^{\tilde{A}} - \gamma_{ij}^{\tilde{A}}$ gives the fuzzy membership value of U. We shall identify a fuzzy soft set with its fuzzy soft matrix and use these two concepts interchangeable. The set of all m×n fuzzy soft matrices over U will be denoted by FSM $_{m\times n}$. For usual fuzzy sets with fuzzy reference function 0, it is obvious to see that $a_{ij}^{\tilde{A}} = [(\mu_{ij}^{\tilde{A}},0)]$ ∀ i , j.

Example 3.1:

Let U = $\{u_1,u_2,u_3\}$ be the universal set and E be the set of parameters given by E = $\{e_1,e_2,e_3\}$

we consider a fuzzy soft set

$(\tilde{F}_A,E) = \{\tilde{F}_A(e_1) = \{(u_1,0.7,0),(u_2,0.3,0),(u_3,0.4,0)\},$

$\tilde{F}_A(e_2) = \{(u_1,0.8,0),(u_2,0.6,0),(u_3,0.9,0)\},$

$\tilde{F}_A(e_3) = \{(u_1,0.5,0),(u_2,0.2,0),(u_3,0.1,0)\}\}$

We would represent this fuzzy soft set in matrix form as

$$[a_{ij}^{\tilde{A}}]_{3\times 3} = \begin{bmatrix} (0.7,0) & (0.8,0) & (0.5,0) \\ (0.3,0) & (0.6,0) & (0.2,0) \\ (0.4,0) & (0.9,0) & (0.1,0) \end{bmatrix}_{3\times 3}$$

3.2 Membership Value Matrix:

The membership value matrix corresponding to the matrix $\tilde{A}$ as MV($\tilde{A}$) = $[\delta_{ij}^{\tilde{A}}]_{m\times n}$, where $\delta_{ij}^{\tilde{A}} = \mu_{ij}^{\tilde{A}} - \gamma_{ij}^{\tilde{A}}$ ∀ $i = 1,2,3,\cdots,m\ and\ j = 1,2,3,\cdots,n$, where $\mu_{ij}^{\tilde{A}}$ and $\gamma_{ij}^{\tilde{A}}$ represent the fuzzy membership function and fuzzy reference function respectively of U in the fuzzy set $\tilde{F}_A(e_j)$.

3.3 Zero Fuzzy Soft Matrices:

Let $\tilde{A}$ = $[a_{ij}^{\tilde{A}}]$∈FSM$_{m\times n}$, where $a_{ij}^{\tilde{A}}$=($\mu_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{A}}$). Then $\tilde{A}$ is called a fuzzy soft zero (or Null) matrix denoted by $[\tilde{0}]_{m\times n}$, or simply by$[\tilde{0}]$, if $\delta_{ij}^{\tilde{A}}$=$\tilde{0}$ for all i and j. For usual fuzzy sets, $\delta_{ij}^{\tilde{A}}$= $\mu_{ij}^{\tilde{A}}$ ∀ i, j.

3.4 Identify Fuzzy Soft Matrices:

Let $\tilde{A}$ = $[a_{ij}^{\tilde{A}}]$∈FSM$_{m\times n}$, where $a_{ij}^{\tilde{A}}$=($\mu_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{A}}$). Then $\tilde{A}$ is called a fuzzy soft unit or identify matrix denoted by $[\tilde{1}]$, if m=n, $a_{ij}^{\tilde{A}}$=($\mu_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{A}}$) for all i≠j and $a_{ij}^{\tilde{A}}$= (1,0) i.e,$\delta_{ij}^{\tilde{A}}$ =1 ∀ i=j.

3.5 Fuzzy Soft Sub Matrix:

Let $\tilde{A}$ = $[a_{ij}^{\tilde{A}}]$, $\tilde{B}$ = $[b_{ij}^{\tilde{B}}]$∈FSM$_{m\times n}$. Then $\tilde{A}$ is said to be a fuzzy soft sub matrix of $\tilde{B}$ denoted by $[a_{ij}^{\tilde{A}}]\tilde{\subseteq}[b_{ij}^{\tilde{B}}]$ if $a_{ij}^{\tilde{A}}\leq b_{ij}^{\tilde{B}}$ ∀ i and j.

3.6 Fuzzy Soft Square Matrix:

Let $\tilde{A}$ = $[a_{ij}^{\tilde{A}}]$∈FSM$_{m\times n}$, where $a_{ij}^{\tilde{A}}$=($\mu_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{A}}$). If m = n, then $\tilde{A}$ is called a fuzzy soft square matrix.

3.7 Fuzzy Soft Rectangular Matrix:

Let $\tilde{A}$ = $[a_{ij}^{\tilde{A}}]$∈FSM$_{m\times n}$, where $a_{ij}^{\tilde{A}}$=($\mu_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{A}}$).If m ≠ n, then $\tilde{A}$ is called a fuzzy soft rectangular matrix.

3.8 Fuzzy Soft Diagonal Matrix:

Let $\tilde{A}$ = $[a_{ij}^{\tilde{A}}]$∈FSM$_{m\times n}$, where $a_{ij}^{\tilde{A}}$=($\mu_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{A}}$). Then $\tilde{A}$ is called fuzzy soft diagonal matrix if m = n and $a_{ij}^{\tilde{A}}$=($\mu_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{A}}$) for all i≠j. In other words for a fuzzy soft diagonal matrix $\tilde{A}$, $\delta_{ij}^{\tilde{A}}$ =0 ∀ i≠j.

3.9 Fuzzy Soft Equal Matrix:

Let the fuzzy soft matrices corresponding to the fuzzy soft sets $(\tilde{F}_A, E)$ and $(\tilde{G}_B$, E) be $\tilde{A}$ = $[a_{ij}^{\tilde{A}}]$, $\tilde{B}$ = $[b_{ij}^{\tilde{B}}]$ ∈ FSM$_{m\times n}$; $a_{ij}^{\tilde{A}}$ =($\mu_{ij}^{\tilde{A}}$, $\gamma_{ij}^{\tilde{A}}$) and $b_{ij}^{\tilde{B}}$ =($\mu_{ij}^{\tilde{B}}$, $\gamma_{ij}^{\tilde{B}}$), i=1,2,3⋯,m; j=1,2,3,⋯,n. Then $\tilde{A}$ and $\tilde{B}$ are called fuzzy soft equal matrices denoted by $\tilde{A}$ = $\tilde{B}$. If $\mu_{ij}^{\tilde{A}}$ = $\mu_{ij}^{\tilde{B}}$ and $\gamma_{ij}^{\tilde{A}}$ = $\gamma_{ij}^{\tilde{B}}$ ∀ i, j.

3.10 Union of Fuzzy Soft Matrices:

Let $\tilde{A}$ = $[a_{ij}^{\tilde{A}}]$, $\tilde{B}$ = $[b_{ij}^{\tilde{B}}]$ ∈ FSM$_{m\times n}$, where $a_{ij}^{\tilde{A}}$=($\mu_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{A}}$) and $b_{ij}^{\tilde{B}}$=($\mu_{ij}^{\tilde{B}}$,$\gamma_{ij}^{\tilde{B}}$). Then the union of $\tilde{A}$ and $\tilde{B}$denoted by $\tilde{A}\,\tilde{\cup}\,\tilde{B}$ is defined as $\tilde{A}\,\tilde{\cup}\,\tilde{B}$= max{$a_{ij}^{\tilde{A}}$,$b_{ij}^{\tilde{B}}$} for all i and j.

3.11 Intersection of Fuzzy Soft Matrices:

Let $\tilde{A}$ = $[a_{ij}^{\tilde{A}}]$, $\tilde{B}$ = $[b_{ij}^{\tilde{B}}]$ ∈ FSM$_{m\times n}$, where $a_{ij}^{\tilde{A}}$=($\mu_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{A}}$) and $b_{ij}^{\tilde{B}}$=($\mu_{ij}^{\tilde{B}}$,$\gamma_{ij}^{\tilde{B}}$). Then the intersection of $\tilde{A}$ and $\tilde{B}$denoted by $\tilde{A}\,\tilde{\cap}\,\tilde{B}$ is defined as $\tilde{A}\,\tilde{\cap}\,\tilde{B}$= min{$a_{ij}^{\tilde{A}}$,$b_{ij}^{\tilde{B}}$} for all i and j.

3.12 Complement of Fuzzy Soft Matrices:

Let $\tilde{A}$ = $[(a_{ij}^{\tilde{A}}, 0)]$∈FSM$_{m\times n}$, where $a_{ij}^{\tilde{A}}$=($\mu_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{A}}$) according to the definition in [8], the representation of the complement of the fuzzy matrix $\tilde{A}$ which is denoted by $\tilde{A}^{\circ}$ and then $\tilde{A}^{\circ}$ is called fuzzy soft complement matrix if $\tilde{A}^{\circ}$= $[(1, a_{ij}^{\tilde{A}})]_{m\times n}$ for all $a_{ij}^{\tilde{A}}$ ∈[0,1]. Then the matrix obtained from so called membership value would be the following $\tilde{A}^{\circ}$= $[a_{ij}^{\tilde{A}}]$ = [(1- $a_{ij}^{\tilde{A}}$)] for all i and j.

3.13 Addition of Fuzzy Soft Matrices:

Let U = $\{u_1, u_2, u_3, \cdots, u_m\}$ be the universal set and E be the set of parameters given by E = $\{e_1, e_2, e_3, \cdots, e_n\}$. Let the set of all m×n fuzzy soft matrices over U be FSM$_{m\times n}$. Let $\tilde{A}$, $\tilde{B}$∈ FSM$_{m\times n}$, Where $\tilde{A}$ = $[a_{ij}^{\tilde{A}}]_{m\times n}$, $a_{ij}^{\tilde{A}}$=($\mu_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{A}}$) and $\tilde{B}$ = $[b_{ij}^{\tilde{B}}]_{m\times n}$, $b_{ij}^{\tilde{B}}$= ($\mu_{ij}^{\tilde{B}}$,$\gamma_{ij}^{\tilde{B}}$). To avoid degenerate cases we assume that min(($\mu_{ij}^{\tilde{A}}$,$\mu_{ij}^{\tilde{B}}$)≥max(($\gamma_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{B}}$) for all i and j. We define the operation 'addition(+)' between $\tilde{A}$ and $\tilde{B}$ as $\tilde{A}+\tilde{B}=\tilde{C}$, where $\tilde{C}$= $[C_{ij}^{\tilde{C}}]_{m\times n}$, $C_{ij}^{\tilde{C}}$= (max($\mu_{ij}^{\tilde{A}}$,$\mu_{ij}^{\tilde{B}}$),min($\gamma_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{B}}$)).

3.14 Product of Fuzzy Soft Matrices:

Let $\tilde{A}$ = $[a_{ij}^{\tilde{A}}]_{m\times n}$, $a_{ij}^{\tilde{A}}$ =($\mu_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{A}}$); where $\mu_{ij}^{\tilde{A}}$ and $\gamma_{ij}^{\tilde{A}}$ represent the fuzzy membership function and fuzzy reference function of u_i, so that $\delta_{ij}^{\tilde{A}} = \mu_{ij}^{\tilde{A}} - \gamma_{ij}^{\tilde{A}}$ gives the fuzzy membership value of u_i. Also let $\tilde{B}$ = $[b_{jk}^{\tilde{B}}]_{n\times p}$, $b_{jk}^{\tilde{B}}$= ($\mu_{jk}^{\tilde{B}}$, $\gamma_{jk}^{\tilde{B}}$); where $\mu_{jk}^{\tilde{B}}$ and $\gamma_{jk}^{\tilde{B}}$ represent the fuzzy membership function and fuzzy reference function of u_i, so that $\delta_{jk}^{\tilde{B}} = \mu_{jk}^{\tilde{B}} - \gamma_{jk}^{\tilde{B}}$ gives the fuzzy membership value of u_i. We now define $\tilde{A}\cdot\tilde{B}$, the product of $\tilde{A}$ and $\tilde{B}$ as $\tilde{A}\cdot\tilde{B}$ = $[d_{ik}^{\tilde{A}\tilde{B}}]_{m\times p}$ = [max min($\mu_{ij}^{\tilde{A}}$, $\mu_{jk}^{\tilde{B}}$), min max($\gamma_{ij}^{\tilde{A}}$, $\gamma_{jk}^{\tilde{B}}$)]$_{m\times p}$, 1≤ i ≤m, 1≤ k ≤p for j = 1,2,3,⋯,n.

3.15 Scalar Multiplication of Fuzzy Soft Matrix:

Let $\tilde{A}$ = $[a_{ij}^{\tilde{A}}]$ ∈FSM$_{m\times n}$. Then scalar multiplication of fuzzy soft matrix $[a_{ij}^{\tilde{A}}]$ by a scalar k denoted by k$[a_{ij}^{\tilde{A}}]$ is defined as k$[a_{ij}^{\tilde{A}}]$ = $[ka_{ij}^{\tilde{A}}]$, 0≤k≤1.

3.16 Transpose of Fuzzy Soft Matrices:

Let $\tilde{A}$ = $[a_{ij}^{\tilde{A}}]_{m\times n}$, $a_{ij}^{\tilde{A}}$ =($\mu_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{A}}$); where $\mu_{ij}^{\tilde{A}}$ and $\gamma_{ij}^{\tilde{A}}$ represent the fuzzy membership function and fuzzy reference function of U, so that $\delta_{ij}^{\tilde{A}} = \mu_{ij}^{\tilde{A}} - \gamma_{ij}^{\tilde{A}}$ gives the fuzzy membership value of U. Then we define $\tilde{A}^T$ = $[a_{ij}^{\tilde{A}}]^{T}_{n\times m}$ ∈FSM$_{n\times m}$ where $[a_{ij}^{\tilde{A}}]^T$ = $[a_{ji}^{\tilde{A}}]$.

3.17 Symmetric Fuzzy Soft Matrix:

Let $\tilde{A}$ = $[a_{ij}^{\tilde{A}}]_{m\times n}$, $a_{ij}^{\tilde{A}}$=($\mu_{ij}^{\tilde{A}}$,$\gamma_{ij}^{\tilde{A}}$). Then $\tilde{A}$ is said to be a fuzzy soft symmetric matrix if $\tilde{A}^T$= $\tilde{A}$.

3.18 Trace of Fuzzy Soft Matrix:

Let $\tilde{A}$ be a square matrix. Then the trace of the matrix $\tilde{A}$ is denoted by tr$\tilde{A}$ and is defined as:

tr$\tilde{A}$ =(max($\mu_{ii}^{\tilde{A}}$), min($\gamma_{ii}^{\tilde{A}}$))

Where $\mu_{ii}^{\tilde{A}}$ stands for the membership functions lying along the principal diagonal and $\gamma_{ii}^{\tilde{A}}$ refers to the reference function of the corresponding membership functions.

Example 3.18:

Let $\tilde{A} = \begin{bmatrix} (0.5,0) & (0.6,0) & (0.2,0) \\ (0.7,0) & (0.4,0) & (0.8,0) \\ (0.3,0) & (0.1,0) & (0.6,0) \end{bmatrix}$

tr$\tilde{A}$ =(max(0.5,0.4,0.6), min(0,0,0)) = (0.6,0).

3.19Principal Diagonal of Fuzzy Soft Matrices:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}]$∈FSM$_{m\times m}$, where $a_{ij}^{\tilde{A}}=(\mu_{ij}^{\tilde{A}},\gamma_{ij}^{\tilde{A}})$, then the elements $a_{11}, a_{22}, a_{33}, \cdots, a_{mm}$ are called the diagonal elements and the line along which they lie is called the principal diagonal of fuzzy soft matrices.

IV. IDEALISTIC PROPERTIES OF FUZZY SOFT MATRICES

In this section, we see the properties of fuzzy soft matrices with some example.

Proposition 4.1:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}]$∈FSM$_{m\times n}$. Then (i) $[[\tilde{A}]^\circ]^\circ = [\tilde{A}]$, (ii) $\tilde{A} \tilde{\cup} \tilde{A} = \tilde{A}$, (iii) $\tilde{A} \tilde{\cap} \tilde{A} = \tilde{A}$, (iv) $\tilde{A} \tilde{\cup} [\tilde{0}] = \tilde{A}$, (v) $\tilde{A} \tilde{\cap} [\tilde{0}] = [\tilde{0}]$, (vi) $[\tilde{0}]^\circ = [\tilde{1}]$.

Proof:

(i) $[[\tilde{A}]^\circ]^\circ = [1-a_{ij}^{\tilde{A}}]^\circ = [1-(1-a_{ij}^{\tilde{A}})] = [a_{ij}^{\tilde{A}}] = \tilde{A}$

(ii) $\tilde{A} \tilde{\cup} \tilde{A} = [\max\{a_{ij}^{\tilde{A}}, a_{ij}^{\tilde{A}}\}] = [a_{ij}^{\tilde{A}}] = \tilde{A}$

(iii) $\tilde{A} \tilde{\cap} \tilde{A} = [\min\{a_{ij}^{\tilde{A}}, a_{ij}^{\tilde{A}}\}] = [a_{ij}^{\tilde{A}}] = \tilde{A}$

(iv) $\tilde{A} \tilde{\cup} [\tilde{0}] = [\max\{a_{ij}^{\tilde{A}}, 0\}] = [a_{ij}^{\tilde{A}}] = \tilde{A}$

(v) $\tilde{A} \tilde{\cap} [\tilde{0}] = [\min\{a_{ij}^{\tilde{A}}, 0\}] = [\tilde{0}]$

(vi) $[\tilde{0}]^\circ = [1-0] = [\tilde{1}]$

Proposition 4.2:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}]$, $\tilde{B} = [b_{ij}^{\tilde{B}}]$∈FSM$_{m\times n}$. Then
(i) $[\tilde{A} \tilde{\cup} \tilde{B}]^\circ = [\tilde{A}]^\circ \tilde{\cap} [B]^\circ$, (ii) $[\tilde{A} \tilde{\cap} \tilde{B}]^\circ = [\tilde{A}]^\circ \tilde{\cup} [B]^\circ$.

Proof:

(i) For all i and j,

$[\tilde{A} \tilde{\cup} \tilde{B}]^\circ = [\max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}]^\circ = [1-\max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}] = [\min\{1-a_{ij}^{\tilde{A}}, 1-b_{ij}^{\tilde{B}}\}] = [\tilde{A}]^\circ \tilde{\cap} [\tilde{B}]^\circ$

(ii) For all i and j,

$[\tilde{A} \tilde{\cap} \tilde{B}]^\circ = [\min\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}]^\circ = [1-\min\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}] = [\max\{1-a_{ij}^{\tilde{A}}, 1-b_{ij}^{\tilde{B}}\}] = [\tilde{A}]^\circ \tilde{\cup} [B]^\circ$.

Proposition 4.3:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}]$, $\tilde{B} = [b_{ij}^{\tilde{B}}]$, $\tilde{C} = [c_{ij}^{\tilde{C}}]$ ∈FSM$_{m\times n}$. Then (i) $\tilde{A} \tilde{\cup} (\tilde{B} \tilde{\cap} \tilde{C}) = (\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cap} (\tilde{A} \tilde{\cup} \tilde{C})$, (ii) $\tilde{A} \tilde{\cap} (\tilde{B} \tilde{\cup} \tilde{C}) = (\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cup} (\tilde{A} \tilde{\cap} \tilde{C})$.

Proof:

(i) $\tilde{A} \tilde{\cup} (\tilde{B} \tilde{\cap} \tilde{C}) = [a_{ij}^{\tilde{A}}] \tilde{\cup} [(\min\{b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}}\})]$

$= [\max(a_{ij}^{\tilde{A}}, \min\{b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}}\})]$

$(\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cap} (\tilde{A} \tilde{\cup} \tilde{C}) = [(\max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\})] \tilde{\cap} [(\max\{a_{ij}^{\tilde{A}}, c_{ij}^{\tilde{C}}\})]$

$= [\min(\max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}, \max\{a_{ij}^{\tilde{A}}, c_{ij}^{\tilde{C}}\})]$

$= [\min(a_{ij}^{\tilde{A}}, \max\{b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}}\})]$

$= [\max(a_{ij}^{\tilde{A}}, \min\{b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}}\})]$

Hence $\tilde{A} \tilde{\cup} (\tilde{B} \tilde{\cap} \tilde{C}) = (\tilde{A} \tilde{\cup} \tilde{B}) \tilde{\cap} (\tilde{A} \tilde{\cup} \tilde{C})$.

(ii) $\tilde{A} \tilde{\cap} (\tilde{B} \tilde{\cup} \tilde{C}) = [a_{ij}^{\tilde{A}}] \tilde{\cap} [(\max\{b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}}\})]$

$= [\min(a_{ij}^{\tilde{A}}, \max\{b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}}\})]$

$(\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cup} (\tilde{A} \tilde{\cap} \tilde{C}) = [(\min\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\})] \tilde{\cup} [(\min\{a_{ij}^{\tilde{A}}, c_{ij}^{\tilde{C}}\})]$

$= [\max(\min\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}, \min\{a_{ij}^{\tilde{A}}, c_{ij}^{\tilde{C}}\})]$

$= [\max(a_{ij}^{\tilde{A}}, \min\{b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}}\})]$

$= [\min(a_{ij}^{\tilde{A}}, \max\{b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}}\})]$

Hence $\tilde{A} \tilde{\cap} (\tilde{B} \tilde{\cup} \tilde{C}) = (\tilde{A} \tilde{\cap} \tilde{B}) \tilde{\cup} (\tilde{A} \tilde{\cap} \tilde{C})$.

Example 4.3:

(i) Let $\tilde{A} = [a_{ij}^{\tilde{A}}]$, $\tilde{B} = [b_{ij}^{\tilde{B}}]$, $\tilde{C} = [c_{ij}^{\tilde{C}}]$ ∈FSM$_{2\times 2}$,

where $\tilde{A} = \begin{bmatrix} (0.6,0) & (0.5,0) \\ (0.2,0) & (0.4,0) \end{bmatrix}$, $\tilde{B} = \begin{bmatrix} (0.4,0) & (0.7,0) \\ (0.9,0) & (0.1,0) \end{bmatrix}$ and $\tilde{C} = \begin{bmatrix} (0.8,0) & (0.2,0) \\ (0.3,0) & (0.6,0) \end{bmatrix}$

$\tilde{B} \tilde{\cap} \tilde{C} = \begin{bmatrix} (0.4,0) & (0.2,0) \\ (0.3,0) & (0.1,0) \end{bmatrix}$; $\tilde{A} \tilde{\cup} \tilde{B} = \begin{bmatrix} (0.6,0) & (0.7,0) \\ (0.9,0) & (0.4,0) \end{bmatrix}$

$\tilde{A} \tilde{\cup} \tilde{C} = \begin{bmatrix} (0.8,0) & (0.5,0) \\ (0.3,0) & (0.6,0) \end{bmatrix}$

$\tilde{A} \tilde{\cup} (\tilde{B} \tilde{\cap} \tilde{C}) = \begin{bmatrix} (0.6,0) & (0.5,0) \\ (0.3,0) & (0.4,0) \end{bmatrix}$;

$$(\tilde{A} \,\tilde{\cup}\, \tilde{B})\, \tilde{\cap} (\tilde{A} \,\tilde{\cup}\, \tilde{C}) = \begin{bmatrix} (0.6,0) & (0.5,0) \\ (0.3,0) & (0.4,0) \end{bmatrix}$$

Hence $\tilde{A} \,\tilde{\cup}\, (\tilde{B} \,\tilde{\cap}\, \tilde{C}) = (\tilde{A} \,\tilde{\cup}\, \tilde{B})\, \tilde{\cap} (\tilde{A} \,\tilde{\cup}\, \tilde{C})$.

(ii)The above said matrices may be considered for the $\tilde{A} \,\tilde{\cap}\, \left(\tilde{B} \,\tilde{\cup}\, \tilde{C}\right) = (\tilde{A} \,\tilde{\cap}\, \tilde{B})\, \tilde{\cup} (\tilde{A} \,\tilde{\cap}\, \tilde{C})$.

$$\tilde{B} \,\tilde{\cup}\, \tilde{C} = \begin{bmatrix} (0.8,0) & (0.7,0) \\ (0.9,0) & (0.6,0) \end{bmatrix};$$

$$\tilde{A} \,\tilde{\cap}\, \tilde{B} = \begin{bmatrix} (0.4,0) & (0.5,0) \\ (0.2,0) & (0.1,0) \end{bmatrix}; \quad \tilde{A} \,\tilde{\cap}\, \tilde{C} = \begin{bmatrix} (0.6,0) & (0.2,0) \\ (0.2,0) & (0.4,0) \end{bmatrix}$$

$$\tilde{A} \,\tilde{\cap}\, (\tilde{B} \,\tilde{\cup}\, \tilde{C}) = \begin{bmatrix} (0.6,0) & (0.5,0) \\ (0.2,0) & (0.4,0) \end{bmatrix};$$

$$(\tilde{A} \,\tilde{\cap}\, \tilde{B})\, \tilde{\cup} (\tilde{A} \,\tilde{\cap}\, \tilde{C}) = \begin{bmatrix} (0.6,0) & (0.5,0) \\ (0.2,0) & (0.4,0) \end{bmatrix}$$

Hence $\tilde{A} \,\tilde{\cap}\, \left(\tilde{B} \,\tilde{\cup}\, \tilde{C}\right) = (\tilde{A} \,\tilde{\cap}\, \tilde{B})\, \tilde{\cup} (\tilde{A} \,\tilde{\cap}\, \tilde{C})$.

Proposition4.4:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}]$, $\tilde{B} = [b_{ij}^{\tilde{B}}]$, $\tilde{C} = [c_{ij}^{\tilde{C}}] \in FSM_{m\times n}$. Then (i) $\tilde{A} \,\tilde{\cup}\, \tilde{B} = \tilde{B} \,\tilde{\cup}\, \tilde{A}$, (ii) $(\tilde{A} \,\tilde{\cup}\, \tilde{B}) \,\tilde{\cup}\, \tilde{C} = \tilde{A} \,\tilde{\cup}\, (\tilde{B} \,\tilde{\cup}\, \tilde{C})$, (iii) $\tilde{A} \,\tilde{\cup}\, [\tilde{1}] = [\tilde{1}]$.

Proof:

(i) For all i and j,

$\tilde{A} \,\tilde{\cup}\, \tilde{B} = [(\max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\})] = [(\max\{b_{ij}^{\tilde{B}}, a_{ij}^{\tilde{A}}\})] = \tilde{B} \,\tilde{\cup}\, \tilde{A}$

(ii) $(\tilde{A} \,\tilde{\cup}\, \tilde{B}) \,\tilde{\cup}\, \tilde{C} = [(\max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\})] \tilde{\cup} [c_{ij}^{\tilde{C}}]$

$= [\max(\max\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}, c_{ij}^{\tilde{C}})]$

$= [\max(a_{ij}^{\tilde{A}}, \max\{b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}}\})]$

$= \tilde{A} \,\tilde{\cup}\, (\tilde{B} \,\tilde{\cup}\, \tilde{C})$

(iii) $\tilde{A} \,\tilde{\cup}\, [\tilde{1}] = [(\max\{a_{ij}^{\tilde{A}}, 1\})] = [\tilde{1}]$.

Proposition 4.5:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}]$, $\tilde{B} = [b_{ij}^{\tilde{B}}]$, $\tilde{C} = [c_{ij}^{\tilde{C}}] \in FSM_{m\times n}$. Then (i) $\tilde{A} \,\tilde{\cap}\, \tilde{B} = \tilde{B} \,\tilde{\cap}\, \tilde{A}$, (ii) $(\tilde{A} \,\tilde{\cap}\, \tilde{B}) \,\tilde{\cap}\, \tilde{C} = \tilde{A} \,\tilde{\cap}\, (\tilde{B} \,\tilde{\cap}\, \tilde{C})$, (iii) $\tilde{A} \,\tilde{\cap}\, [\tilde{1}] = \tilde{A}$.

Proof:

(i) For all i and j,

$\tilde{A} \,\tilde{\cap}\, \tilde{B} = [(\min\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\})] = [(\min\{b_{ij}^{\tilde{B}}, a_{ij}^{\tilde{A}}\})] = \tilde{B} \,\tilde{\cap}\, \tilde{A}$

(ii) $(\tilde{A} \,\tilde{\cap}\, \tilde{B}) \,\tilde{\cap}\, \tilde{C} = [(\min\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\})] \tilde{\cap} [c_{ij}^{\tilde{C}}]$

$= [(\min(\min\{a_{ij}^{\tilde{A}}, b_{ij}^{\tilde{B}}\}, c_{ij}^{\tilde{C}}))]$

$= [(\min(a_{ij}^{\tilde{A}}, \min\{b_{ij}^{\tilde{B}}, c_{ij}^{\tilde{C}}\}))]$

$= \tilde{A} \,\tilde{\cap}\, (\tilde{B} \,\tilde{\cap}\, \tilde{C})$

(iii) $\tilde{A} \,\tilde{\cap}\, [\tilde{1}] = [(\min\{a_{ij}^{\tilde{A}}, 1\})] = \tilde{A}$.

Proposition 4.6:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}]$, $\tilde{B} = [b_{ij}^{\tilde{B}}]$, $\tilde{C} = [c_{ij}^{\tilde{C}}] \in FSM_{m\times n}$, where $[a_{ij}^{\tilde{A}}] = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]$, $[b_{ij}^{\tilde{B}}] = [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})]$, $[c_{ij}^{\tilde{C}}] = [(\mu_{ij}^{\tilde{C}}, \gamma_{ij}^{\tilde{C}})]$. Then the following results hold.

(i) $\tilde{A}+\tilde{B} = \tilde{B}+\tilde{A}$(commutative law)

(ii) $(\tilde{A}+\tilde{B})+\tilde{C} = \tilde{A}+(\tilde{B}+\tilde{C})$(Associative law).

Proof:

(i) Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})]$

Now $\tilde{A}+\tilde{B} = [(\max(\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}), \min(\gamma_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{B}}))]$

$= [(\max(\mu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{A}}), \min(\gamma_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{A}}))]$

$= \tilde{B}+\tilde{A}$.

(ii) Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})]$, $\tilde{C} = [(\mu_{ij}^{\tilde{C}}, \gamma_{ij}^{\tilde{C}})]$

Now $(\tilde{A}+\tilde{B})+\tilde{C} = [(\max(\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}), \min(\gamma_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{B}}))] + [(\mu_{ij}^{\tilde{C}}, \gamma_{ij}^{\tilde{C}})]$

$= [(\max((\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}), \mu_{ij}^{\tilde{C}}), \min((\gamma_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{B}}), \gamma_{ij}^{\tilde{C}}))]$

$= [(\max(\mu_{ij}^{\tilde{A}}, (\mu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{C}})), \min(\gamma_{ij}^{\tilde{A}}, (\gamma_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{C}})))]$

$= \tilde{A}+(\tilde{B}+\tilde{C})$.

Example 4.6:

(i) Let $\tilde{A} = \begin{bmatrix} (0.6,0) & (0.3,0) \\ (0.5,0) & (0.2,0) \end{bmatrix}$, $\tilde{B} = \begin{bmatrix} (0.1,0) & (0.7,0) \\ (0.4,0) & (0.8,0) \end{bmatrix}$ and $\tilde{C} = \begin{bmatrix} (0.9,0) & (0.2,0) \\ (0.6,0) & (0.5,0) \end{bmatrix}$

$$\tilde{A}+\tilde{B} = \begin{bmatrix} (0.6,0) & (0.7,0) \\ (0.5,0) & (0.8,0) \end{bmatrix}, \quad \tilde{B}+\tilde{A} = \begin{bmatrix} (0.6,0) & (0.7,0) \\ (0.5,0) & (0.8,0) \end{bmatrix}$$

Hence $\tilde{A}+\tilde{B} = \tilde{B}+\tilde{A}$

(ii) The above said matrices may be considered for the $(\tilde{A}+\tilde{B})+\tilde{C} = \tilde{A}+(\tilde{B}+\tilde{C})$.

$$\tilde{A}+\tilde{B} = \begin{bmatrix}(0.6,0) & (0.7,0)\\(0.5,0) & (0.8,0)\end{bmatrix},\quad \tilde{B}+\tilde{C} = \begin{bmatrix}(0.9,0) & (0.7,0)\\(0.6,0) & (0.8,0)\end{bmatrix},$$

$$(\tilde{A}+\tilde{B})+\tilde{C} = \begin{bmatrix}(0.6,0) & (0.7,0)\\(0.5,0) & (0.8,0)\end{bmatrix} + \begin{bmatrix}(0.9,0) & (0.2,0)\\(0.6,0) & (0.5,0)\end{bmatrix}$$

$$= \begin{bmatrix}(0.9,0) & (0.7,0)\\(0.6,0) & (0.8,0)\end{bmatrix}$$

$$\tilde{A}+(\tilde{B}+\tilde{C}) = \begin{bmatrix}(0.6,0) & (0.3,0)\\(0.5,0) & (0.2,0)\end{bmatrix} + \begin{bmatrix}(0.9,0) & (0.7,0)\\(0.6,0) & (0.8,0)\end{bmatrix}$$

$$= \begin{bmatrix}(0.9,0) & (0.7,0)\\(0.6,0) & (0.8,0)\end{bmatrix}$$

Hence $(\tilde{A}+\tilde{B})+\tilde{C} = \tilde{A}+(\tilde{B}+\tilde{C})$.

Property 4.7:

The product of two fuzzy soft matrices $\tilde{A}$ and $\tilde{B}$ representing fuzzy soft sets over the same initial universe is defined only when the matrices are square matrices. Then $\tilde{A}.\tilde{B}$ and $\tilde{B}.\tilde{A}$ exist and matrices are of same type, it is not necessary that $\tilde{A}.\tilde{B} = \tilde{B}.\tilde{A}$. Let the following illustration may be taken into account for proof.

Example 4.7:

Let $\tilde{A} = \begin{bmatrix}(0.5,0) & (0.4,0)\\(0.7,0) & (0.6,0)\end{bmatrix}$,

$\tilde{B} = \begin{bmatrix}(0.3,0) & (0.2,0)\\(0.1,0) & (0.8,0)\end{bmatrix}$

be two fuzzy soft square matrices representing two fuzzy soft sets defined over the same initial universe.

Then $\tilde{A}\,.\,\tilde{B} = \begin{bmatrix}(0.3,0) & (0.4,0)\\(0.3,0) & (0.6,0)\end{bmatrix}$, $\tilde{B}.\tilde{A} = \begin{bmatrix}(0.3,0) & (0.3,0)\\(0.7,0) & (0.6,0)\end{bmatrix}$

From the example we obtained that $\tilde{A}.\tilde{B} \neq \tilde{B}.\tilde{A}$

Property 4.8:

If the product $\tilde{A}.\tilde{B}$ is defined then $\tilde{B}.\tilde{A}$ may not be defined.

Let the following illustration may be considered.

Example 4.8:

Let $\tilde{A} = \begin{bmatrix}(0.6,0) & (0.1,0) & (0.5,0) & (0.9,0)\\(0.3,0) & (0.7,0) & (0.2,0) & (0.4,0)\\(0.5,0) & (0.4,0) & (0.8,0) & (0.3,0)\end{bmatrix}$ and $\tilde{B} = \begin{bmatrix}(0.4,0) & (0.5,0)\\(0.6,0) & (0.1,0)\\(0.3,0) & (0.2,0)\\(0.7,0) & (0.8,0)\end{bmatrix}$

We have $\tilde{A}.\tilde{B} = \begin{bmatrix}(0.7,0) & (0.8,0)\\(0.6,0) & (0.4,0)\\(0.4,0) & (0.5,0)\end{bmatrix}$

But here $\tilde{B}.\tilde{A}$ is not defined as number of columns in $\tilde{B}$ = 2, Whereas number of row in $\tilde{A}$ = 3.

Proposition 4.9:

Let $\tilde{A} = [a_{ij}^{\tilde{A}}]$, $\tilde{B} = [b_{ij}^{\tilde{B}}] \in FSM_{m\times n}$, where $[a_{ij}^{\tilde{A}}] = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]$, $[b_{ij}^{\tilde{B}}] = [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})]$. Then the following results hold.

(i) $(\tilde{A}^T)^T = \tilde{A}$, (ii) $(\tilde{A}+\tilde{B})^T = \tilde{A}^T+\tilde{B}^T$.

Proof:

(i) Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]$

Here $\tilde{A}^T = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]^T$

$= [(\mu_{ji}^{\tilde{A}}, \gamma_{ji}^{\tilde{A}})]$

$(\tilde{A}^T)^T = [(\mu_{ji}^{\tilde{A}}, \gamma_{ji}^{\tilde{A}})]^T$

$= [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]$

$= \tilde{A}$

Hence $(\tilde{A}^T)^T = \tilde{A}$

(ii) $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})]$

Here $\tilde{A}+\tilde{B}$ = $[(\max(\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}), \min(\gamma_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{B}}))]$

$(\tilde{A}+\tilde{B})^{T} = [(\max(\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}), \min(\gamma_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{B}}))]^{T}$

$= [(\max(\mu_{ji}^{\tilde{A}}, \mu_{ji}^{\tilde{B}}), \min(\gamma_{ji}^{\tilde{A}}, \gamma_{ji}^{\tilde{B}}))]$

$= [(\mu_{ji}^{\tilde{A}}, \gamma_{ji}^{\tilde{A}})] + [(\mu_{ji}^{\tilde{B}}, \gamma_{ji}^{\tilde{B}})]$

$= [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]^{T} + [(\mu_{ij}^{\tilde{B}}, \gamma_{ij}^{\tilde{B}})]^{T}$

$= \tilde{A}^{T}+\tilde{B}^{T}$

Hence $(\tilde{A}+\tilde{B})^{T} = \tilde{A}^{T}+\tilde{B}^{T}$.

Example 4.9:

(i) Let $\tilde{A} = \begin{bmatrix} (0.3,0) & (0.6,0) & (0.4,0) \\ (0.5,0) & (0.8,0) & (0.1,0) \\ (0.7,0) & (0.2,0) & (0.6,0) \end{bmatrix}$ and

$\tilde{B} = \begin{bmatrix} (0.2,0) & (0.4,0) & (0.5,0) \\ (0.3,0) & (0.1,0) & (0.8,0) \\ (0.6,0) & (0.7,0) & (0.4,0) \end{bmatrix}$

$(\tilde{A}^{T}) = \begin{bmatrix} (0.3,0) & (0.5,0) & (0.7,0) \\ (0.6,0) & (0.8,0) & (0.2,0) \\ (0.4,0) & (0.1,0) & (0.6,0) \end{bmatrix}$;

$(\tilde{A}^{T})^{T} = \begin{bmatrix} (0.3,0) & (0.6,0) & (0.4,0) \\ (0.5,0) & (0.8,0) & (0.1,0) \\ (0.7,0) & (0.2,0) & (0.6,0) \end{bmatrix} = \tilde{A}$

Hence $(\tilde{A}^{T})^{T} = \tilde{A}$.

(ii) The above said matrices may be considered for the $(\tilde{A}+\tilde{B})^{T} = \tilde{A}^{T}+\tilde{B}^{T}$.

$\tilde{B}^{T} = \begin{bmatrix} (0.2,0) & (0.3,0) & (0.6,0) \\ (0.4,0) & (0.1,0) & (0.7,0) \\ (0.5,0) & (0.8,0) & (0.4,0) \end{bmatrix}$;

$\tilde{A}+\tilde{B} = \begin{bmatrix} (0.3,0) & (0.6,0) & (0.5,0) \\ (0.5,0) & (0.8,0) & (0.8,0) \\ (0.7,0) & (0.7,0) & (0.6,0) \end{bmatrix}$

$(\tilde{A}+\tilde{B})^{T} = \begin{bmatrix} (0.3,0) & (0.5,0) & (0.7,0) \\ (0.6,0) & (0.8,0) & (0.7,0) \\ (0.5,0) & (0.8,0) & (0.6,0) \end{bmatrix}$;

$\tilde{A}^{T}+\tilde{B}^{T} = \begin{bmatrix} (0.3,0) & (0.5,0) & (0.7,0) \\ (0.6,0) & (0.8,0) & (0.7,0) \\ (0.5,0) & (0.8,0) & (0.6,0) \end{bmatrix}$

Hence $(\tilde{A}+\tilde{B})^{T} = \tilde{A}^{T}+\tilde{B}^{T}$.

Proposition 4.10:

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy soft square matrices each of order n. Then $tr(\tilde{A}+\tilde{B}) = tr(\tilde{A})+tr(\tilde{B})$.

Proof:

From the proposed definition of trace of fuzzy soft matrices, we have obtained

$tr\tilde{A} = (\max(\mu_{ii}^{\tilde{A}}),\min(\gamma_{ii}^{\tilde{A}}))$ and $tr\tilde{B}=(\max(\mu_{ii}^{\tilde{B}}),\min(\gamma_{ii}^{\tilde{B}}))$

Then $\tilde{A}+\tilde{B} = \tilde{C}$, where $\tilde{C} = [c_{ij}^{\tilde{C}}]$

Adopting the definition of addition of two fuzzy soft matrices, we have

$C_{ij}^{\tilde{C}} = (\max(\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}),\min(\gamma_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{B}}))$

According to definition 3.18 the trace of fuzzy soft matrix based on reference function would be :

$tr(\tilde{C}) = (\max\{\max(\mu_{ii}^{\tilde{A}}, \mu_{ii}^{\tilde{B}})\},\min\{\min(\gamma_{ii}^{\tilde{A}}, \gamma_{ii}^{\tilde{B}})\})$

$= (\max\{\max(\mu_{ii}^{\tilde{A}}),\max(\mu_{ii}^{\tilde{B}})\},\min\{\min(\gamma_{ii}^{\tilde{A}}),\min(\gamma_{ii}^{\tilde{B}})\})$

$= tr(\tilde{A})+tr(\tilde{B})$

Conversely,

$tr(\tilde{A})+tr(\tilde{B}) = (\max\{\max(\mu_{ii}^{\tilde{A}}),\max(\mu_{ii}^{\tilde{B}})\},\min\{\min(\gamma_{ii}^{\tilde{A}}),\min(\gamma_{ii}^{\tilde{B}})\})$

$= (\max\{\max(\mu_{ii}^{\tilde{A}}, \mu_{ii}^{\tilde{B}})\},\min\{\min(\gamma_{ii}^{\tilde{A}}, \gamma_{ii}^{\tilde{B}})\})$

$= tr(\tilde{A}+\tilde{B})$

Hence $tr(\tilde{A}+\tilde{B}) = tr(\tilde{A})+tr(\tilde{B})$.

Example 4.10:

Let $\tilde{A} = \begin{bmatrix} (0.6,0) & (0.5,0) & (0.3,0) \\ (0.4,0) & (0.7,0) & (0.9,0) \\ (0.2,0) & (0.8,0) & (0.1,0) \end{bmatrix}$ and

$\tilde{B} = \begin{bmatrix} (0.4,0) & (0.3,0) & (0.1,0) \\ (0.7,0) & (0.9,0) & (0.2,0) \\ (0.6,0) & (0.5,0) & (0.3,0) \end{bmatrix}$

The addition of two fuzzy soft matrices would be:

$$\tilde{A}+\tilde{B} = \begin{bmatrix} (0.6,0) & (0.5,0) & (0.3,0) \\ (0.7,0) & (0.9,0) & (0.9,0) \\ (0.6,0) & (0.8,0) & (0.3,0) \end{bmatrix}$$

Using the definition of trace of fuzzy soft matrices, we find the following results:

$\mathrm{tr}\tilde{A}$ = (max{0.6,0.7,0.1},min{0,0,0})= (0.7,0)

$\mathrm{tr}\tilde{B}$= (max{0.4,0.9,0.3},min{0,0,0}) = (0.9,0)

Thus we have $\mathrm{tr}(\tilde{A})+\mathrm{tr}(\tilde{B})$ = (max{0.7,0.9},min{0,0,0}) = (0.9,0)

And $\mathrm{tr}(\tilde{A}+\tilde{B})$ = (max{0.6,0.9,0.3},min{0,0,0}) = (0.9,0)

Hence $\mathrm{tr}(\tilde{A}+\tilde{B}) = \mathrm{tr}(\tilde{A})+\mathrm{tr}(\tilde{B})$.

Proposition 4.11:

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]$ be fuzzy soft square matrices each of order n. Then $\mathrm{tr}(\tilde{A}) = \mathrm{tr}(\tilde{A}^{\mathrm{T}})$, where $\tilde{A}^{\mathrm{T}}$ is the transpose of $\tilde{A}$.

Proof:

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \gamma_{ij}^{\tilde{A}})]$, then $\tilde{A}^{\mathrm{T}} = [(\mu_{ji}^{\tilde{A}}, \gamma_{ji}^{\tilde{A}})]$

$\mathrm{tr}(\tilde{A}^{\mathrm{T}}) = (\max(\mu_{ii}^{\tilde{A}}), \min(\gamma_{ii}^{\tilde{A}})) = \mathrm{tr}(\tilde{A})$

Hence $\mathrm{tr}(\tilde{A}) = \mathrm{tr}(\tilde{A}^{\mathrm{T}})$.

Example 4.11:

Let $\tilde{A} = \begin{bmatrix} (0.5,0) & (0.3,0) & (0.8,0) \\ (0.6,0) & (0.4,0) & (0.9,0) \\ (0.2,0) & (0.7,0) & (0.1,0) \end{bmatrix}$;

$$\tilde{A}^{\mathrm{T}} = \begin{bmatrix} (0.5,0) & (0.6,0) & (0.2,0) \\ (0.3,0) & (0.4,0) & (0.7,0) \\ (0.8,0) & (0.9,0) & (0.1,0) \end{bmatrix}$$

$\mathrm{tr}(\tilde{A})$ = (max{0.5,0.4,0.1},min{0,0,0}) = (0.5,0)

$\mathrm{tr}(\tilde{A}^{\mathrm{T}})$ = (max{0.5,0.4,0.1},min{0,0,0}) = (0.5,0)

Hence $\mathrm{tr}(\tilde{A}) = \mathrm{tr}(\tilde{A}^{\mathrm{T}})$.

V. CONCLUSION

In this paper, we extend the concept with regard to the matrices of union, intersection, complement, addition, multiplication and trace of fuzzy soft matrices. For doing the said types of matrices with the help of reference function which are different from the existing definition by establishing some of its properties with example. We put forward that our work would enrich the scope of characterization of fuzzy soft matrices.

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