Side Slip Angle Control of Reusable Launch Vehicle using Backstepping

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***Abstract*— All physical systems are non-linear in nature. Backstepping is a control method for non-linear control system design. Backstepping designs by breaking down complex non-linear systems into smaller subsystems. Then designing Control Lyapunov Functions (CLF) and virtual controls for these subsystems and finally integrating these individual controllers by "stepping-back" through the subsystem and reassembling it from its component subsystems. The major advantage of backstepping is that it can avoid the cancellation of useful non-linearities which help in stabilization and tracking. Here, the X-38 reusable launch vehicle model is considered for the analysis. Since this model is nonlinear, a nonlinear control technique is more effective than other linearization methods. In this paper, a Lyapunov based non-linear backstepping controller is proposed for lateral dynamics. The controller exhibits stable and good tracking performance. Simulation under various initial conditions shows stability as expected in the CLF analysis.**

Keywords—Backstepping, Nonlinear Control

I. INTRODUCTION

Recent years have witnessed a rapid development of techniques for feedback control of non-linear systems. The backstepping is a powerful design tool, for non-linear and linear systems in the pure feedback and strict feedback forms [1]. This is a systematic design method which can be applied to wide variety of non-linear and linear systems. Unfortunately, its application fails in systems which do not appear (or are not transformable) in either of the above two forms.

Presently, the focus in the area of control theory has shifted from linear to non-linear systems, providing control algorithms for systems that are both more general and more realistic. Virtually, all physical systems are non-linear in nature. No physical system belongs to the class of linear time invariant system. Sometimes it is possible to describe the operation of a physical system by a linear model, such as ordinary linear differential equations. This is the case when mode of operation of the physical system does not deviate too much from the nominal set of operating conditions. Thus the analysis of linear systems occupies an important place in system theory. But in analyzing the behavior of any physical system, one often encounters situations where the linearised model is inadequate or inaccurate.

The effects of non-linear components can be avoided by restricting the operation of the systems over a small limited range, i.e., during the design of the controller, the plant is linearised about some specific equilibrium point and a linear controller is designed. Being designed for a specific equilibrium point it may not function well for other equilibrium points and so different controllers may be required as the operating conditions changes.

Backstepping is used in various applications such as flight path angle control [2], path tracing of mobile robots [3] and position and speed tracking dc motor and induction motor [4]. It is also used in re-entry vehicles as well as in missiles [5]. Backstepping is a design tool for a class of non-linear dynamic systems, which is based on Lyapunov stability theory. This can solve stabilization and tracking problems. Feedback linearization is also an approach to non-linear control design. In feedback linearization, exact plant model is required, whereas, in the latter, it is not compulsory to know the plant fully.

Structured singular value synthesis (µ-synthesis) [7] is a multi-variable control technique that provides an effective way to guarantee robust performance in the face of plant uncertainty. Unfortunately, µ-synthesis controllers are extremely difficult to gain schedule. Due to the initial high order of the controllers generated using µ-synthesis, some form of model order reduction is usually necessary before the controllers can be implemented. Finding a correspondence between the states of two different reduced-order controllers can be extremely difficult, if not impossible. One modern control methodology that seeks to eliminate the gain-scheduling problem is dynamic inversion [8]. But, dynamic inversion by itself cannot guarantee any level of robustness to unmodeled dynamics or other plant uncertainties which is the main disadvantage of this technique. Backstepping control technique does not require gain scheduling, which is its main advantage.

This paper is organized as follows. In the following section the design of backstepping is explained in detail. Section 3 and 4 deals with the system modeling and backstepping controller design for RLV respectively. Section 5 presents the simulation results and discussion. The conclusions of the work are included in Section 6.

II. BACKSTEPPING

A. Theory

Backstepping designs a controller by breaking down the complex nonlinear system into smaller subsystems. The design of Control Lyapunov Function(CLF) and virtual controllers for these subsystems is the next step. In the final step all these individual controllers are integrated by stepping back through the subsystems. If a control Lyapunov function exists, a control law which makes the system globally asymptotically stable can be found. Backstepping is a procedure which finds both a control Lyapunov function and a control law simultaneously.

For applying Backstepping, the system should be either in pure feedback form or strict feedback form. Many physical systems cannot be written in this form. Therefore, to apply backstepping some of the physical properties are neglected when modeling the system. But it is to be ensured that the neglected physical property does not affect the stability of the closed loop system.

$$\begin{aligned}
\dot{x} &= f(x) + g(x)\xi_1 \\
\dot{\xi}_1 &= f_1(x,\xi_1) + g_1(x,\xi_1)\xi_2 \\
\dot{\xi}_2 &= f_2(x,\xi_1,\xi_2) + g_2(x,\xi_1,\xi_2)\xi_3 \\
&\vdots \\
\dot{\xi}_k &= f_k(x,\xi_1,\ldots,\xi_k) + g_1(x,\xi_1,\ldots,\xi_k)u
\end{aligned} \tag{1}$$

B. Design Procedure

To show how to find a control Lyapunov function and a control law, a short design example is considered [1][9]. The system that is to be controlled is given below.

$$\begin{aligned}
\dot{x} &= f(x) + g(x)\xi \\
\dot{\xi} &= a(x,\xi) + b(x,\xi)u
\end{aligned} \tag{2}$$

where $x \in R^n$ and $\xi \in R$ are state variables and $u \in R$ is the control input. First ξ is regarded as a control input for the x-subsystem. ξ can be chosen in any way to make the x-subsystem globally asymptotically stable. The choice $\xi^{des}(x)$ is called a virtual control law. For the x-subsystem a control Lyapunov function, $V_1(x)$, can be chosen so that with the virtual control law, the time derivative of Lyapunov function becomes negative definite.

$$\dot{V}_1(x) = V_{1x}\dot{x} = V_{1x}(x)(f(x) + g(x)\xi^{des}(x)) < 0, \quad x \neq 0 \tag{3}$$

A new state is introduced which represents the error variable

$$\tilde{\xi} = \xi - \xi^{des}(x) \tag{4}$$

The system shown in equation (2) is then written in terms of these new variables

$$\begin{aligned}
\dot{x} &= f(x) + g(x)(\tilde{\xi} + \xi^{des}(x)) \\
\dot{\tilde{\xi}} &= a(x,\tilde{\xi} + \xi^{des}(x)) + b(x,\tilde{\xi} + \xi^{des}(x))u - \\
&\quad \frac{\partial \xi^{des}(x)}{\partial x}(f(x) + g(x)(\tilde{\xi} + \xi^{des}(x)))
\end{aligned} \tag{5}$$

For the system given above a control Lyapunov function is constructed from $V_1(x)$ by adding a quadratic term which penalizes the error variable $\tilde{\xi}$,

$$V_2(x,\tilde{\xi}) = V_1(x) + \frac{1}{2}\tilde{\xi}^2 \tag{6}$$

Differentiating $V_2(x, \tilde{\xi})$ with respect to time

$$\begin{aligned}
\dot{V}_2(x,\xi) &= V_{1x}(x)\left(f(x) + g(x)\xi^{des}(x) + g(x)\tilde{\xi}\right) + \\
&\quad \tilde{\xi}\left(a(x,\xi + \xi^{des}(x))\right) + \tilde{\xi}\{b(x,\xi + \xi^{des}(x))u \\
&\quad - \frac{\partial \xi^{des}(x)}{\partial x}(f(x) + g(x)(\tilde{\xi} + \xi^{des}(x)))\}
\end{aligned} \tag{7}$$

Equation (7) can be rewritten in the following way if the variables that the functions depend on are omitted. To guarantee stability $\dot{V}_2$ has to be negative definite. This can be achieved by choosing the control input, u in (7) as

$$u = \frac{1}{b}\left(\frac{\partial \xi^{des}(x)}{\partial x}(f + g(\tilde{\xi} + \xi^{des}(x))) - a - V_1 g - k\tilde{\xi}\right) \tag{8}$$

where k > 0. Then $\dot{V}_2$ becomes

$$\dot{V}_2 = V(f + g\xi^{des}) - k\tilde{\xi}^2 \leq 0 \tag{9}$$

If u is not the actual control input but a virtual control law consisting of state variables, then the system can be further expanded by starting over again. Hence the backstepping design procedure is recursive.

III. REUSABLE LAUNCH VEHICLE

Reusable launch vehicles typically include ascent and descent phases of flight. It effectively reduces the cost of accessing the space. The control of a reusable launch vehicle (RLV) is a very difficult task, since the vehicle dynamics changes dramatically as altitude and Mach number vary from atmospheric entry at hypersonic speeds to subsonic approach and landing. During descent phase, the aerodynamic forces are comparable with the gravitational forces. Consequently, the axial and transverse loads acting on the vehicle becomes important issues while designing the system. Therefore, the vehicle has to be equipped with a high performance and reliable flight control system. Because of the nonlinear model of the re-entry vehicle, the choice of control design technique is a major concern in any flight control system.

A. X38 Vehicle Model

The system considered here for backstepping design is a Reusable Launch Vehicle (RLV) during its reentry phase[6]. The X-38 vehicle has two sets of control surfaces: a pair of elevon control surfaces, located on the lower rear of the vehicle; and a pair of rudders, one at the top of each of the vertical fins.

Each surface is deflected independently to provide the required control authorities. The elevon deflections are averaged to give the total elevon angle or elevator angle for pitch control.

$$\delta_e = \frac{\delta_{eL} + \delta_{eR}}{2} \tag{10}$$

The average of the difference gives aileron angles for roll control.

$$\delta_a = \frac{\delta_{eL} - \delta_{eR}}{2} \tag{11}$$

Similarly, the rudder deflections are averaged to give total rudder for yaw control.

$$\delta_r = \frac{\delta_{rL} + \delta_{rR}}{2} \tag{12}$$

IV. BACKSTEPPING CONTROLLER DESIGN

A Lyapunov based non-linear backstepping controller is designed for the reusable launch vehicle. The controller is designed for the lateral dynamics of the vehicle. The lateral dynamics are made out of a subsystem with the states

$$x = [\beta \quad r]^T \tag{13}$$

and the input vector

$$u = \delta_r \tag{14}$$

The lateral dynamics of the RLV consists of side slip angle
β and yaw rate *r*. The objective of the controller is to regulate and track the commanded slide slip angle. The equations of motion for the lateral dynamics are as follows.

$$\begin{aligned}\dot{\beta} &= \frac{Y_\beta}{V_t}\sin\beta + \frac{Y_p}{V_t}p + \left[\frac{Y_r}{V_t} - 1\right]r + \frac{g}{V_t}\phi + \frac{Y_{\delta r}}{V_t}\sin\delta_r \\ \dot{r} &= \frac{1}{I_z}[N_\beta \sin\beta + N_p p + N_r r + N_{\delta a}\sin\delta_a + N_{\delta r}\sin\delta_r]\end{aligned} \tag{15}$$

The equations are represented in terms of aerodynamic forces and moments, where *Y* and *N* are the side force and yawing moment respectively. Here, ϕ is the roll angle and *p* is the roll rate. The coefficients related to the motion variables, *β, p* and *r*, are called stability derivatives, while those related to the control inputs, δ_a and δ_r, are termed control derivatives. I_z represents the moment of inertia in the *z* direction, *g* is the acceleration due to gravity and V_t is the vehicle velocity.

For applying backstepping, the system should be either in pure feedback form or in strict feedback form. Hence only the effect of rudder is taken into consideration. Even otherwise the moment due to ailerons, $N_{\delta a}$ is negligible compared to moment due to rudders. Then the above equation becomes.

$$\begin{aligned}\dot{\beta} &= \frac{Y_\beta}{V_t}\sin\beta + \frac{Y_p}{V_t}p + \left[\frac{Y_r}{V_t} - 1\right]r \\ \dot{r} &= \frac{1}{I_z}[N_\beta \sin\beta + N_r r + N_{\delta r}\sin\delta_r]\end{aligned} \tag{16}$$

The control Lyapunov functions are selected as

$$V_1 = \frac{1}{2}x_1^2 \tag{17}$$

$$V_2 = V_1 + \frac{1}{2}z^2 \tag{18}$$

Where *z* is the error variable and x_2^{des} is the virtual control law for the first subsystem.

$$z = x_2 - x_2^{des} = x_2 - c_1 x_1 \tag{19}$$

And the derivative of control Lyapunov functions become

$$\dot{V}_1 = \frac{Y_\beta}{V_t}x_1 \sin x_1 - c_1 x_1^2 \tag{20}$$

$$\begin{aligned}\dot{V}_2 &= \frac{Y_\beta}{V_t}x_1 \sin x_1 - c_1 x_1^2 - x_1 z + z[\frac{N_\beta}{I_z}\sin x_1 + \\ &\frac{N_{\delta r}}{I_z}\sin u - \frac{Y_\beta}{V_t}c_1 \sin x_1 + c_1 x_2]\end{aligned} \tag{21}$$

The desired control law to make $\dot{V}_2$ negative definite is given by

$$\begin{aligned}u^{des} &= \sin^{-1}\frac{I_z}{N_{\delta r}}[\frac{-N_\beta}{I_z}\sin x_1 + \frac{Y_\beta}{V_t}c_1 \sin x_1 - \\ &(c_1 + c_2)x_2 + (1 + c_1 c_2)x_1]\end{aligned} \tag{22}$$

The variables c_1 and c_2 are positive design constants and are chosen arbitrarily. Then the derivative of V_2 becomes negative definite as follows

$$\dot{V}_2 = \frac{Y_\beta}{V_t}x_1 \sin x_1 - c_1 x_1^2 - c_2 z^2 \tag{23}$$

V. SIMULATION RESULTS

The control law given in equation 22 is simulated in MATLAB/SIMULINK platform and the simulation results for RLV with backstepping controller are shown in the following figures.

The initial side slip angle given is 1 degree and yaw rate is 1 degree/sec. From figure 1 and 2 we can see that the side slip angle and yaw rate are regulated to zero as desired. It is clear that when the values of c_1 and c_2 increases the response becomes faster i.e., when these constants increases the

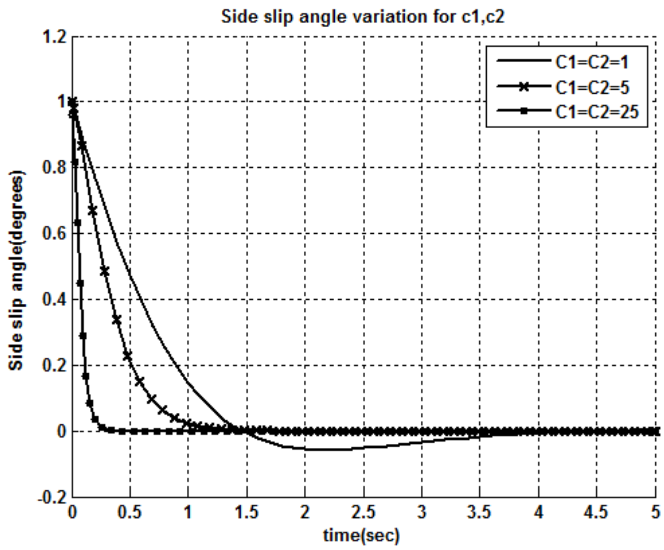

Fig. 1. Stabilization of Side Slip Angle

derivative of Lyapunov function becomes more negative definite. Lyapunov function is a representation of energy. Hence the energy decreases and the system response become faster.

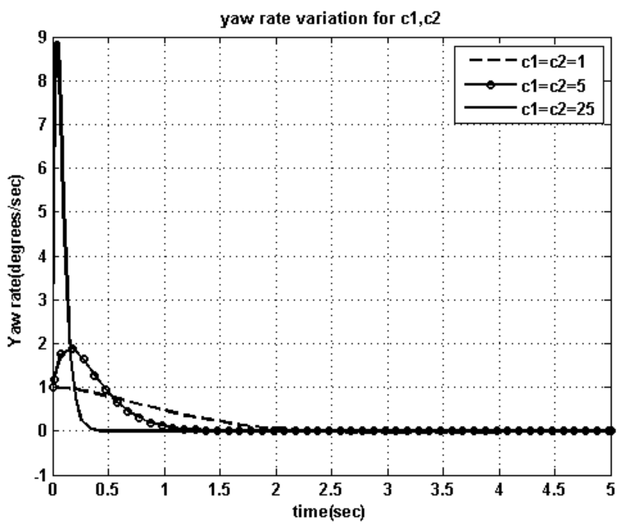

Fig. 2. Yaw Rate Variation with c_1 and c_2

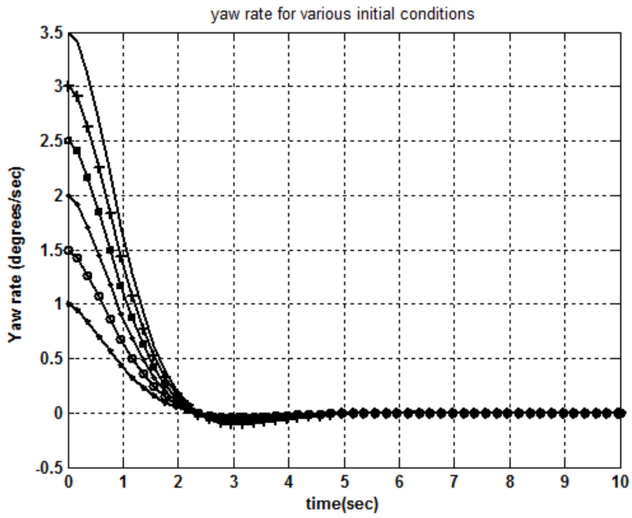

Fig. 3. Stabilization of Yaw Rate

The yaw rate for various initial conditions is plotted in figure 3. From the figure we can see that the yaw rate is regulated to zero irrespective of the initial conditions also.

In addition to the stabilization problem, the backstepping controller exhibits good tracking performance. In figure 4 the side slip angle follows the commanded sinusoidal signal and for higher values of c_1 and c_2 better tracking performance is obtained.

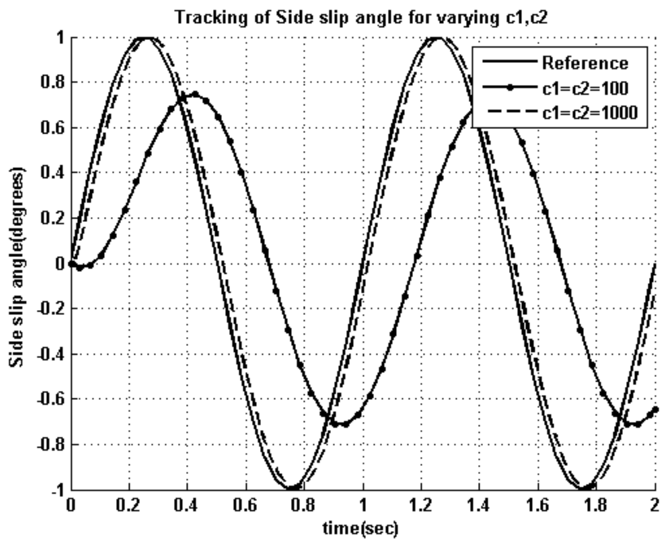

Fig. 4. Tracking of sinusoidal signal

The phase plots for 6 different initial conditions were plotted in figure 5. The points are given by (-8,-6), (-8,6),(-4,4),(4,3), (6,5) and (8,-7).It can be seen that the phase portrait converges to the equilibrium point (0,0) irrespective of the operating point. As it converges to the equilibrium point the system is rendered with asymptotic stability. It shows that the backstepping controller ensures asymptotic stability.

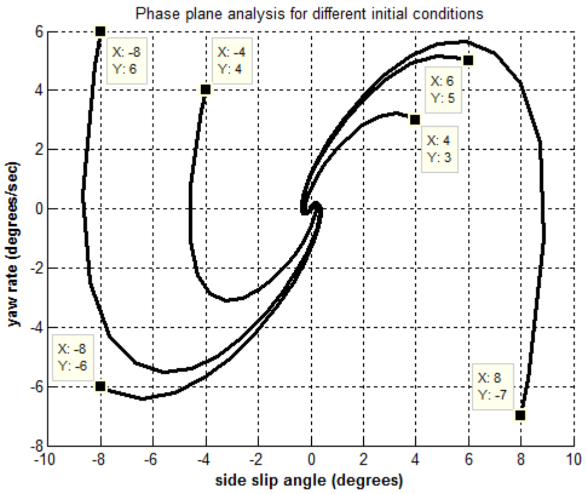

Fig. 5. Phase portrait with different initial conditions

VI. CONCLUSIONS

Backstepping is a recursive procedure for global stabilization by state feedback. It is based on Lyapunov theory which is used to guarantee stability. Here backstepping technique applied to RLV and the control law has been designed for the lateral dynamics of the vehicle. The design objective is to regulate and track the side slip angle. The corresponding vehicle model with backstepping controller is simulated using SIMULINK and the results are analyzed. The side slip angle and yaw rate are regulated as desired and good tracking is obtained. Here the controller tracks the commanded sinusoidal signal and the stabilization is achieved. From the phase portrait analysis it can be seen that the system converges to a single equilibrium point irrespective of the initial conditions. According to Lyapunov's main stability theorem, the negative definiteness of derivative of Lyapunov function ensures stability.

The overall system performance of the lateral dynamics of the reusable launch vehicle using backstepping is analyzed in this paper. The backstepping control law is computationally much simpler, and is globally stabilizing. In addition, the simulation results have clearly illustrated that the proposed non-linear backstepping controllers are quite effective and efficient for the design of RLV.

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