

# Sequential Quadratic Programming Algorithm Based Optimization of Shell and Tube Type Heat Exchangers

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**Abstract**-Shell and Tube type heat exchangers are having special importance in boilers, oil coolers, condensers and pre-heaters. These are also widely used in process applications as well as the refrigeration and air conditioning industry. The robustness and medium weighted shape of Shell and Tube type heat exchangers make them well suited for high pressure operations. The basic configuration, the thermal analysis and design of such exchangers form an included part of the mechanical, thermal and chemical engineering scholars for their curriculum and research activity.

Traditional design approaches using graph sheets are time consuming, these may not considered all the variables and constraints simultaneously. On the other hand some new evolutionary algorithms viz. Genetic Algorithm (GA), Particle swarm optimization (PSO), Imperialist competitive algorithm (ICA) are not simple to understand by every designer and are not easy to be implemented. Therefore, in present work, a new shell and tube heat exchanger optimization design approach is discussed based on sequential quadratic programming (SQP). The SQP algorithm has some good features in reaching to the global minimum in comparison to other evolutionary algorithms. In present study, SQP technique has been applied to minimize the total cost which includes capital investment and total discounted operating cost. The design variables considered in the present work are tube outer diameter, shell diameter and baffle spacing. A matlab code is developed based on SQP for optimal design of shell and tube heat exchangers. The different test cases are solved using code to demonstrate the effectiveness and accuracy of the proposed algorithm. The results using developed

code are compared to those obtained from previous literatures. It is found that the SQP algorithm is simple and it can be successfully applied for optimal design of shell and tube heat exchangers with higher accuracy.

**Key Words:** *Shell and tube type heat exchangers, Optimal Design, Sequential Quadratic Programming.*

## 1. INTRODUCTION

A shell and tube heat exchanger is a class of heat exchanger designs. It is the most common type of heat exchanger in oil refineries and other large chemical processes, and is suited for higher-pressure applications. As its name implies, this type of heat exchanger consists of a shell (a large pressure vessel) with a bundle of tubes inside it. One fluid runs through the tubes, and another fluid flows over the tubes (through the shell) to transfer heat between the two fluids. The set of tubes is called a tube bundle, and may be composed of several types of tubes: plain, longitudinally finned, etc.

Shell and tube type heat exchanger is probably the most used and widespread type of the heat exchanger's classification. It is used most widely in various fields such as oil refineries, thermal power plants, chemical industries and many more. This high degree of acceptance is due to the comparatively large ratio of heat transfer area to volume and weight, easy cleaning methods, easily replaceable parts etc. Shell and tube type heat exchanger consists of a number of tubes through which one fluid flows. Another fluid flows through the shell which encloses the tubes and other supporting items like baffles, tube header sheets, gaskets etc. The heat exchange between the two fluids takes through the wall of the tubes. A schematic diagram of shell and tube type heat exchanger is given below: [1,2]

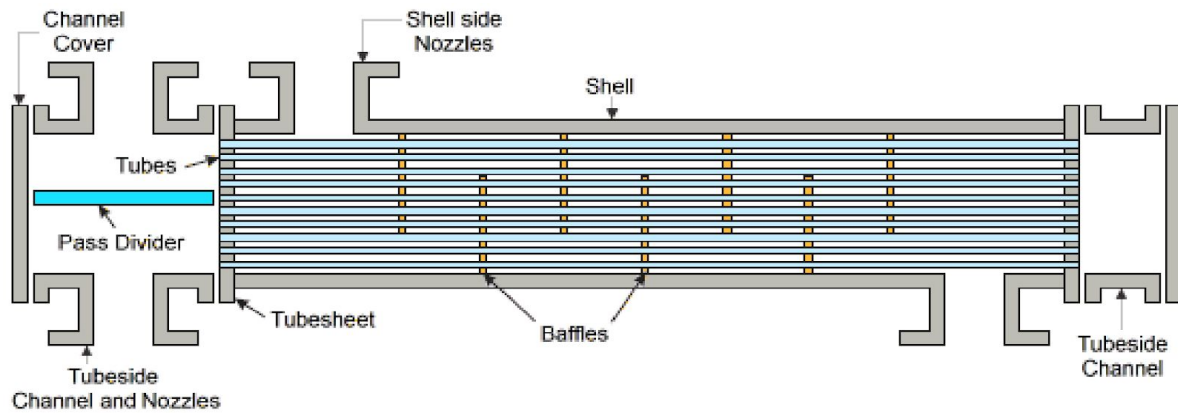


Fig 1.1: A Shell and Tube Type Heat Exchanger [1]

## NOMENCLATURE

$a_1, a_2, a_3$	numerical constant	$N_t$	number of tubes
$a_s$	cross sectional area normal to flow direction ( $m^2$ )	$P$	pumping power (W)
$B$	baffles spacing (m)	$Pr$	Prandtl number
$Cl$	clearance (m)	$P_t$	tube pitch (m)
$C_p$	specific heat ( $kJ/kg \cdot K$ )	$R_f$	fouling resistance ( $m^2 \cdot K/W$ )
$C_i$	capital investment (Rs)	$Q$	heat duty (W)
$C_e$	energy cost (Rs /kW hr)	$Re$	Reynolds number
$C_o$	annual operating cost (Rs/yr)	$S$	heat transfer surface area ( $m^2$ )
$C_{od}$	total discounted operating cost (Rs)	$T$	temperature ( $^{\circ}C$ )
$C_{tot}$	total annual cost (Rs)	$U$	overall heat transfer coefficient ( $W/m^2 \cdot K$ )
$d_o$	tube diameter (m)	$v$	fluid velocity (m/s)
$D_s$	shell diameter (m)	<b>Greek symbols</b>	
$f$	friction factor	$\Delta P$	pressure drop (Pa)
$F$	correction factor	$\Delta T$	logarithmic mean temperature difference ( $^{\circ}C$ )
$h$	heat transfer coefficient ( $W/m^2 \cdot K$ )	$\mu$	dynamic viscosity (Pa s)
$H$	annual operating time (hr/yr)	$\nu$	kinematic viscosity ( $m^2/s$ )
$i$	annual discount rate (%)	$\rho$	density ( $kg/m^3$ )
$k$	thermal conductivity ( $W/m \cdot K$ )	$\eta$	overall pumping efficiency
$K_1$	numerical constant	<b>Subscripts</b>	
$L$	tubes length (m)	$i$	inlet
$m$	mass flow rate (kg/s)	$o$	outlet
$n$	number of tubes passages	$s$	belonging to shell
$ny$	equipment life (year)	$t$	belonging to tube
$n_1$	numerical constant		

## 2. LITERATURE REVIEW

The basic configuration of shell and tube heat exchangers, the thermal analysis and design of such exchangers form an included part of the mechanical, thermal, chemical engineering scholars for their curriculum and research activity. In recent past year, the improvements in computing cost have increased the interest of engineers and researchers to simulate their problems with computational and numerical methods. A lot of computational tools and methods have been developed in the last decades to analyse fluid dynamics, combustion, and different modes of heat transfer.

Srivastava A.K., Dubey V.V.P., Verma R.R., Verma P.S. have presented an overview of shell and tube

type heat exchanger, constructional details, design methods and the reasons for the wide acceptance of shell and tube type heat exchangers [1]. Shah R.K. and Sekulic D.R. have given the classification of shell and tube type heat exchangers based on heat transfer process, constructional features and flow arrangements [2]. Sinnot R.K. has presented various chemical processing equipment theory and design, (e.g. heat exchanger,) [3]. Kern D.Q. has discussed various types of heat transfer processes and design of engineering equipment explained [4].

Caputo A.C., Pelagagge M.P., Saline P., have presented a procedure for optimal design for shell and tube heat exchangers which utilized a genetic algorithm to minimize the total discounted cost of the equipment

including the capital investment and pumping related annual energy expenditures [5]. Taal M., Bulatov I., Klemes J., Stehlik P. have given the most common methods used for cost estimation of heat exchange equipment in the process industry and the sources of energy price projections and considered ten methods for heat exchanger costing procedure [6]. Peters MS, Timmerhaus K.D. have presented methods of plant design and economics. Further these methods are used for calculation of heat exchanger's total annual cost and pressure drop at shell side [7]. Philip G.E., Laurent J.O., Michael L.W., Linda P.R., Sharmad V., have proposed a sequential quadratic programming (SQP) method for the optimal control of large-scale dynamical systems and various steps of sequential quadratic programming method algorithm is also discussed [8]. Philip G.E., Wong E., have proposed the sequential quadratic programming (SQP) method for the solution of constrained nonlinear optimization problems and also compared with other optimization methods [9]. Patel V.K., Rao R.V., have discussed a non-traditional optimization technique; called particle swarm optimization (PSO), for design optimization of shell and tube heat exchangers from economic view point and minimization of total annual cost is considered as an objective function [10]. Hadidi A., Hadidi A., Nazari A., have presented a new design approach for shell and tube heat exchangers using imperialist competitive algorithm (ICA) from economic point of view. ICA technique has been applied to minimize the total cost of the equipment including capital investment and the sum of discounted annual energy expenditures related to pumping of shell and tube heat exchanger. Finally the results are compared to those obtained by other literature approaches [11].

### 3.1 SEQUENTIAL QUADRATIC PROGRAMMING ALGORITHM

Sequential Quadratic Programming (SQP) is one of the most successful methods for the numerical solution of constrained nonlinear optimization problems. It relies on a strong theoretical foundation and provides powerful algorithmic tools for the solution of large-scale technologically relevant problems. The problem which is considered to solve is to minimize some objective or cost function,  $f(x)$ , subject to constraints  $a_i(x) = 0$  for  $i = 1, 2, \dots, p$  and  $c_j(x) > 0$  for  $j = 1, 2, \dots, q$ . The  $f(x)$  can be a linear or nonlinear objective function.  $a_i(x)$  and  $c_j(x)$  are constraints which are functions of  $x$  and can be nonlinear.  $f(x)$ ,  $a_i(x)$  and  $c_j(x)$  are assumed to be continuous and have continuous second partial derivatives, and the feasible region of this problem is assumed to be nonempty. A solution of the such type of problem generally requires an iterative procedure to establish a direction of search at each major

iteration. This is usually achieved by the solution of an LP, a QP, or an unconstrained subproblem. The quadratic programming subproblem is created using initial objective function as quadratic and linearizing constraints about a starting point. This method uses constraints steepest descent (CSD) method for search direction. The solution of quadratic programming problem is used as starting point for next iteration; therefore this method is called Sequential Quadratic Programming method.

It is important to note that method work equally well when initiated from feasible or infeasible points. It can also treat equality and inequality constraints.

### 4.1 NUMERICAL DATA AND RESULTS

The effectiveness and validity of the suggested approach in this work is assessed by analyzing some relevant case studies taken from the literature, in order to have reliable reference sizing data for the sake of comparison. The following two different test cases, representative of a wide range of possible applications, are considered. The first case study of this work is a heat exchanger for methanol and brackish water, taken from [3]. The heat load is 4.34 MW. This heat exchanger has two tube side passages with triangle pitch pattern and one shell side passage. The second case study is taken from [4] is a heat exchanger which transfers a heat load of 0.46 MW between distilled water–raw water heat exchanger and has two tube side passages with triangle pitch pattern and one shell side passage. The same configuration of above cases is retained in the present approach. For each case the original design specifications, shown in Table [2,4], were taken as input to the optimization algorithm and the resulting optimal exchangers design parameters given by the SQP method were compared with the original design suggested in literatures [3] and [4].

The following upper and lower bounds for the optimization variables were imposed: Tubes outside diameter  $d_o$  ranging from 0.01 m to 0.051 m; Shell internal diameter  $D_s$  ranging between 0.1 m and 1.5 m; Baffles spacing  $B$  ranging from 0.05 m to 0.5 m. All values of discounted operating costs were computed with  $n_y=10$  years, annual discount rate=10%, Energy cost  $C_e = 0.12$  Rs/kW hr, And annual amount of work hours =7000 hr/yr similar to other researches. [3, 7, 10, 11]

**4.2 Case study 1:** This case study is taken from Sinnott [3] and the process inputs and physical properties are given in table 4.1:

Table 4.1: The process input and physical properties for case study 1

Case-1	m (kg/s)	T <sub>i</sub> (°C)	T <sub>o</sub> (°C)	ρ (kg/m <sup>3</sup> )	C <sub>p</sub> (kJ/kg)	μ (Pa-s)	k (W/mK)	R <sub>f</sub> (m <sup>2</sup> K/W)
Shell side: methanol	27.8	95	40	750	2.84	0.00034	0.19	0.00033
Tube side: sea water	68.9	25	40	995	4.2	0.00080	0.59	0.00020

Table 4.2: Parameters of the optimal shell and tube heat exchangers for case study 1 using different optimization methods

Parameters	Literature [3]	GA [5]	PSO [10]	ICA [11]	SQP [present work]
d <sub>o</sub> (m)	0.02	0.016	0.015	0.015	0.015
D <sub>s</sub> (m)	0.894	0.83	0.81	0.879	0.786
B (m)	0.356	0.5	0.424	0.5	0.5
L (m)	4.83	3.379	3.115	3.107	3.2115
P <sub>t</sub> (m)	0.025	0.02	0.0187	0.01875	0.0188
Cl (m)	0.005	0.004	0.0037	0.00375	0.0037
D <sub>e</sub> (m)	0.014	0.011	0.0107	0.011	0.0107
N <sub>t</sub>	918	1567	1658	1752	1550
v <sub>t</sub> (m/s)	0.75	0.69	0.67	0.699	0.7885
Re <sub>t</sub>	14925	10936	10503	10429	11769
Pr <sub>t</sub>	5.7	5.7	5.7	5.7	5.7
h <sub>t</sub> (W/m <sup>2</sup> K)	3812	3762	3721	3864	4814.5
f <sub>t</sub>	0.028	0.031	0.0311	0.031	0.030
ΔP <sub>t</sub> (Pa)	6251	4298	4171	5122	7449.8
a <sub>s</sub> (m <sup>2</sup> )	0.032	0.0831	0.0687	0.0879	0.0786
v <sub>s</sub> (m/s)	0.58	0.44	0.53	0.42	0.4718
Re <sub>s</sub>	18381	11075	12678	9917	10928
Pr <sub>s</sub>	5.1	5.1	5.1	5.1	5.1
h <sub>s</sub> (W/m <sup>2</sup> K)	1573	1740	1950.8	1740	1957
f <sub>s</sub>	0.33	0.357	0.349	0.362	0.3569
ΔP <sub>s</sub> (Pa)	35789	13267	20551	12367	14318
U (W/m <sup>2</sup> K)	615	660	713.9	677	740.4033
S (m <sup>2</sup> )	278.6	262.8	243.2	256.6	234.4616
C <sub>i</sub> (Rs)	3863025	3694425	3483975	3627750	3389100
C <sub>o</sub> (Rs/year)	158325	71025	77902.5	73125	82897.5
C <sub>od</sub> (Rs)	972975	436350	508365	449625	509362.5
C <sub>o</sub> (Rs)	4836000	4130775	3992332.5	4077450	3898462.5

Fig 4.1: Cost comparison for case study 1

Table 4.3: The process input and physical properties for case study 2

Case-2	m (kg/s)	T <sub>i</sub> (°C)	T <sub>o</sub> (°C)	ρ (kg/m <sup>3</sup> )	C <sub>p</sub> (kJ/kg)	μ (Pa-s)	k (W/mK)	R <sub>f</sub> (m <sup>2</sup> K/W)
Shell side: distilled water	22.07	33.9	29.4	995	4.18	0.00080	0.62	0.00017
Tube side: raw water	35.31	23.9	26.7	999	4.18	0.00092	0.62	0.00017

Table 4.4: Parameters of the optimal shell and tube heat exchangers for case study 2 using different optimization methods

Parameters	Literature[4]	GA[5]	PSO[10]	ICA[11]	SQP[present work]
d <sub>o</sub> (m)	0.019	0.016	0.0145	0.015	0.015
D <sub>s</sub> (m)	0.387	0.62	0.59	0.66	0.576
B (m)	0.305	0.440	0.423	0.5	0.5
L (m)	4.880	1.548	1.45	1.467	1.717
P <sub>t</sub> (m)	0.023	0.020	0.0181	0.01875	0.0187
Cl (m)	0.004	0.004	0.0036	0.00375	0.0037
D <sub>e</sub> (m)	0.013	0.015	0.0103	0.011	0.0107
N <sub>t</sub>	160	803	894	897	781
v <sub>t</sub> (m/s)	1.76	0.68	0.74	0.745	0.8001
Re <sub>t</sub>	36409	9487	9424	10390	10425
Pr <sub>t</sub>	6.2	6.2	6.2	6.2	6.2
h <sub>t</sub> (W/m <sup>2</sup> K)	6558	6043	5618	5412	4489.8
f <sub>t</sub>	0.023	0.031	0.0314	0.031	0.0311
ΔP <sub>t</sub> (Pa)	62812	3673	4474	3497	4442
a <sub>s</sub> (m <sup>2</sup> )	0.0236	0.0541	0.059	0.0657	0.0576
v <sub>s</sub> (m/s)	0.94	0.41	0.375	0.36	0.3851
Re <sub>s</sub>	16200	8039	4814	5130	10059
Pr <sub>s</sub>	5.4	5.4	5.4	5.4	5.4
h <sub>s</sub> (W/m <sup>2</sup> K)	5735	3476	4088.3	5239	6337.9
f <sub>s</sub>	0.337	0.374	0.403	0.3998	0.3614
ΔP <sub>s</sub> (Pa)	67684	4365	4721	4696	5022.9
U (W/m <sup>2</sup> K)	1471	1121	1177	1243	1221.5
S (m <sup>2</sup> )	46.6	62.5	59.15	62.05	63.163
C <sub>i</sub> (Rs)	1241175	1437225	1396050	1431975	1378230
C <sub>o</sub> (Rs/year)	334950	20400	20700	20475	21121.515
C <sub>od</sub> (Rs)	2058000	125325	127200	125925	129780
C <sub>tot</sub> (Rs)	3299175	1562550	1523250	1557900	1508010

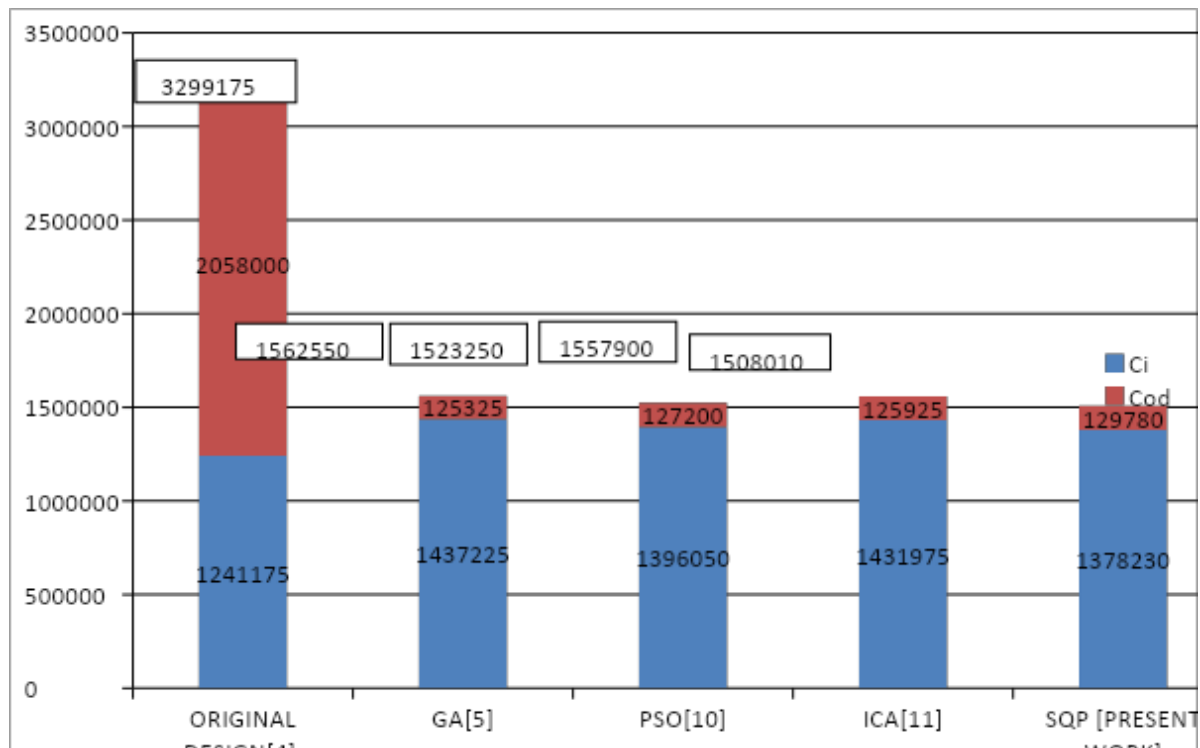


Fig 4.2: Cost comparison for case study 2

The figure shows results and graphs of case study 2 by SQP used in optimization tool of Matlab. It may be observed that the objective function converges within 9 iterations for case

2. The first graph shows the optimization variables

### 5.1 CONCLUSIONS AND FUTURE SCOPES

Identifying the best and cheapest heat exchanger for a specific heat duty is a tough decision making task. The present work focuses upon total cost minimization of shell and tube type heat exchanger. The total cost includes capital investment cost and discounted operating cost. The design variables tube diameter, shell diameter and baffle spacing along with bounds and a nonlinear constraint have been considered. The resulting optimization problem has been solved using sequential quadratic programming (SQP)

values, second graphs shows the optimization function value with respect to the iterations and third graph shows maximum constraint violation with respect to the iterations.

algorithm, which is simple and easy for implementation.

A code has been implemented in Matlab for optimization purposes. Two cases are considered from previous literatures. The code developed in present work using SQP converges to optimum value of the objective function within quite few iterations. This feature signifies the importance of SQP. In each case study, starting point may either be feasible or infeasible, optimization problem converges to same optima.

CASE STUDY1: lower bound = [0.015 0.1 0.05] and upper bound = [0.051 1.5 0.5]

STARTING POINT	FINAL POINT	IMPROVEMENT IN TOTAL COST WITH RESPECT TO			
		ORIGINAL DESIGN[3]	GA[5]	PSO[10]	ICA[11]
FEASIBLE POINT [0.02 0.2 0.1]	[0.015 0.786 0.5]				
INFEASIBLE POINT [0.01 0.2 0.1]	[0.015 0.786 0.5]	19.38%	5.62%	2.35%	4.389%



CASE STUDY 2: lower bound = [0.015 0.1 0.05] and upper bound = [0.051 1.5 0.5]

STARTING POINT	FINAL POINT	IMPROVEMENT IN TOTAL COST WITH RESPECT TO			
FEASIBLE POINT [0.02 0.2 0.1]	[0.015 0.576 0.5]	ORIGINAL DESIGN[4]	GA[5]	PSO[10]	ICA[11]
INFEASIBLE POINT [0.01 0.2 0.1]	[0.015 0.576 0.5]	54.291%	3.49%	1.00%	3.202%

It may be concluded that total cost of a shell and tube type heat exchanger is decreased by using SQP in each case study and the obtained results show improvement as compare to those presented in previous literatures. In present work, the total cost of shell and tube type heat exchanger is optimized considering three design variables, two tube side passages and one shell side passage using SQP. In future, number of design variables such as

[12] length, pitch, tube side passages and shell side passages may be increased, for the optimization of total cost. Further, the algorithm used in present work may also be utilised in some other applications like in maximizing the total revenue by a hydroelectric power plant.

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