Special Issue - 2015

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
NACTET-2015 Conference Proceedings

Sensor Fault Diagnostics of a Flexible Link Robot using Linear Matrix Inequalities

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***Abstract*—A stable observer design for Lipschitz non-linear systems like flexible link Robot is considering using Linear Matrix Inequalities. The methodology is applied to the system with three or more sensors in which state is observable through any one of the sensor measurements. By using this observer gain, we can find the residues, which are used for fault identification. Distance to unobservabilty and its relation to observer design is also considering here. A stable Observer is also designed for the flexible link robot by this method.**

I. Introduction

Observer design for non–linear systems has been a very active field of research during the last decade. The introduction of geometric techniques led to great success in the development of controllers for non–linear systems. Many attempts have been made to achieve results of equally wide applicability for state estimation. But the observer problem found to be more difficult than the controller problem. Krener and Respondek (1985), Krener and Isidori (1983) and Xiao-Hua and Gao (1985) attempted a coordinate transformation so that the state estimation error dynamics were linear in the new coordinates. Necessary and sufficient conditions for the existence of such a coordinate transformation have been established but in practice are extremely difficult to satisfy. Even if these conditions are satisfied, the construction of the observer still remains a difficult task due to the need to solve a set of simultaneous partial differential equations to obtain the actual transformation function.

This present paper assumes partial state measurement and concentrates on developing a fault diagnostic methodology for systems with Lipschitz non-linear dynamics. A continuous system is said to be globally Lipschitz if the derivative at all points of the function has absolute value, which is always less than a definite real number. That definite real number is called Lipschitz constant γ. The solution is unique and existing if the non–inear function $\phi(x,u)$ satisfies the Lipschitz condition given by,

$$\frac{\|\phi(x,u)-\phi(\hat{x},u)\|}{\|x-\hat{x}\|} \leq \gamma \tag{1}$$

l A fault diagnostic technique using Linear Matrix Inequality(LMI)for Robotic system with Lipchitz non–linear dynamics is presented here. The system use three sensors, in which the state is observable through any one of the sensor measurements. The observer gain matrix is calculated for finding the residues in the system. Fault is determined using these residues. The paper not only provides sufficient conditions on the observer gain matrix for enabling isolation of faults but also develops an explicit and systematic numerical procedure for design of the observer gain. This paper also presents the observer design for flexible link robot using distance to un–observability technique.

II. Flexible Link Robot

Consider a one–link manipulator with revolute joints actuated by a motor as shown in The elasticity of the joint can be well-modelled by a linear torsional spring. The elastic coupling of the motor shaft to the link introduces an additional degree of freedom. The states of this system are motor position and velocity, and the link position and velocity.

$$\dot{\theta_m} = \omega_m \tag{2}$$

$$\dot{\theta_1} = \omega_1 \tag{3}$$

$$\dot{\omega_m} = \frac{K}{J_m}(\theta_1 - \theta_m) - \frac{B}{J_m}\omega_m + \frac{K_\tau}{J_m}u \tag{4}$$

$$\dot{\omega_1} = -\frac{K}{J_1}(\theta_1 - \theta_m) - \frac{mgh}{J_1}\sin\theta_1 \tag{5}$$

where, θ_1 is the link angle, θ_m is the motor angular position, ω_1 is the link angular velocity and ω_m is the motor angular velocity.

The system dynamics are non–linear and of the form

$$\dot{x} = Ax + \phi(x,u) + g(y)u \tag{6}$$

$$y = Cx \tag{7}$$

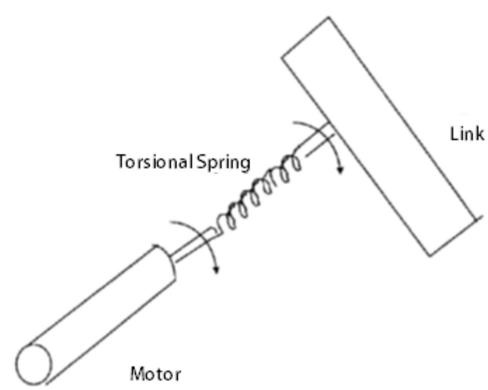

Fig. 1. Robotic system with flexible joint

where $x = \begin{bmatrix} \theta_m \\ \omega_m \\ \theta_1 \\ \omega_m \end{bmatrix}$, $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -19.5 & 0 \end{bmatrix}$

Here x is the state vector and A is the system matrix.

$$g(y) = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix}, \phi(x,u) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3.33\sin(x_3) \end{bmatrix}$$

and $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

The three sensors are

$$y_1 = C_1 x + v_1 \tag{8}$$

$$y_2 = C_2 x + v_2 \tag{9}$$

$$z = Ex + v_z \tag{10}$$

If there is no fault in the system, then both residues are zero[1]. In the presence of a fault in the sensor z, both residues R_1 and R_2 are non–zero.

Assumptions

1) The non–linearity $\phi(x,u)$ is Lipschitz with a Lipschitz constant γ.
2) Only one of the sensors fail at a given time.
3) The signals $v_1(t)$, $v_2(t)$ and $v_3(t)$ are assumed to be zero, when the corresponding sensors are healthy.

Theorem 1

If the observer gain matrix L is chosen[1] such that,

1) LMI of equation (11) is satisfied
2) $M_1 L \neq 0$, $M_2 L \neq 0$
3) The non–linear function $\phi(x,u)$ doesn't span the entire subspace that is spanned by the vector L orthogonal to null space of M_1 and M_2. Then the residues R_1 and R_2 grow in the unique subspace corresponding to each fault in any of the three sensors.

III. Linear Matrix Inequality (LMI) Formulation

The basic Linear Matrix Inequality (LMI) form is

$$\begin{pmatrix} -\sum_{i=1}^{q+s} x_i(A^T P_i + P_i A) + \sum_{l=1}^{r+p} \beta_1(E^T u_l^T + u_l E) - 1 & -Q \\ -Q & 1 \end{pmatrix} > 0 \tag{11}$$

Where $Q = \gamma \sum_{i=1}^{q+s} x_i P_i$, $L = \sum_{j=1}^{r+p} \alpha_i v_j$, $\alpha = H^{-1} b$, $H = (pv_1 \quad pv_2 \quad Pv_3)$ and $P = \sum_{i=1}^{q+s} x_i p_i$

In the equation 11 the matrices P and L are written in terms of their basis vectors.
The variables r, q, s, p are such that

$$dim(p_i v_j)_{i=1,2,\cdots q, j=1,2,\cdots r} = r$$

$$dim(p_i v_j)_{i=1,2,\cdots q+s, j=1,2,\cdots r+p} = r + p$$

$$\Longrightarrow r = 3, q = 3 \text{ and } r + p = 4 \Longrightarrow p = 1, s = 2$$

Also

$$M_1 = \begin{pmatrix} C_1 \\ C_1 A \\ C_1 A^2 \\ C_1 A^3 \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 0 & 0 \\ 60.75 & 1.5625 & -60.75 & 0 \\ -76 & 59 & 76 & 60.75 \end{bmatrix} \tag{12}$$

$$M_2 = \begin{pmatrix} C_2 \\ C_2 A \\ C_2 A^2 \\ C_2 A^3 \end{pmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 60.75 & 50.2 & 60.75 & 48.6 \\ 3387.42 & 2 & -3387.42 & 60.75 \end{bmatrix} \tag{13}$$

Two vectors orthogonal to null space of both M_1 and M_2 are

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

The other vectors can be used in the LMI formulation are,

$$v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The basis matrices for P are

$$P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, P_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, P_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$P_5 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus

$$P_1 v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, P_1 v_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, P_1 v_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, P_1 v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_2 v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, P_2 v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, P_2 v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, P_3 v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_3 v_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, P_3 v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, P_3 v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, P_4 v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_4 v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, P_4 v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, P_4 v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, P_5 v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_5v_2 = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}, \; P_5v_3 = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

The basis vectors for this space are

$$u_1 = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \; u_2 = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \; u_3 = \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \; u_4 = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

$$P = \sum_{i=1}^{5} x_i p_i = \begin{pmatrix} x_1 & 0 & 0 & 0 \\ 0 & x_2 & x_4 & 0 \\ 0 & x_4 & x_3 & 0 \\ 0 & 0 & 0 & x_5 \end{pmatrix} \tag{14}$$

Thus the matrix P is positive definite if

$$x_1 > 0 \,,\, x_2 > 0 \,,\, x_3 > 0 \,,\, x_5 > 0 \,,\, x_2x_3 > 0 \,,\, x_4^2 > 0$$

A. To find the new Lipchitz constant γ

The Lipschitz condition is

$$\frac{\|\phi(x,u) - \phi(\hat{x},u)\|}{\|x - \hat{x}\|} =$$

$$\frac{0.333|\sin(x_3 - \sin(\hat{x}_3))|}{\sqrt{(x_1 - \hat{x}_1)^2) + (x_2 - \hat{x}_2)^2) + (x_3 - \hat{x}_3)^2)}} <$$

$$\frac{0.333\left|\sin(\frac{(x_3 - \hat{x}_3)}{2})\cos(\frac{(x_3 + \hat{x}_3)}{2})\right|}{\left|\frac{x_3 - \hat{x}_3}{2}\right|} < 0.333 = \gamma$$

Equation 11 is reduced into

$$P = \begin{pmatrix} 4\beta - 1 - \gamma^2 x_1^2 & \beta_1 + 3\beta_2 - x_1 + 48.6x_2 & D & G \\ \beta_1 + 2\beta_2 - x_1 + 48.6x_2 & A & B & C \\ \beta_1 + 2\beta_3 + 48.6x_4 & B & E & H \\ \beta_1 + 2\beta_4 - 19.5x_5 & C & F & I \end{pmatrix} > 0 \tag{15}$$

Where $A = 2.5x_2 + 2\beta_2 - 1 - \gamma^2(x_2^2 + x_4^2)$, $B = \beta_3 + \beta_2 + 1.25x_4 - 48.6x_2 - \gamma^2 x_4(x_2 + x_3)$, $C = \beta_4 + \beta_2 - x_4$, $D = \beta_1 + 2\beta_3 + 48.6x_4 + \beta_2$, $E = 2\beta_3 - 1 - 97.2x_4 - \gamma^2(x_3^2 + x_4^2)$, $F = \beta_3 + \beta_4 - x_3 + 19.5x_5$, $G = \beta_1 + 2\beta_4 - 19.5x_5 + \beta_2$, $H = \beta_3 + \beta_4 + 19.5x_5 - x_3$, $I = 2\beta_4 - \gamma^2 x_5^2 - 1$

Equation 15 implies that

$$4\beta_1 - 1 - \gamma^2 x_1^2 > 0 \Rightarrow \beta_1 > \frac{1.1678}{4} = 0.2919$$

Let $\beta_1 = 0.3$, $x_1 = 0.1230$, $x_2 = 1$, $x_4 = 0.25$

Then $x_2x_3 - x_4^2 > 0 \Rightarrow x_3 = 0.07$

The second and third order principal minors are positive implies

$$-24.341 < \beta_2 < -16.3 \tag{16}$$

$$\beta_2 = -16\beta_3^2 - 36.25\beta_3 - 74.5 > 0 \Rightarrow \beta_3 > 38.20$$

$$\text{or } \beta_3 < -1.95 \tag{17}$$

Select $\beta_2 = -16.50$

$$\text{Thus, } \beta_1 u_1 + \beta_2 u_2 + \beta_3 u_3 + \beta_4 u_4 = \begin{pmatrix} 0.30 \\ -16.5 \\ \beta_3 \\ \beta_4 \end{pmatrix} \tag{18}$$

Select the observer gain matrix L such that the matrix ALE is stable

ie, $\lambda^4 + (b_1 + c_1 + d_1 + 6.13)\lambda^3 + (2b_1 + 27.02c_1 + 2.25d_1 + 6.526)\lambda^2 + (117.262c_1 + 98.45d_1 + 42.33b_1 + 247)\lambda + 65.16b_1 - 161.755c_1 + 145.8d_1 - 34.325 = 0$

This is possible if $d_1 > 22.60$ or $d_1 < -162.48$

$$d_1 > 142.86 \text{ or } 3.35 < d_1 < 5.723$$

$$d_1 > 60.15 \text{ or } d_1 < 5.1311$$

$$d_1 > 14.205 \text{ or } d_1 > 10.85$$

Select $d_1 = 143$, $c_1 = 1$, $b_1 = -16.5 - 0.25c_1 = -16.75$

Also the fourth order leading principle minor is positive

$$\Rightarrow x_5^2 - 2.483x_5 + 0.3608 < 0 \Rightarrow 0.1549 < x_5 < 2.329$$

Select $x_5 = 0.42 \Rightarrow \beta_4 = x_4\alpha_4 = 143 \times 0.42 = 60.06$

$$\text{Hence, } x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1.123 \\ 1 \\ 0.07 \\ 0.25 \\ 0.42 \end{pmatrix}, \; \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix} = \begin{pmatrix} 0.30 \\ -16.5 \\ -4.11 \\ 60.06 \end{pmatrix}$$

$$\text{and } L = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} 2.44 \\ -16.72 \\ 1 \\ 143 \end{pmatrix}$$

For simulation, set frequency $f = 1Hz$, so that $u = 0.5\sin(2\pi t)$. The actual and observed link angle, link velocity, motor angular position and angular velocity are given below.

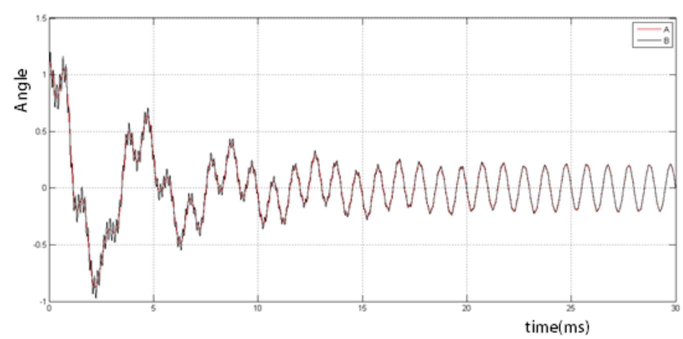

Fig. 2. Actual and Observed link axis Vs time

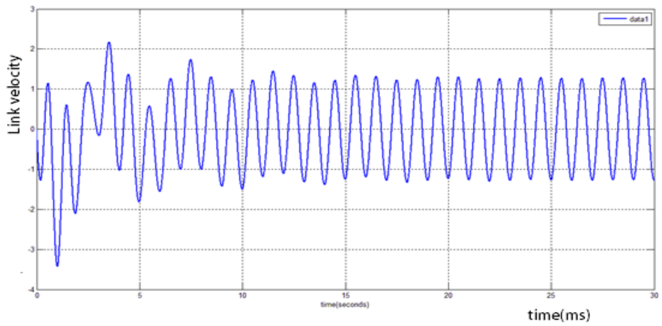

Fig. 3. Actual and Observed link axis Vs time

The observed motor angular rotation is obtained as

$$\hat{\theta_m}(t) = 0.1727\cos(6.28t - 1.4743) + 1.2113e^{-0.193t}\cos(1.5015t - 0.4634) - 0.3653e^{-0.4322t}\cos(8.276t - 0.4281) - 0.00098e^{-0.10138t}\sin(41.668t - 1.506) - 0.00613e^{-117.828t} - 0.002145e^{-15.526t}$$

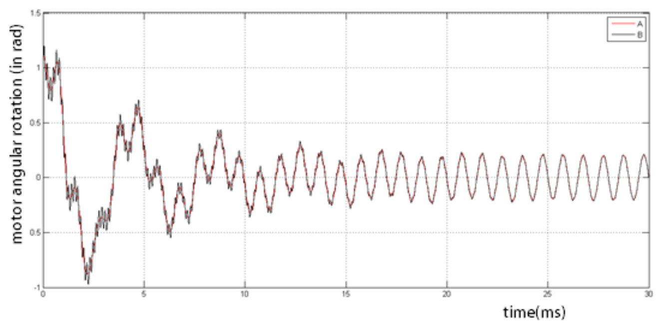

Fig. 4. Actual and Observed motor angular position VS time

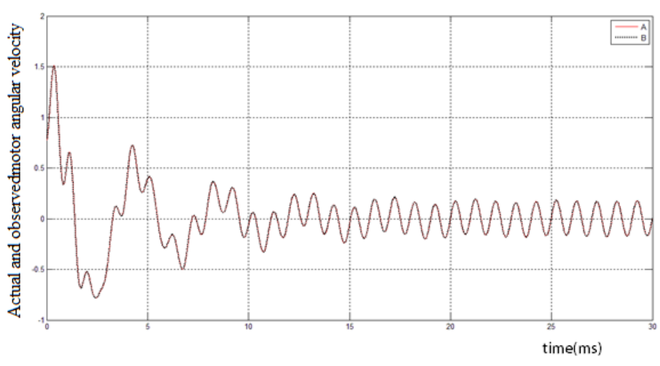

Fig. 5. Actual and Observed motor angular velocity VS time

When fault occurs, the signal $v_z(t)$ cannot be zero. Set $v_z(t) = 1.5\sin(500t)$. Thus under fault condition, the motor angular position and velocity are estimated are estimated and plotted as shown below. In this case, the residues are non–zero.

IV. Distance to Unobservability

The distance to unobservability[2] of a pair (A,C) is defined as the smallest perturbation (E,F) that makes the pair (F+A,

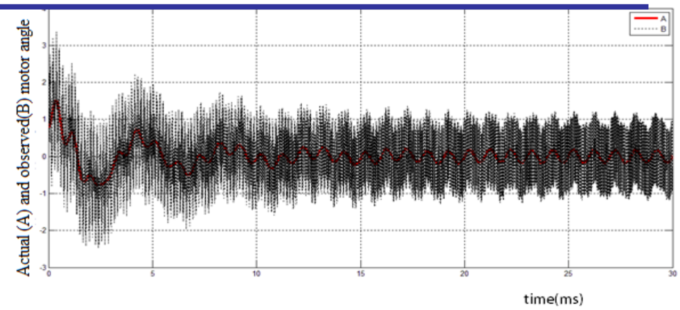

Fig. 6. Actual and Observed motor angular position VS time under fault condition

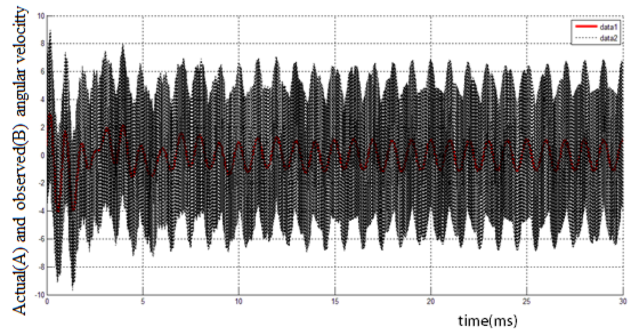

Fig. 7. Actual and Observed motor angular velocity VS time under fault condition

F+C) unobservable. It is defined mathematically as

$$\delta(A, C) = min_{\omega \varepsilon R}\sigma_{min}\begin{pmatrix} j\omega I - A \\ C \end{pmatrix} \quad (19)$$

A. Distance to Unobservability of Original System

The minimum singular value,

$$\sigma_{min}\begin{pmatrix} j\omega I - A \\ C \end{pmatrix} = \sigma_{min}((j\omega I - A)^*(j\omega I - A) + C^T C) \quad (20)$$

We can find the minimum singular value for different values of ω. Solve the equation,

$$|\lambda I - ((j\omega I - A)^*(j\omega I - A) + C^T C)| = 0 \quad (21)$$

That implies (a) when $\omega = 0, \sigma_{min} = 0.47$, (b) when $\omega = 0.5, \sigma_{min} = 0.5031$, (c) when $\omega = 1.2, \sigma_{min} = 0.7921$, (d) when $\omega = 3, \sigma_{min} = 1.1347$, (e) when $\omega = 5, \sigma_{min} = 1.06654$, (f) when $\omega = 8, \sigma_{min} = 0.9035$, (g) when $\omega = 12, \sigma_{min} = 1.3486$, (h) when $\omega = 20, \sigma_{min} = 4.2866$

B. Distance to Unobservability of the New system obtained by Similarity Transformation

The transformation $X = T_0 z$ is used to obtain a new system with new Lipschitz constant. From the original state equation.

$$\dot{z} = T_0^{-1}AT_0 z + T_0^{-1}\phi(T_0 Z, u) + T_0^{-1}g(y)u \quad (22)$$

is obtained, where

$$T_0 = diag(1, 1, 1, \beta)$$

Let $\beta = 20$, then

$$\gamma' = \frac{1}{20} \times 3.33 = 0.1665$$

Let $A_1 = T_0^{-1} A T_0$ Solve'

$$|\lambda I - ((j\omega I - A_1)^*(j\omega I - A_1) + C^T C)| = 0 \tag{23}$$

That implies (a) when $\omega = 0, \sigma_{min} = 0.7104$, (b) when $\omega = 0.5, \sigma_{min} = 0.7314$, (c) when $\omega = 1.2, \sigma_{min} = 0.8216$, (d) when $\omega = 3, \sigma_{min} = 0.9874$, (e) when $\omega = 5, \sigma_{min} = 1.0531$, (f) when $\omega = 8, \sigma_{min} = 1.0717$, (g) when $\omega = 12, \sigma_{min} = 1.3739$, (h) when $\omega = 20, \sigma_{min} = 4.2793$ The distance to unobservability for orginal and new system VS ω is shown in

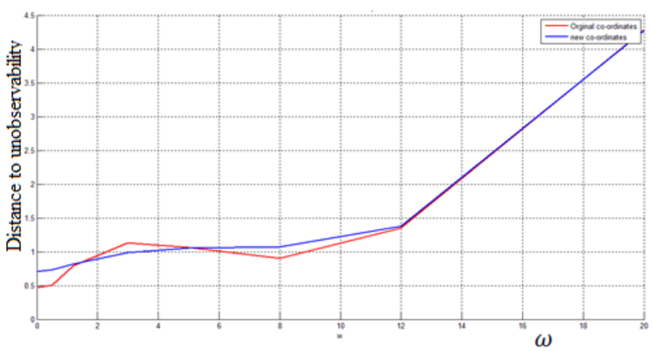

Fig. 8. Distance to unobservability VS ω

V. Observer design by Similarity Transformation

In this section a method for designing a stable observer for robot by new method[3] is presented. The method is based on distance to observability. Similarity transformation is used to reduce the Lipschitz constant and increase the distance to unobservability.
Let $x = T_0 x'$ so that Then the original system is transformed into

$$\begin{aligned} x' &= bT_0^{-1}(A - L_1 C)T_0 x' + T_0^{-1}\phi(T_0 x', u) \\ &\quad + T_0^{-1} g(y)u + T_0^{-1} L_1 y \end{aligned} \tag{24}$$

$$y = CT_0 x' = C' x' \tag{25}$$

The new Lipschitz constant is determined from

$$\frac{\|T_0^{-1}\phi(T_0 x', u) - T_0^{-1}\phi(T_0 x', u)\|}{\|x - \hat{x}\|} < \gamma \tag{26}$$

The observer for the above system is given by

$$\begin{aligned} \hat{x}' &= A'\hat{x}' + T_0^{-1}\phi(T_0 x', u) + T_0^{-1} g(y)u + T_0^{-1} L_1 y \\ &\quad + ({\gamma'}^2 + \varepsilon_0) L'(y - C'x')/\|C'\|^2 \end{aligned} \tag{27}$$

$$\hat{x} = C'\hat{x}' \tag{28}$$

The observer gain L' is chosen to be $L' = \dfrac{P^{-1}C'^T}{2}$ under the following conditions,

1) AL_1C is stable
2) ${\gamma'}^2 + \varepsilon_0 \geq 0$
3) $\eta_0 > max(\varepsilon_0, 0)$
4) $A'^T P + PA' + PP + ({\gamma'}^2 + \eta_0)I - \dfrac{({\gamma'}^2 + \varepsilon_0)C'^T C'}{\|C'\|^2} = 0$
Where P is the symmetric positive definite solution of the above algebraic ricatti equation
5) ${\gamma'}^2 + \eta_0 < \delta^2 \left(A', \sqrt{({\gamma'}^2 + \varepsilon_0)} \dfrac{C'}{\|C'\|^2}\right)$

To find the new Lipchitz constant

The new Lipschitz condition is

$$\frac{\|T_0^{-1}\phi(T_0 x, u) - T_0^{-1}\phi(T_0 \hat{x}, u)\|}{\|x - \hat{x}\|} =$$

$$\frac{0.333|\sin(x_3 - \sin(\hat{x}_3))|}{\sqrt{(x_1 - \hat{x}_1)^2) + (x_2 - \hat{x}_2)^2) + (x_3 - \hat{x}_3)^2)}} <$$

$$\frac{0.333\left|\sin(\dfrac{(x_3 - \hat{x}_3)}{2})\cos(\dfrac{(x_3 + \hat{x}_3)}{2})\right|}{\left|\dfrac{x_3 - \hat{x}_3}{2}\right|} < 0.333$$

Thus the new Lipschitz constant is 0.333.
Let,

$$L_1 = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \\ a_4 & b_4 \end{pmatrix} \tag{29}$$

and

$$P = \begin{pmatrix} 4.21 & -0.02 & 0.02 & 0.001 \\ -0.02 & x & -2.3 & 1 \\ 0.02 & -2.3 & y & h \\ 0.001 & 1 & h & z \end{pmatrix} \tag{30}$$

The matrix P is positive definite. The elements of P is chosen such that 4.21 is an eigen value of P.
Let $T_0 = diag(1, 1, 1, 10)$
Thus from the Algebraic Ricatti Equation (ARE),

$$a_{33} = a_{43} = a_{44} = a_{34} = 0 \Rightarrow h^2 + 20h + z^2 + 1.11 + \eta_0 = 0$$

$$h^2 - 3.90h + y^2 - 218.159 + \eta_0 = 0$$

$$(h + 10)y + z(h - 1.95) = -46.3$$

Where

$$A' = \begin{pmatrix} -a_1 & 1 - b_1 & 0 & 0 \\ -48.6 - a_2 & -1.25 - b_2 & 48.6 & 0 \\ -a_3 & -b_3 & 0 & 10 \\ 1.95 - 0.1a_4 & -0.1b_4 & -1.95 & 0 \end{pmatrix} \tag{31}$$

Solve the algebraic Ricatti equation for A' with $\eta_0 = 16$, $\varepsilon_0 = -0.0123$, z and y are obtained as $z = 8.63587$, $y = 11.1242$. Select x such that the matrix P is positive definite. From this $x = 10.5$ is obtained.

Solve for the remaining variables, $a_1 = -17.1349$, $a_2 = -10214.9583$, $a_3 = -5449.3247$, $a_4 = -53324.6597$, $b_1 = 23646.6245$, $b_2 = 52.2998$ and $b_3 = 9.21073$, are found.
Also $\delta^2 \left(A', \sqrt{{\gamma'}^2 + \varepsilon_0} \dfrac{c'}{\|c'\|}\right) - {\gamma'}^2 - \eta_0 = 10.1119^2 -$

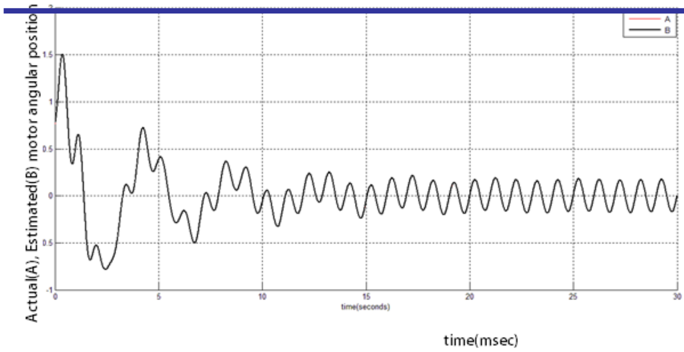

Fig. 9. Actual and Observed motor angular position

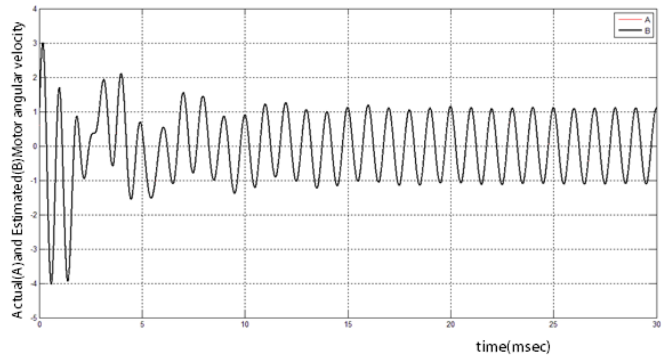

Fig. 10. Actual and estimated motor angular velocity

$0.333^2 - 16 = 86.139 > 0$ is satisfied.
Thus,

$$L_1 = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \\ a_4 & b_4 \end{pmatrix} = \begin{pmatrix} -17.1349 & 23646.624 \\ -10214.9529 & 52.2999 \\ -5449.3247 & 9.2107 \\ -53324.6597 & 1.0000 \end{pmatrix} \quad (32)$$

Define $\hat{x} = T_0 \hat{x}'$ as the estimate of x, then

$$\dot{\hat{x}} = A\hat{x} + \phi(\hat{x}, u) + g(y)u + L(y - C\hat{x}) \quad (33)$$

Where,

$$L = L_1 + \frac{(\gamma'^2 + \varepsilon_0) T_0 L'}{\|C'\|^2} = \begin{pmatrix} 0.11904 & 0.00017 \\ 0.000167 & 0.050267 \\ -0.0004236 & 0.0141 \\ 0.00035547 & 0.005805 \end{pmatrix} \quad (34)$$

By using this observer gain matrix, actual and observed motor angular rotation and velocity are found.

$$\hat{\theta_m}(t) = 0.1727\cos(6.28t - 1.4743) + 1.2113e^{-0.193t}\cos(1.5015t - 0.4634) - 0.3653e^{-0.4322t}\cos(8.276t - 0.4281) - 0.00484e^{-5.1858t}\sin(15.8409t - 1.4193) - 0.00478e^{-13.036t}\cos(15504.4744)$$

$$\hat{\omega_m}(t) = 1.10685\sin(6.28t - 1.47425) - 1.83369e^{-0.193t}\cos(1.5015t - 1.9056) + 3.0276e^{-0.4322t}\cos(8.276t - 1.946) - 0.00031346e^{-13.0364t}\sin(15504.4743t)$$

$$\theta_m(t) = 1.17265\cos(6.28t - 1.47425) + 1.211128e^{-0.193t}\cos(1.5015t - 0.46341) - 0.36533e^{-0.4322t}\cos(8.276t - 0.4281)$$

$$\omega_m(t) = 1.10685\sin(6.28t - 1.47425) - 1.83369e^{-0.193t}\cos(1.5015t - 1.9056) + 3.0276e^{-0.4322t}\cos(8.276t - 1.946)$$

VI. Conclusion

A stable observer is designed for flexible link robot using LMI. By using this observer gain matrix, we can identify the fault in the system. Distance to unobservability is studied and estimated for orginal and transformed system. A stable observer is designed for robot by using this method. Two theorems based on Algebraic riccati equation are found.

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