Published by :
http://www.ijert.org
International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 06, June-2016

Self Balancing of Unicycle Robot

Divya . A
PG Scholar
Electrical & Electronics Dept
Mar Baselios college of Engineering,
Thiruvananthapuram, Kerala, India

Shyju Susan Mathew
Assistant Professor
Electrical & Electronics Dept.
Mar Baselios college of Engineering,
Thiruvananthapuram, Kerala, India

***Abstract**:-* **This paper deals with the control of Unicycle mobile robot which is a vehicle which touches the ground with one wheel. It have many advantages over multi wheeled mobile robot that it can have higher degree of mobility and less space as it has only one wheel to move. The system consists of three parts which are the lower, upper and middle parts. The lower part is composed of a wheel which is moving back and forth to stabilize the pitch angle. Meanwhile, the upper part consists of a balance weight and middle is main frame that functions to stabilize the roll angle and yaw angle of the unicycle system. At first (Linear Quadratic Regulator) was used to control the pitch, yaw and roll angles. Then SMC was used to study the roll angle characteristics .At last a combination of LQR (pitch and yaw control) and SMC (roll control) is proposed. The dynamic model of the unicycle mobile robot is to be developed and verification is done through simulation using MATLAB software.**

Index Terms-Unicycle Robot, Dynamic Modelling, LQR, SMC

I. INTRODUCTION

In the near future personal mobile robots will be providing a better life not only to common people but especially to elderly and impaired. In particular wheeled robots will be expected to provide many convenient and user friendly transport solutions for both people and objects. The importance of the wheeled mobile robots have long been recognized by the robotics research community as shown by the numerous robotic competitions and research projects run worldwide in the last decades. The importance of the subject motivated and continues motivating many projects.

Unicycle is a vehicle consists of one wheel, driven by pedals and also known as monocycle. The unstable dynamics of the unicycle has attracted a lot of researchers to analyze and design unicycle robot and its controller[1]. An autonomous unicycle in the form of mobile robot is quite unique because of the challenge posted by the robot to balance its position either in static condition or when it is moving. The class of unicycle type (mobile) robots, i.e.robots having some forward speed but zero instantaneous lateral motion, is frequently selected for designing and modelling robots.

Idea of having unicycle mobile robot is enlightened from huma riding unicycle and researchers use inverted pendulum to achiev the goal. Human riding unicycle stabilizes roll angle by moving hi arm, wrist and body together, while pitch angles stabilized by controlling the speed and the position of the wheel using his legs. The pictorial representation of unicycle robot is given as Fig.1

Fig : 1 Unicycle

This paper is intended to study the dynamics of unicycle mobile robot focusing on stabilizing the lateral and longitudinal position of the robot[2]. The proposed unicycle robot consists of a wheel, a frame and a balance weight as shown in Fig. 2. The 3D unicycle robot system can be characterized by three tilt angles namely the roll the pitch and the yaw angles. The robot can reach longitudinal stability by appropriate control of the wheel (control of pitch angle) and lateral stability by applying appropriate torque generated by the rotating disc (control of roll angle). In order to do that the system requires at least three motors to balance the robot so as to achieve the longitudinal and lateral stability.

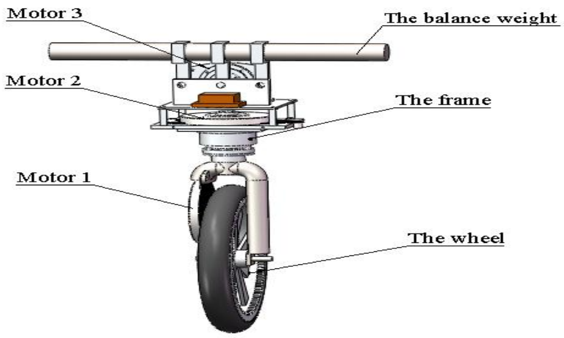

Fig :2 Parts of Unicycle Mobile Robot System

Unicycle mobile robot system is an underactuated system since it possesses fewer control inputs than the total number of degree of freedom (DOF) . The robot is also considered as nonholonomic system because it has nonholonomic constraint

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 06, June-2016

Unicycle mobile robot have many advantages over multi wheeled mobile robot such as

a) higher degree of mobility

b) less space as it only has one wheel to move.

It is a vehicle performing missions in fixed or uncertain environments.

The organization of this paper can be summarized as follows. The dynamic modelling of the Unicycle Robot are explained in Section II. Different controllers are implemented and compared in this section. Conclusion based on the experimental work is given in Section III.

II. DYNAMIC MODELLING

The Unicycle Robot can be considered as a system having three major parts, a wheel, a frame and a balance weight. The pitching balance is controlled by the motor driving the wheel. Balance weight controls the rolling balance and rotation of motor on the body/frame controls the yaw angle.

The unicycle robot is an under-actuated, nonlinear and unstable system. The Routh equation is the basis for the dynamic modeling of the robot .This equation is very close to the second kind Lagrange equations. But it introduces the Lagrange multiplier, and has the advantages of Lagrange the first equation. It could solve the non holonomic system or c holonomic system with redundant coordinates and is applicable in practical engineering.

The assumptions are made as following: the unicycle robot components are all rigid body, the wheel is a hollow ring and its centroid is in the center of the ring, the wheel is pure roll on the ground and other frictions and external disturbances are ignored.

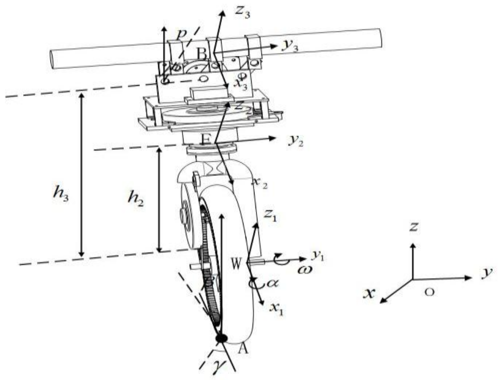

Fig :3. Dimensional Details of Unicycle Robot

As per Fig. 3, ω is the wheel rotation angle .The body roll angle is α . The pitch angle is β . The yaw angle is γ . The balance weight is connected with the rod of pendulum, the pendulum angle is p The o-xyz is the fixed coordinate system. W-x1 y1 z1 , F-x2 y2 z2 and B-x3 y3 z3 are the moving coordinate system of the wheel, frame and the balance weight respectively. In accordance with the principle of dynamics, we set each rotation matrix as follows:

$$R_{(x,\alpha)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha & -sin\alpha \\ 0 & sin\alpha & cos\alpha \end{bmatrix} \quad (1)$$

$$R_{(y,\beta)} = \begin{bmatrix} \cos\beta & 0 & sin\beta \\ 0 & 1 & 0 \\ -sin\beta & 0 & cos\beta \end{bmatrix} \quad (2)$$

$$R_{(z,\gamma)} = \begin{bmatrix} cos\gamma & -sin\gamma & 0 \\ sin\gamma & cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

The coordinate the wheel on the ground is(x_c y_c,0). The wheel is pure roll on the ground. The constraint equation of the wheel center is given as (4).

$$\begin{Bmatrix} \dot{x_c} - r_1\dot{\omega}cos\gamma = 0 \\ \dot{y_c} - r_1 sin\gamma = 0 \end{Bmatrix} \quad (4)$$

A. *The Analysis of the Wheel Kinetic Energy and Potential Energy*

(x_1 , y_1 , z_1) is the coordinate of the wheel center. Assume that the quality of wheel is m_1 and the radius is r_1 . By the coordinate transformation eqn (5) can be obtained. The wheel also has the rotation kinetic energy during the translation is given by eqn (6). Consequently, the translational kinetic energy T_{11} , rotation kinetic energy T_{12} and the gravitational potential energy V_{11} given by eqn (7),(8) and (9)

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} + R_{(z,\gamma)}R_{(x,\alpha)}\begin{bmatrix} 0 \\ 0 \\ r_1 \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} w_{1x} \\ w_{1y} \\ w_{1z} \end{bmatrix} = \begin{bmatrix} \dot{\alpha} \\ \dot{\omega} + \dot{\gamma}sin\alpha \\ \dot{\gamma}cos\alpha \end{bmatrix} \quad (6)$$

$$T_{11} = {}^1/_2\, m_1\dot{x}_1^2 + {}^1/_2\, m_1\dot{y}_1^2 + {}^1/_2\, m_1\dot{z}_1^2 \quad (7)$$

$$T_{12} = {}^1/_2 J_{11}{w_{1x}}^2 + {}^1/_2 J_{12}{w_{1y}}^2 + {}^1/_2 J_{13}w_{1z}^2 \quad (8)$$

$$V_{11} = m_1 g r_1 cos\alpha \quad (9)$$

$$J_{11} = m_1 r_1^2,\ J_{12} = J_{13} = {}^1/_2\, m_1 r_1^2$$

B. *The Analysis of the Frame Kinetic Energy and Potential Energy*

(x_2 ,y_2 , z_2) is the coordinate of the centroid of frame is given by eqn no (10). The frame is equivalent to a cylinder. The quality of frame is m2 ,the length is l_2 and the radius is r_2 . The distance between the centroid of frame and the centerof the wheel is h_2 .

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + R_{(z,\gamma)}R_{(x,\alpha)}R_{(y,\beta)}\begin{bmatrix} 0 \\ 0 \\ h_2 \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} w_{2x} \\ w_{2y} \\ w_{2z} \end{bmatrix} = \begin{bmatrix} \dot{\alpha}cos\beta - \dot{\gamma}cos\alpha sin\beta \\ \dot{\beta} + \dot{\gamma}sin\alpha \\ \dot{\alpha}sin\beta - \dot{\gamma}cos\alpha cos\beta \end{bmatrix} \quad (11)$$

$$T_{21} = {}^1/_2\, m_2\dot{x}_2^2 + {}^1/_2\, m_2\dot{y}_2^2 + {}^1/_2\, m_2\dot{z}_2^2 \quad (12)$$

$$T_{22} = {}^1/_2 J_{21}{w_{2x}}^2 + {}^1/_2 J_{22}{w_{2y}}^2 + {}^1/_2 J_{23}w_{2z}^2 \quad (13)$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 06, June-2016

$$V_{21} = m_2 g(r_1 + h_2 cos\beta)cos\alpha \qquad (14)$$

$$J_{21} = J_{22=} {}^{m_2}/_{12}\,(3r_2^{\acute{2}} + l_2^2, J_{23} = {}^1/_2\, m_2 r_2^{\ 2}$$

The frame also has the rotation kinetic energy during the translation is given by eqn(11). Consequently, we could get the translational kinetic energy T_{21}, rotation kinetic energy T_{22} and the gravitational potential energy V_{21} are given by eqns (12),(13)and (14).

C. The Analysis of the Balance Weight Kinetic Energy and Potential Energy

(x_3 , y_3 , z_3) is the coordinate of the centroid of balance weight is given by eqn (15). The balance weight is equivalent to a long cylinder. The quality of balance weight is m_3 , the length is l_3 and the radius is r_3 .The pendulum connects the frame and the balance weight. The distance between the centroid of the balance weight and the connection point of the frame top is s_3 . The distance between the frame top and the center of the wheel is h_3. The balance weight also has the rotation kinetic energy during the translation is given eqn (16). Consequently, we could get the translational kinetic energy T_{31} , rotation kinetic energy T_{32} and the gravitational potential energy V_{31} are given by eqns (17),(18) and (20)

$$\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + R_{(z,\gamma)} R_{(x,\alpha)} R_{(y,\beta)} \begin{bmatrix} 0 \\ -s_3 sinp \\ h_3 + cosp \end{bmatrix} \qquad (15)$$

$$\begin{bmatrix} w_{3x} \\ w_{3y} \\ w_{3z} \end{bmatrix} = \begin{bmatrix} \dot{\alpha}cos\beta - \dot{\gamma}cos\alpha sin\beta + \dot{p} \\ \dot{\beta} + \dot{\gamma}sin\alpha \\ \dot{\alpha}sin\beta + \dot{\gamma}cos\alpha cos\beta \end{bmatrix} \qquad (16)$$

$$T_{31} = {}^1/_2\, m_3 \dot{x}_3^2 + {}^1/_2\, m_3 \dot{y}_3^2 + {}^1/_2\, m_3 \dot{z}_3^2 \qquad (17)$$

$$T_{22} = {}^1/_2 J_{31}(\,\dot{\alpha} - \dot{p})^2 \ + {}^1/_2 J_{32} \dot{\beta}^2 \qquad (18)$$

$$V_{31} = m_3 g(r_1 + (h_3 + s_3 cosp)cos\beta]cos\alpha \qquad (19)$$

$$J_{21} = J_{22=} {}^{m_3}/_{12}\,(3r_3^{\acute{2}} + l_3^2, J_{32} = {}^1/_2\, m_3 r_3^{\ 2}$$

D. The Dynamics Modeling of the robot

The Routh equation defined as following

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j + \sum_{\beta=1}^{r} \mu_\beta A_{\beta j}{}^{,}$$

The Total KE is

$$T = T_{11} + T_{12} + T_{21} + T_{22} + T_{31} + T_{32} \qquad (20)$$

The Total PE is

$$V = V_{11} + V_{21} + V_{31} \qquad (21)$$

Where

$q = (x_c, y_c, \omega, \beta, \alpha, p, \gamma)$

$$A_{27} = \begin{bmatrix} 1 & 0 & -r_1 \cos\gamma & 0 & 0 & 0 & 0 \\ 0 & 1 & -r_1 \sin\gamma & 0 & 0 & 0 & 0 \end{bmatrix}$$

The generalized force equation is

$$Q = [0\ 0\ \tau_\omega\ -\tau_\omega\ 0\ \ \tau_p\ \tau_\gamma]^T$$

From eqn (4) the following eqn can be deduced

$$\{\ddot{x_c} = r_1 \ddot{\omega} cos\gamma - r_1 \dot{\omega}\dot{\gamma} sin\gamma\} \qquad (22)$$

$$\{\ddot{y_c} = r_1 \ddot{\omega} sin\gamma + r_1 \dot{\omega}\dot{\gamma} cos\gamma\}$$

The dynamic equation,

$$x = [\omega\ \beta\ \alpha\ p\ \gamma]^T$$

On simplification kinetic equation can be obtained

$$M(x)\ddot{x} + G(x, \dot{x}) = N\tau \qquad (23)$$

Where

$$M(x) = \begin{bmatrix} M_{11} & M_{12} & 0 & 0 & 0 \\ M_{21} & M_{22} & 0 & 0 & 0 \\ 0 & 0 & M_{33} & M_{34} & 0 \\ 0 & 0 & M_{43} & M_{44} & 0 \\ 0 & 0 & 0 & 0 & M_{55} \end{bmatrix}$$

$$G(x, \dot{x}) = [0\ G_2\ G_3\ 0\ 0]^T$$

$$N = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 06, June-2016

$M_{11} = J_{12} + r_1^2(m_1 + m_2 + m_3), M_{12} = M_{21} = r_1(m_2h_2 + m_3h_3 + m_3s_3)$
$M_{22} = J_{22} + J_{32} + 2m_3h_3s_3 + m_2h^2{}_2 + m_3h^2{}_3 + m_3s_3^2$
$M_{33} = J_{11} + J_{21} + J_{31} + 2m_3(h_3s_3 + r_1s_3 + r_1h_3) + m_3(h_3^2 + s_3^2) + r_1(m_3 + m_3)$
$M_{34} = M_{43} = -J_{31} - m_3(r_3s_3 + s_3^2 + h_3s_3)$
$M_{55} = J_{13} + J_{23} + J_{33} + 2m_2r_1h_2 + m_2(h_2^2 + r_1^2)$
$G_2 = -g\beta(m_2h_2 + m_3h_3 + m_3s_3)$
$G_3 = -g\alpha(m_1r_1 + m_2(h_2 + r_1) + m_3(h_3 + s_3 + r_1))$
$\tau = [\tau_{w,}\tau_{p,}, \tau]^T$

Table 1 Robot parameters

Symbol	Parameter	Quantity
m_1, m_2, m_3	Mass of wheel, frame, and balance weight	1kg,3.7kg,2.66kg
r_1, r_2	Radius of wheel and frame	0.12m,0.04m
l_2	Length of frame	0.2m
G	Gravity of acceralation	9.806m/s^2
h_2	The distance b/w centroid of frame and the centre of the wheel	0.22m
r_3, l_3	Radius and length of balance weight	0.015 m &0.4 m
h_3	The distance b/w of frame top and the centre of the wheel	0.32m
s_3	The distance b/w the centroid of balance weight and the connection point of the frame top	0.070m

III. The State Equation Of The Unicycle Robot

The state equation of a continuous time system is

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du ....(24)$$

The M in the equation (23) is a block diagonal matrix and the unicycle robot could be divided into three subsystems.
The state equations of the subsystems are as below.

Subsystem 1: The state equation of pitching direction is as follows.

$$A^{(1)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & A_{23} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & A_{43} & 0 \end{bmatrix}, \quad B^{(1)} = \begin{bmatrix} 0 \\ B_2^{(1)} \\ 0 \\ B_4^{(1)} \end{bmatrix}$$

$$C^{(1)} = [1, \quad 0, \quad 1, \quad 0]^T$$

$$D^{(1)} = 0, x^{(1)} = [\omega \quad \dot{\omega} \quad \beta \quad \dot{\beta}]$$

$$u^{(1)} = \tau_{\omega}$$

$$A_{23}^{(1)} = -220.385, A_{43}^{(1)} = -112.2754,$$
$$B_2^{(1)} = -4.5005, B_4^{(1)} = -6.7932$$

Subsystem 2: The state equation of rolling direction is as follows.

$$A^{(2)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ A_{21}^{(2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ A_{41}^{(2)} & 0 & 0 & 0 \end{bmatrix}, B^{(2)} = \begin{bmatrix} 0 \\ B_2^{(2)} \\ 0 \\ B_4^{(2)} \end{bmatrix}$$

$$C^{(2)} = [1, \quad 0, \quad 1, \quad 0]^T, D^{(2)} = 0, x^{(2)} = [\alpha \quad \dot{\alpha} \quad p \quad \dot{p}]^T$$
$$u^{(2)} = \tau_p, A_{21}^{(2)} = -1.1406, A_{41}^{(2)} = -9.3131,$$
$$B_2^{(2)} = 0.3475, B_4^{(2)} = 20.4918$$

Subsystem 3: The state equation of yawing direction is as follows.

$$A^{(3)} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B^{(3)} = \begin{bmatrix} 0 \\ B_2^{(3)} \end{bmatrix}, C^{(3)} = [1, \quad 0]^T, D^{(3)} = 0$$

$$x^{(3)} = [\gamma \quad \dot{\gamma}] u^{(3)} = \tau_{\gamma,} B_2^{(3)} = -2.1164$$

The ranks of the controllability discrimination matrix of Subsystem 1, the Subsystem 2 and the Subsystem 3 are 4, 4, 2, and the same as the rank of the observability criterion matrix through the calculation. The three subsystems all have controllability and observability but they are instable. Therefore, the effective controllers are need to be designed to control the system.

III. THE DESIGN OF UNICYCLE ROBOT CONTROLLER

A.LINEAR QUADRATIC REGULATOR

The theory of optimal control is concerned with operating a dynamic system at minimum cost. The case where the system dynamics are described by a set of linear differential equations and the cost is described by a quadratic function is called the LQ problem. One of the main results in the theory is that the solution is provided by the linear-quadratic regulator (LQR).

The LQR have certain advantages as:

a) Provides a systematic method of calculating the state feedback gain matrix

b) The designed system is always stable.

According to the principle of the LQR regulator, there is state equation $\dot{x} = Ax + Bu$. The mimimum control performance indicator

$$J = \int_0^\infty (x^T Qx + u^T Ru)dt \qquad (25)$$

through determining the matrix K in the feedback control $u(t) = -Kx(t)$ .Q is a positive definite or positive semi definite matrix or a real symmetric matrix. R is a positive definite or a real symmetric matrix. Q and R are the weighted matrix of x and u. The Riccati equation is given by:

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \quad (26)$$

The matrix P , is obtained by solving this equation. If there is a positive definite matrix P , the system would be stable. Then take the matrix P into the

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 06, June-2016

$$K = R^{-1}B^T P \qquad (27)$$

The matrix K is obtained.

B. SLIDING MODE CONTROL (SMC):

Sliding mode control (SMC) is a nonlinear control technique featuring remarkable properties of accuracy, robustness, and easy tuning and implementation.

SMC systems are designed to drive the system states onto a particular surface in the state space, named sliding surface. Once the sliding surface is reached, sliding mode control keeps the states on the close neighbourhood of the sliding surface. Hence the sliding mode control is a two part controller design. The first part involves the design of a sliding surface so that the sliding motion satisfies design specifications. The second is concerned with the selection of a control law that will make the switching surface attractive to the system state .

There are two main advantages of sliding mode control. First is that the dynamic behaviour of the system may be tailored by the particular choice of the sliding function. Secondly, the closed loop response becomes totally insensitive to some particular uncertainties. This principle extends to model parameter uncertainties, disturbance and non-linearity that are bounded.

From a practical point of view SMC allows for controlling nonlinear processes subject to external disturbances and heavy model uncertainties.

The SMC is widely recognized control strategy in the control system engineering which is to be insensitive to plant parametric uncertainties and external disturbances. For better understanding of SMC the Fig.4, it gives two phase plane plots showing the behaviour of SMC.

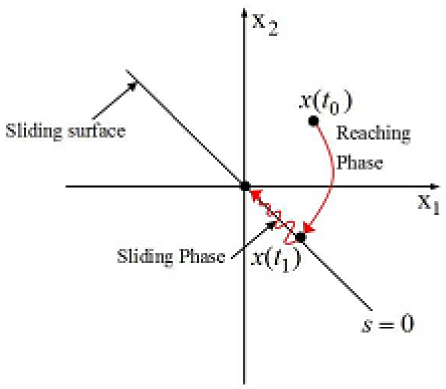

Fig 4 Phase plot for SMC

$$\begin{aligned}
\dot{V} &= \sigma\dot{\sigma} \\
&= \sigma[-a_1x_1 - a_2x_2 + bu] \\
&= \rho|\sigma| \\
u &= b^{-1}[-a_1x_1 - (a_2 - \lambda)x_2 - \rho\frac{|\sigma|}{\sigma}] \\
&= b^{-1}[-a_1x_1 - (a_2 - \lambda)x_2 - \rho sign(\sigma)] \\
&= u_{eq} + u_{sw}
\end{aligned}$$

The plant is defined as

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \text{f} + \text{u, with the sliding variable}$$

$$\sigma = \lambda x_1 + x_2$$

The simplest SMC controller $u = ksign\sigma$ is applied.

In the phase plot, after the initial reaching period, the controller is forcing the trajectories to stay on the line $\sigma = \lambda x_1 + x_2$ which corresponds to exponential decay of with the rate defined by the control parameter λ. The system has been reduced from being two-dimensional to be of just one dimension, i.e., the sliding surface along which system slides. When the hysteresis tends to zero width, any bounded disturbance will be rejected if k is chosen sufficiently large. This disturbance rejection is known as the invariance property. These are the main advantages of SMC. The chattering in the sliding phase. Deviations are due to the hysteresis which is an imperfection .The strength of SMC lies in its robustness against plant parameter. High speed switching feedback gain is necessary to achieve these goals. In SMC, a high speed switching gain is attained using a control signal with a discontinuous element (sign function). The simple discontinuous control law , u=-ksig(σ) will switch the controller output from k to – k or vice versa for the slightest change across. Such a control law is theoretically realizing an infinite feedback gain with a finite-valued control signal. The discontinuous element implements high theoretically infinite gain that is the conventional mean to suppress the influence of disturbances and uncertainties in system behaviour and unlike systems with continuous control. The invariance is attained using finite control actions. Consider a SISO system with its state space model as:.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -a_1x_1 - a_2x_2 + bu$$

Where

and $0 < a < \bar{a}, 0 < b < \bar{b}$ are the known constraints. For design of SMC of above mentioned system, let us choose sliding surface as:

$\sigma = \lambda x_1 + x_2$, where λ is positive.

For above sliding surface, a control law should be designed so that the sliding surface is reached to zero, i.e. made attractive towards the origin. This can be achieved by Lyapunov stability technique. Let us consider a Lyapunov function as: $V = \frac{1}{2}\sigma^2$

Where

$$sign(\sigma) = \frac{|\sigma|}{\sigma}$$

$$u_{eq} = b^{-1}[-a_1x_1 - (a_2 - \lambda)x_2]$$

$$u_{sw} = -b^{-1}\rho sign(\sigma)$$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 5 Issue 06, June-2016

IV. The SIMULATION EXPERIMENT

For LQR, Matlab the "care" command gives solution to the algebraic riccati equation (ARE) and determines the optimal control gain matrix

eg: care(A,B,Q,R)

In this paper the command calculates the optimal feedback gain matrix k such that the feedback control law , u=-kx minimizes the performance.

Here the weighted matrices Q and R for Subsytem 1 are assumed as, Q_1 = diag(500,1,1,1) , R_1 = 0.01)

The K value for subsystem 1 (Pitching Direction) is obtained as follows:

$$K_1 = [223.6068\ 36.5055 - 828.9929 - 42.7414]$$

In subsystem 2(Rolling Direction), the weighted matrices Q and R are assumed as

Q2=diag(200,200,200,200),R2=1 and K2 is obtained as

$$K_2 = [13.7343\ \ -14.1015\ \ \ 14.1421\ \ \ 14.4328]$$

Similarly in Subsytem 3 (Yawing Direction), the weighted matrices Q and R are assumed as Q3=diag(10,1) and R3=0.01 the gain K3 is obtained as

$$K_3 = [31.6228\quad 11.3966]$$

The body roll angle is assumed initially as α= 0.05 rad, the pitch angle β=0.1 rad, the yaw angle as ϒ=0.1 rad for the intial moment. There is a disturbance at the fifth second.

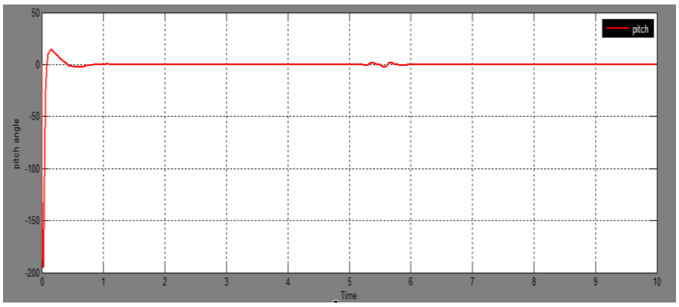

Fig 5 LQR control of pitch angle

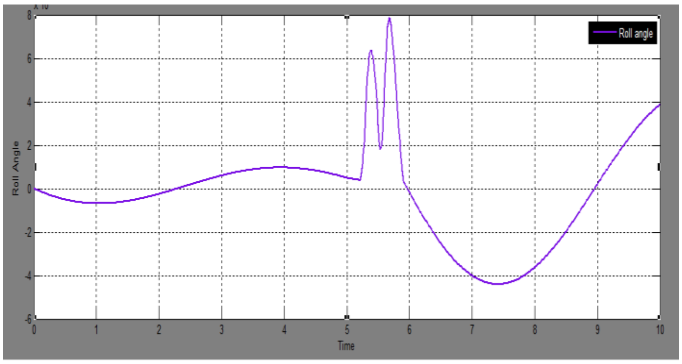

Fig :6. LQR control of roll angle

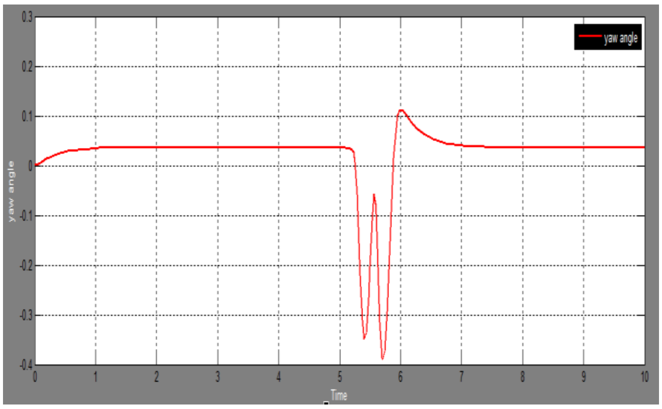

Fig 7: LQR control of yaw angle

Next the roll angle control is carried out using Sliding Mode control, using the gain value K_2 obtained using the roll angle control using LQR. Using Matlab program the sliding surface "S "can be calculated. The value of sliding surface is obtained as

$$S = [\ 0 \qquad 0 \quad -0.0170 \quad -0.0170]$$

Using this sliding surface the control law can be calculated which is fed back to the subsystem2 as input. A signal is build using signal generator with a disturbance on the 5sec and result is obtained in such a way that after the oscillations caused due to disturbance the system retrieved back to stability.

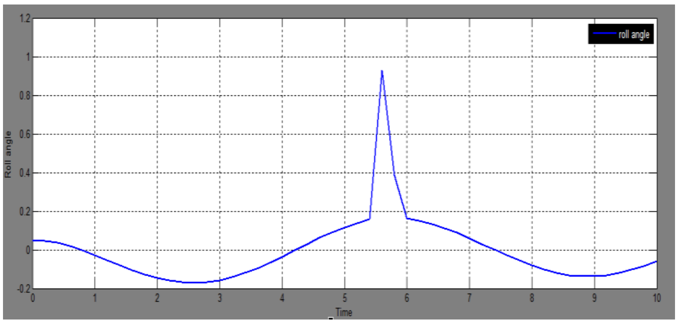

Fig 8: SMC control of roll angle

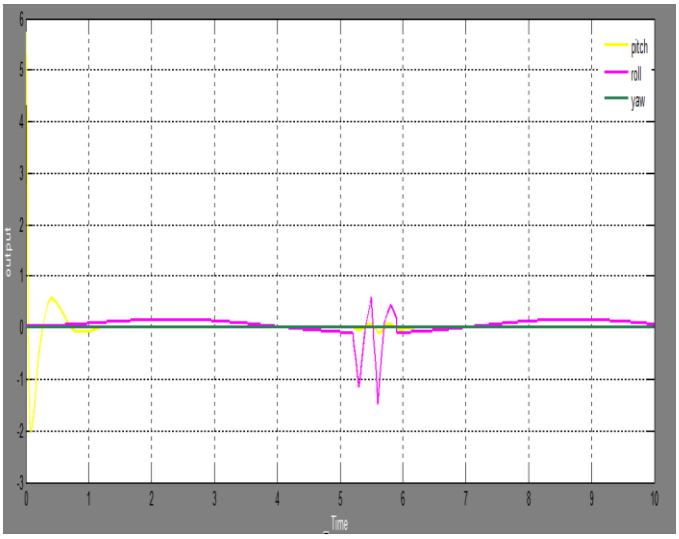

Fig 9:The angle curves for LQR(pitch &yaw) and SMC(roll) control

V. CONCLUSION

The study of nonlinear, unstable under actuated non holonomic system of unicycle robot is carried out. The dynamic modeling of the system using Lagrange Approach is studied and being used here. The robot angles are initially simulated and studied using Linear Quadratic Regulator . Then a Sliding Mode control was used to study the roll axis. The simulation studies were carried out to investigate the performance of the controller. The roll angle control using SMC shows that the error when compared to LQR control is considerably reduced .So for roll control the better option of control is using SMC. The system is assembled with pitch and yaw angles controlled using LQR controller and roll angle using SMC. The designed controllers are reasonable and reliable, and consistent with the physical significance. All the simulation results show that the controllers basically achieve expected results. This paper could lay a foundation for the motion control of the robot prototype.

REFERENCES

[1] S. H. Kim, J. O. Lee, J. Hwang, B. Ahn, and J. M. Lee, "Dynamic modeling and performance improvement of a unicycle robot," J.Inst.Control, Robot. Syst., vol. 16, no. 11, pp. 1174–1181, Nov. 2010.

[2] Seong I. Han, Member, IEEE, and Jang M. Lee, Senior Member, IEEE, Balancing and Velocity Control of a Unicycle Robot Based on the Dynamic Model 406" IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS," VOL. 62, NO. 1, JANUARY 2015

[3] S. Kim, J. Lee, J. Hwang, B. Ahn, and J. Lee, "Dynamic modeling and performance improvement of a unicycle robot," J. Inst. Contr. Robot. Syst., vol. 16, no. 11, pp. 1074–1081, 2010.

[4] I. Fantoni, R. Lozano, and M. Spong, "Energy based control of the Pendubot," IEEE Trans. Autom. Control, vol. 45, no. 4, pp. 725–729, Apr. 2000.

[5] J. Constantin, C. Nasr, and D. Hamad, "Control of robot manipulator and pendubot system using artificial neural networks," Robotica, vol. 23, no. 6, pp. 781–784, Nov. 2005.

[6] C. C. Tsai, C. K. Chan, S. C. Shih, and S. C. Lin, "Adaptive nonlinear control using RBFNN for an an electric unicycle," in Proc. IEEE Int. Conf. SMC, 2008, pp. 2343–2348.