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Selective Harmonic Elimination in Multilevel Inverter Using Real Coded Genetic Algorithm Initialized Newton Raphson Method

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Abstract – This paper presents a novel two-phase hybrid optimization algorithm called Real Coded Genetic Algorithm Initialized Newton Raphson (GAIN) method for solving the transcendental nonlinear equations characterizing harmonics in multilevel converters. The proposed hybrid Real GAIN method is developed in such a way that Real Coded Genetic Algorithm (RCGA) is the primary optimizer exploiting its global search capabilities by directing the search towards the optimal region, and Newton Raphson method is then employed as a local search method to fine tune the best solution provided by RCGA in each evolution. The proposed method is implemented for the offline computation of the optimum switching angles in an 11-level inverter so that the required fundamental voltage is produced while the low order harmonics specifically the 5th, 7th, 11th and 13th harmonics which are more harmful and more difficult to remove with filters are eliminated. Computational and MATLAB simulation results clearly demonstrate the effectiveness and high spectral performance of the proposed algorithm.

Keywords- Multilevel inverter, Selective Harmonics Elimination (SHE), Real Coded Genetic Algorithm (RCGA), Newton Raphson method

INTRODUCTION

Multilevel voltage source inverters have recently become very popular in medium and high power applications such as large motor drives and electric utilities due to their ability to meet the increasing demand of power ratings and high power quality. By synthesizing the desired ac output voltage of a multilevel inverter from several levels of dc input voltages, staircase waveforms are produced, which approach the sinusoidal waveform with low harmonic distortion. In comparison with the hard-switched two-level pulse width modulation (PWM) inverters, multilevel inverters have a lower dv/dt per switching, lower electro-magnetic interference (EMI), considerably reduced switching loss and higher efficiency [1, 2].

Due to their high spectral performance and ability to attain a higher voltage with low harmonics without the use of transformers, multilevel inverters have drawn tremendous interest in applications such as industrial motor drives, High Voltage Direct Current (HVDC) transmission, flexible AC transmission system (FACTs) and utility interface for renewable energy systems because several batteries, fuel cells, solar cells, or rectified wind turbines or microturbines can be connected through a multilevel inverter to feed a load or interconnect to the ac grid without voltage balancing problems [1, 2].

Basically, there are three multilevel converter topologies and they are as follows: Diode-Clamped Multilevel Converter (DCMC) which is based on the neutral-point-clamped (NPC) inverter topology introduced by Nabae, et al, in 1981 [3], Capacitor-Clamped Multilevel Converter (CCMC) also known as flying capacitor or multicell converter proposed by Meynard and Foch in 1992 [3], and Cascaded Multicell Converter (CMC) otherwise known as Cascaded H-bridge Multilevel Converter (CHBMLC) [5, 6]. However many varieties of each topology as well as hybrid of the fundamental topologies such as Generalized P2 Converter, Mixed-Level Hybrid Converter, Asymmetric Hybrid converter have been developed but with the same underlying principle [7-11].

Several modulation techniques and control paradigms have been developed for multilevel converters among which are Sinusoidal Pulse Width Modulation (SPWM), Selective Harmonic Elimination (SHE) method, Space Vector Control (SVC), and Space Vector Pulse Width Modulation (SVPWM) [1, 2,]. Selective Harmonics Elimination (SHE) method at fundamental switching frequency arguably gives the best spectral performance. The main challenge associated with the SHE method is how to obtain analytical solutions of the nonlinear transcendental equations that contain trigonometric terms [12].

The traditional methods used for solving this kind of optimization problems include derivative-dependent method like Newton Raphson method which is very fast and accurate but risks being trapped at a local optimum and diverges if the arbitrarily chosen initial values are not sufficiently close to the roots [13-16]. Evolutionary algorithms like Ant Colony Optimization (ACO) [17], Particle Swarm Optimization (PSO) [18] and the conventional Binary Coded Genetic Algorithm (BCGA) [19-21] are derivative free and are successful in locating the optimal solution with any arbitrarily chosen initial values, but they are usually slow in convergence and require much computing time. Also, they minimize rather than eliminate the unwanted low order harmonics Chiasson et al [22] proposed a method based on Elimination theory using resultants of polynomials to determine the solutions of the SHE equations. A difficulty with this approach is that as the number of levels increases, the order of the polynomials becomes very high, thereby making the computations of solutions of these polynomial equations very complex. Another approach uses Walsh functions [23-25] where solving linear equations, instead of non-linear transcendental equations, optimizes the switching angle. The method results in a set of algebraic matrix equations and the calculation of the optimal switching angles is a complex and time-consuming operation.

II. CASCADED H-BRIDGE MULTILEVEL INVERTERS

Cascaded H-bridge multilevel converters consist of a number of H-bridge power conditioning cells, each supplied by an isolated source on the DC side and series-connected on the AC side [1, 2, 6, 8, 16]. The structure of a single phase cascaded H-bridge multilevel converter is shown in Figure 1.

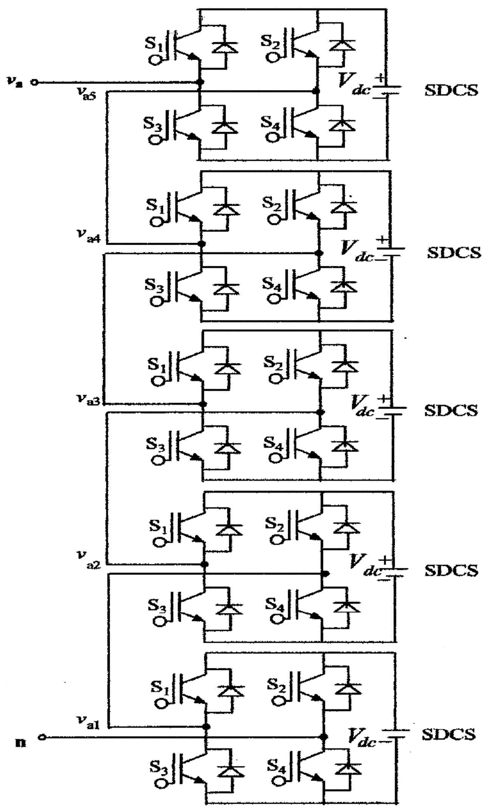

Fig. 1. Configuration of an 11-level single-phase cascaded H-bridge multilevel converter

The number of output phase voltage levels in a cascaded H-Bridge inverter is defined by $n = 2s + 1$, where s is the number H-bridges per phase connected in cascade. Each H-bridge switch can generate three different voltage levels: $+V_{dc}$, 0, and $-V_{dc}$ by connecting the DC source to the AC output by different combinations of the four switches S_1, S_2, S_3, and S_4 shown in the figure. To obtain $+V_{dc}$, switches S_1 and S_4 are turned on, whereas $-V_{dc}$ can be obtained by turning on switches S_2 and S_3. By turning on S_1 and S_2, or S_3 and S_4, the output voltage is zero. The outputs of H-bridge switches are connected in series such that the synthesized AC voltage waveform is the summation of all voltages from the cascaded H-bridge cells [22].

III. PROBLEM FORMULATION

The Fourier series expansion of the staircase output voltage waveform shown in Figure 2 is expressed in equation (1).

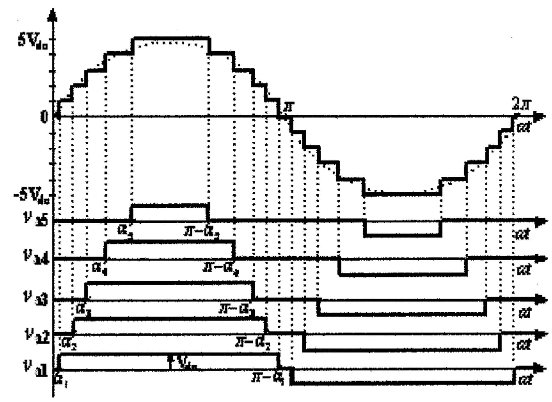

Fig. 2. Output voltage waveform of an 11-level cascaded H-bridge multilevel converter using fundamental frequency switching scheme.

$$V(\omega t) = H_n(\alpha)\sin(n\omega t) \tag{1}$$

Where

$$H_n(\alpha) = \frac{4V_{dc}}{n\pi}\sum_{k=1}^{s}\cos(n\alpha_k)\text{, for odd n} \tag{2}$$

$$H_n(\alpha) = 0\text{, for even n} \tag{3}$$

In three-phase power system, the triplen harmonics in each phase need not be cancelled as they automatically cancel in the line-to-line voltages as a result only non-triplen odd harmonics are present in the line-to-line voltages [22]

Combining equations 1, 2 and 3

$$v(\omega t) = \sum_{n=1,3,5\ldots}^{\infty}\frac{4V_{dc}}{n\pi}(\cos(n\alpha_1) + \cos(n\alpha_2) + \ldots + \cos(n\alpha_s))\sin n\omega t) \tag{4}$$

Subject to $0 < \alpha_1 < \alpha_2 < \ldots \alpha_s \le \pi/2$

Generally for *s* number of switching angles, one switching angle is used for the desired fundamental output voltage V_1 and the remaining (*s*-1) switching angles are used to eliminate certain low order harmonics that dominate the Total Harmonic Distortion (THD) such that equation (4) becomes

$$V(\omega t) = V_1\sin(\omega t) \tag{5}$$

From equation (4), the expression for the fundamental output voltage V_1 in terms of the switching angles is given by

$$V_1 = \frac{4V_{dc}}{\pi}\left(\cos(\alpha_1) + \cos(\alpha_2) + ... + \cos(\alpha_s)\right) \quad (6)$$

The relation between the fundamental voltage and the maximum obtainable fundamental voltage V_{1max} is given by modulation index. The modulation index, m_i, is defined as the ratio of the fundamental output voltage V_1 to the maximum obtainable fundamental voltage V_{1max}. The maximum fundamental voltage is obtained when all the switching angles are zero [16]. From equation (6),

$$V_{1\max} = \frac{4sV_{dc}}{\pi} \quad (7)$$

$$\therefore m_i = \frac{V_1}{V_{1\max}} = \frac{\pi V_1}{4sV_{dc}} \quad (0 < m_i \leq 1) \quad (8)$$

To develop an 11-level cascaded multilevel inverter, five SDCSs are required. The modulation index and switching angles that result in the synthesis of AC waveform with the least Total Harmonic Distortion (THD) can be found by solving the following nonlinear and transcendental equations characterizing the harmonics derived from equations (1), (2) and (4) [16, 22] :

$$\frac{4V_{dc}}{\pi}(\cos(\alpha_1) + \cos(\alpha_2) + ... + \cos(\alpha_5)) = V_1$$
$$\cos(5\alpha_1) + \cos(5\alpha_2) + ... + \cos(5\alpha_5) = 0$$
$$\cos(7\alpha_1) + \cos(7\alpha_2) + ... + \cos(7\alpha_5) = 0 \quad (9)$$
$$\cos(11\alpha_1) + \cos(11\alpha_2) + ... + \cos(11\alpha_5) = 0$$
$$\cos(13\alpha_1) + \cos(13\alpha_2) + ... + \cos(13\alpha_5) = 0$$

The correct solution must satisfy the condition

$$0 \leq \alpha_1 < \alpha_2 < ... < \alpha_5 \leq \pi/2 \quad (10)$$

Equation (8) in equation (9) yields:

$$\cos(\alpha_1) + \cos(\alpha_2) + ... + \cos(\alpha_5) = 5m_i$$
$$\cos(5\alpha_1) + \cos(5\alpha_2) + ... + \cos(5\alpha_5) = 0$$
$$\cos(7\alpha_1) + \cos(7\alpha_2) + ... + \cos(7\alpha_5) = 0 \quad (11)$$
$$\cos(11\alpha_1) + \cos(11\alpha_2) + ... + \cos(11\alpha_5) = 0$$
$$\cos(13\alpha_1) + \cos(13\alpha_2) + ... + \cos(13\alpha_5) = 0$$

Generally equation (11) can be written as

$$F(\alpha) = B(m_i) \quad (12)$$

The Total Harmonic Distortion (THD) is computed as shown in equation (13) :

$$THD = \frac{\sqrt{\sum_{i=5,7,11,13.....}^{49} V_i^2}}{V_1} \quad (13)$$

IV. REVIEW OF REAL CODED GENETIC ALGORITHM AND NEWTON RAPHSON METHOD

A. REAL CODED GENETIC ALGORITHM

The genetic algorithm proposed by Holland in 1975 is an evolutionary algorithm that was inspired by the study of genetics [26]. He proposed a Binary-Coded Genetic Algorithm (BCGA) modeled on Darwinian principles of survival of the fittest, with a random but structured exchange of information. A random population of individuals, or potential solutions to the problem called strings or chromosomes, is created, and in turn the parameters of these solutions are modified by the genetic operators (selection, crossover and mutation) to create new (and hopefully better) population of solutions. This process is repeated for a number of generations until the desired solution is obtained. Due to the inexact nature of genetic algorithm, its performance depends on the population size as well as the choice and values of the genetic operators used. Population size has to be chosen in such a way that there is balance between the execution time and accuracy, which means that an increase in the accuracy of a solution can only come at the expense of the convergent speed and vice versa.

For real valued numerical optimization problems, Real-Coded Genetic Algorithm (RCGA), whose chromosomes comprise real numbers outperforms binary-coded genetic algorithms. The obvious advantages of RCGA include global search capability, enhanced convergent speed resulting from a reduced computational effort (BCGA uses binary code, which needs a lot of time to code and decode the values). In this research work, with the population size set at 40, Real Coded Genetic Algorithm using floating-point representation together with the tournament selection, heuristic crossover at the rate of 0.8, dynamic or non-uniform mutation at the rate of 0.02 and generational replacement strategy is proposed. Each chromosome (potential solution) of the nonlinear and transcendental equations is encoded as a vector of floating-point valued or real numbers of the same length as the dimension of the search space. For each chromosome (potential solution), the fitness function is calculated as follows [21]:

$$f = \min_{\alpha_i} \left[\left(100 \frac{V_1^* - V_1}{V_1^*} \right)^4 + \sum_{s=2}^{5} \frac{1}{h_s} \left(50 \frac{V_{hs}}{V_1} \right)^2 \right] \quad (14)$$

$i = 1, 2 \ldots 5$

subject to $0 \le \alpha_1 < \alpha_2 \ldots < \alpha_5 \le \pi/2$

Where:

V_1^* = desired fundamental output voltage, S = number of switching angles = the number of DC sources = 5, h_s= order of the s^{th} viable harmonic at the output of a three phase multilevel converter. For example, $h_2 = 5$, $h_4 = 11$.

In this work, the GA for each state is run twice, because it may fall into local minima. The least fitness function between both runs is chosen. By increasing the number of runs, the probability of reaching the global minimum increases but the convergent speed decreases due to the increase in the execution time. Also, the default number of generations is 100 but sometimes, GA converges to a solution much before 100 generations are completed. In order to save time, generations are halted if the result remains unchanged for 50 generations.

B. NEWTON RAPHSON ITERATIVE METHOD

Newton-Raphson (NR) method is one of the fastest iterative methods. This method begins with an arbitrary initial approximation and generally converges at a zero of a given system of nonlinear equations [27]. However, in this work, the NR method is used to compute the switching angles for the system of SHE equations using the best solutions returned by RCGA as the initial approximation. The Switching angles producing the desired fundamental voltage along with elimination of 5^{th}, 7^{th}, 11^{th}, and 13^{th} harmonic components are computed for complete range of modulation. Different solution sets are obtained for the range of modulation index where they exist.

The hybrid Real Coded Genetic Algorithm Initialised Newton Raphson (Real GAIN) method was developed in such a way that Real-Coded Genetic Algorithm (RGA) with step size of 0.005 is the primary optimizer exploiting its global search capabilities by directing the search towards the optimal region, Newton-Raphson (NR) method with step size of 0.02 is then employed as a local search method to fine tune the best solution provided by RGA in each evolution as follows:

Step 1: Formulate the SHE problem.

Step 2: RCGA proceeds by randomly generating a population of potential solutions.

Step 3: i) Assesses the population fitness using the objective function.

ii) Ranking is carried out.

iii) Selection is employed to pick the best individuals as members.

iv) Creation of offsprings based on discrete recombination (crossover and mutation).

v) Elitism is employed and a new generation is created.

vi) Repeat steps (i) to (v) for sufficient number of iterations to attain the stopping criterion.

Step 4: The solution from step 3 is fine tuned with Newton-Raphson method as follows;

i) A solution set of $RCGA_{best}$ is used as initial values for the switching angles (i.e. $RGA_{best} = \alpha_{initial} = \alpha_0$)

ii) Set $m_i = 0$.

iii) Calculate $F(\alpha_0)$, $B(m_i)$, and Jacobian $J(\alpha_0)$

iv) Compute correction $\Delta\alpha$ during the iteration using relation,

$$\Delta\alpha = J^{-1}(\alpha_0)\left[B(m_i) - F(\alpha_0)\right]$$

v) Update the switching angles i.e.

$$\alpha(k+1) = \alpha(k) + \Delta\alpha(k)$$

vi) Perform transformation to bring switching angles in feasible range.

$$\alpha(k+1) = \cos^{-1}\left[abs[(\cos(\alpha(k+1))]\right]$$

vii) Repeat steps (iii) to (vi) for sufficient number of iterations to attain error goal.

viii) Increment m_i by a fixed step.

ix) Repeat steps (ii) to (viii) for the whole range of m_i

Step 5: Plot the switching angles as a function of m_i. Different solution sets would be obtained.

Step 6: Take one solution set at a time and compute the complete solution set for the range of m_i where it exits.

By following the above steps, all possible solutions when they exist, can be computed.

V. COMPUTATIONAL RESULTS

A personal computer (1.83GHz Intel dual core processor with 2.00 GB Random Access Memory and 286 GB Hard disk drive) running MATLAB R2009a on Windows 7 Ultimate edition was used to carry out the calculations. A plot of the switching angles for values of modulation indices ranging from 0 to 1.0 is shown in Figure 3. As the plot shows, solutions do not exist at the lower end [0, 0.3764] and upper end [0.915, 1] of the modulation indices, isolated solution sets are only found for modulation index in the intervals [0.3764, 0.3779] and [0.9149]. In the subinterval [0.4871, 0.5395], [0.58], and [0.6179, 0.6586] two sets of solution exist. For those values of modulation indices that multiple solution sets exist, the set with the least Total Harmonic Distortion (THD) is chosen.

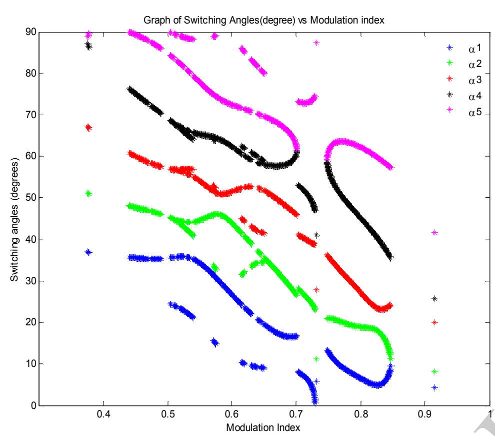

Fig.3. Optimal switching angles versus modulation index for 11-level CHBMLI

The plots of Total Harmonic Distortion (THD) computed out to the 13^{th} order labeled Low Order Harmonic Distortion (LOHD), and Total Harmonic Distortion (THD) computed out to the 49^{th} order versus modulation index are shown in Figure 4.

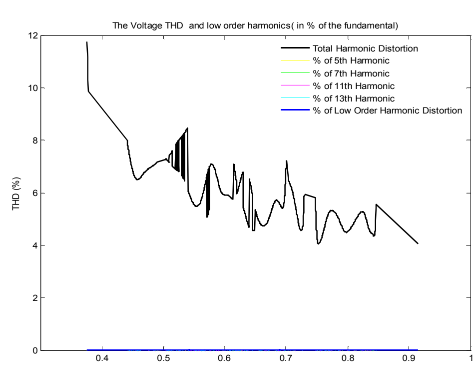

Fig. 4. Plot of THDs versus modulation index for 11-level CHBMLI

From the plots, it is observed that the low order harmonics are completely eliminated. Also, the best solution set found at modulation index of 0.9149 has Total Harmonic Distortion (THD) of 4.04% and corresponding Low Order Harmonic Distortion (LOHD) of 0.00%.

VI. SIMULATION RESULTS

In order to validate the observed analytical results, an 11-level single-phase Cascaded H-Bridge inverter was modelled in MATLAB-SIMULINK using SimPower System block set. In each of the five H-Bridges in the 11-level single-phase Cascaded H-Bridge inverter, 12V dc source is the SDCS, and the switching device used is Insulated Gate Bipolar Transistor (IGBT). Simulations were performed using the best solution set of the hybrid real GAIN algorithm calculated offline. The switching scheme adopted in this research work is the Fundamental frequency switching scheme because of its simplicity and low switching losses. The Fast Fourier Transform analysis of the simulated phase voltage waveforms was done using the FFT block to show the harmonic spectrum of the 11-level single-phase output AC voltage synthesized at the fundamental frequency (f = 50Hz) producing fundamental voltage of 69.91V (peak) at modulation index of 0.9149 which agrees closely with the analytical value of 69.89V (peak) calculated using equation (8).

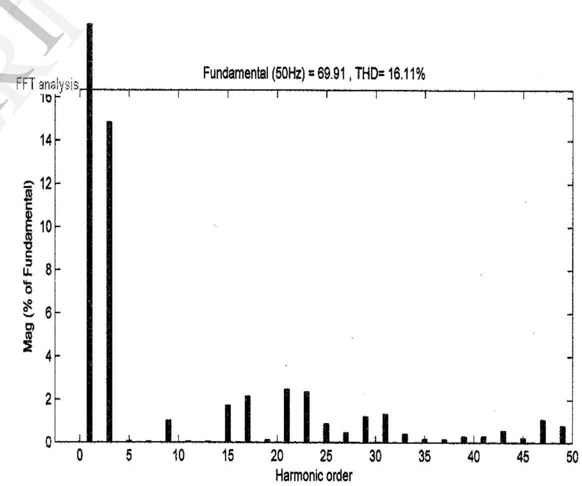

Fig.5. Harmonic spectrum for 11-level CHBMLI at modulation index, $m_i = 0.9149$

TABLE I

Analytical and Simulation values of THDs for $m_i = 0.9149$

Orders of THD	Analytical Results	Simulation Results
13^{th}	0.00%	0.13%
49^{th}	4.04%	4.06%

The analytical and simulation values of THD computed up to 13th order and 49th order are shown in Table I for comparison purpose. It can be seen from the Table I that the analytical and simulation values of THD are in close agreement thereby validating the analytical results. It should be noted that THD value of 16.11% is shown in Figure 6; the reason for this is that the THD shown is for phase voltages which include triplen harmonic components while analytical value is for line voltages which exclude triplen harmonic components.

VII. CONCLUSION

The selective harmonic elimination method at fundamental frequency switching scheme has been implemented for computing the switching angles that eliminate the low order harmonics in 11-level inverter using Real Coded Genetic Algorithm Initialized Newton Raphson (GAIN) method. The proposed algorithm combines the global search capability and improved convergence with random initial values exhibited by RCGA with speed and accuracy of NR while the initial condition problem of NR is avoided by using RCGA to provide multiple and good starting points for NR. Also, the hybrid algorithm mitigates the detrimental effects that poorly selected evolutionary parameters can have on the final results by using NR for final computation. It is shown here there are solutions in regions previously thought to be infeasible to find solution demonstrating the global search capability of the proposed method. Computational results are validated with MATLAB simulation, and both results satisfy the maximum THD limit of 5% specified by IEEE-519 standard which shows that the method is efficient for elimination of the undesired low order harmonics as well as minimization of THD.

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